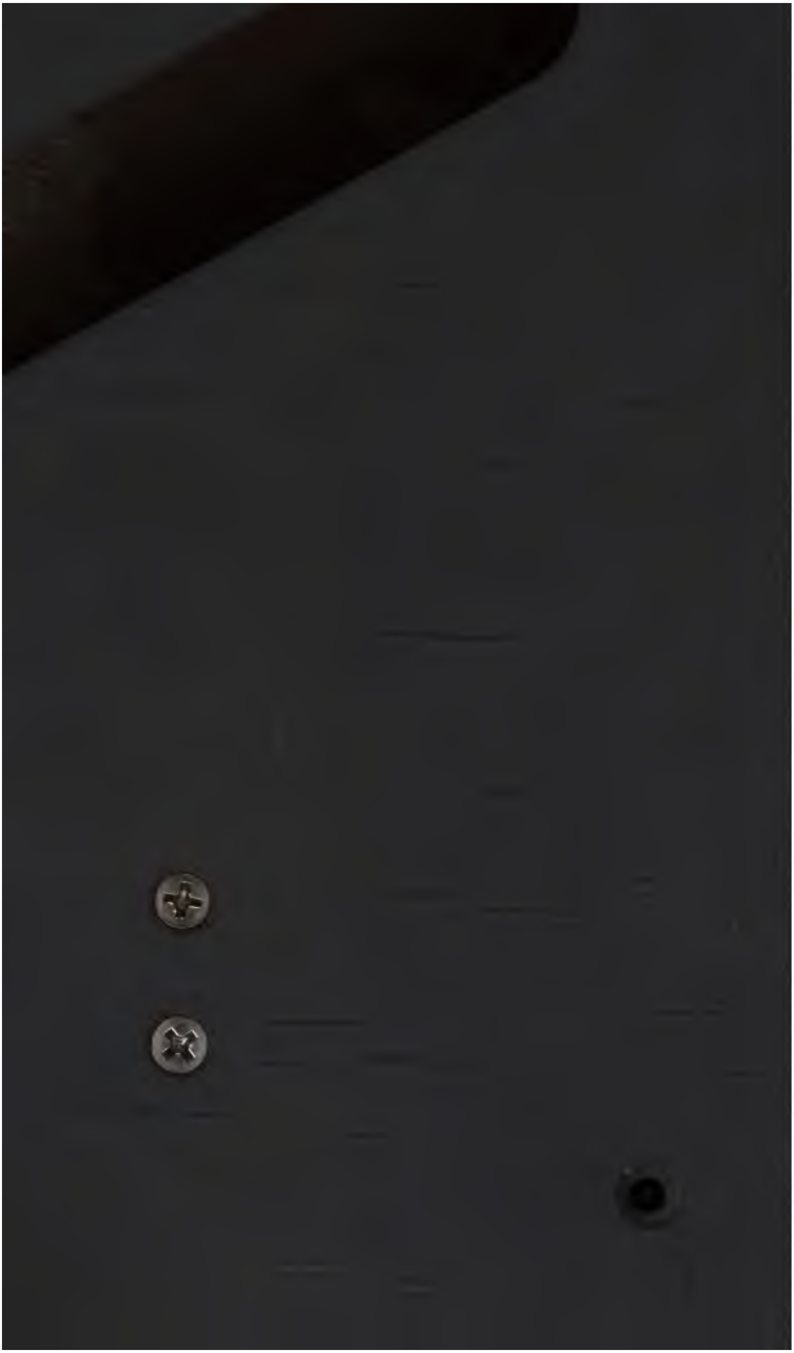

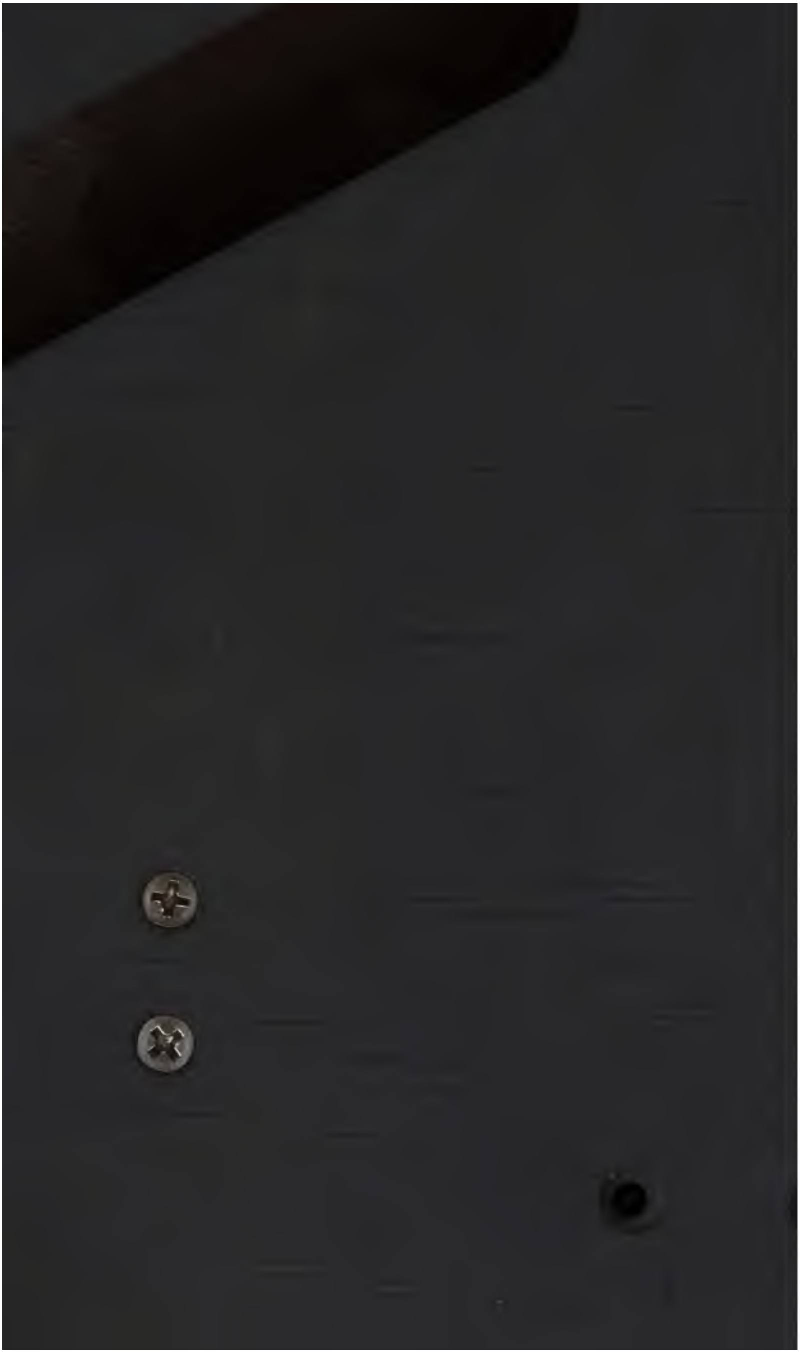

NUMBERS
APPLIED

APPLETONS' STANDARD ARITHMETICS

NUMBERS APPLIED

A COMPLETE ARITHMETIC

BY

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NEW YORK, BOSTON, AND CHICAGO
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PREFACE.

THIS work is not the result of any ambition on the part of the publishers to add another title to their already long list of text-books, but of a desire to meet a wide-spread and growing demand for a treatise on arithmetic adapted to the objective methods of instruction now so common in all educational institutions which have been reached directly or indirectly by the influence of normal schools, teachers' institutes, etc.

In its preparation the author has kept steadily in view these two thoughts: (1) That words are useless in the ratio that they fail to call up in the mind vivid images of the things signified. Hence the aim to vitalize the relation of words and things by the aid of the best practicable illustrations at every point; and (2) That, to the learner, the operations of arithmetic are apt to be but manipulations of figures after prescribed models, unless he realizes the fact that they are representative of processes that may be applied to material objects.

The book is intended to be put into the hands of the learner as soon as he has completed a course in primary arithmetic; but it would be well for him to begin the study of it with the first chapter, that he may get a better technical knowledge of the fundamental rules and their relations to each other, and that he may become rapid and reliable in computations involving integers before he takes up the more complicated subject of fractions.

Great care has been taken to adapt the work as far as possible to the needs of the great number of children who are withdrawn from school before a full course in arithmetic can be completed. With this object in view, the more useful business applications of elementary principles are made as soon as they are learned. Thus, familiar measures are introduced before reduction is mentioned; federal money before decimals; many practical measurements before mensuration; and questions even in percentage and interest are to be met with before those subjects are reached in due course. The conditions of these problems are so presented as to be within

the easy understanding of the pupil, while their solution requires only such arithmetical operations as he has already learned.

Attention is respectfully called

1. To the simple treatment of the decimal system of notation, and the great number of exercises intended to familiarize the pupil with the facilities for calculation which it affords.

2. To the multiplicity of short exercises that can be performed without the aid of the pencil, or that require but few figures in their solution. Longer ones are not wanting to test the perseverance of the pupil.

3. To the directions to the pupil, having in view the formation of right habits of computation. The "making up" method of subtraction, and the so-called "continental" method in division, though not obtrusively presented, are worthy of the attention of teachers. The latter furnishes an excellent mental exercise.

4. To the suggestions for original problems, now so commonly resorted to by the best teachers to stimulate the interest of their pupils, and to give them a better understanding of the subjects to which they relate. Their usefulness as short practical exercises in penmanship, spelling, and composition, will be appreciated by all.

5. To the simple and direct methods of treating the fundamental rules, common and decimal fractions, percentage, interest, proportion, square and cube roots, the problems of mensuration, etc.

6. To the rigorous adherence throughout the work to the inductive methods of instruction.

The number and variety of exercises and problems in this work are so great as to supersede any necessity for a supplementary book of exercises.

It is earnestly recommended that the pictured illustrations may be regarded as merely suggestive of the objective demonstrations which the student should be encouraged to get up for himself. As far as possible, let the learner furnish all the apparatus needed. While he is engaged in preparing it, the principles to be illustrated will present themselves to his mind more forcibly than in the repetition of definitions and rules in which he can take but slight interest till he appreciates their significance. If this course be taken, the pupil will, in most cases, be able to make out his own analyses. These may be crude at first, but they will be the better for being his own. Observation and experience will guide to better forms. Such a method will give him a mastery of the subject, develop mental power, and cultivate a taste for independent investigation.

New York City, May 15, 1886.

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APPENDIX.

APPLETONS' Standard Arithmetic.

CHAPTER I.

NOTATION AND NUMERATION.

The Writing and Reading of Numbers.

OBJECTS.	NUMBER.		OBJECTS.	NUMBER.	
●	*One*	1	●●●●●●	*Six*	6
●●	*Two*	2	●●●●●●●	*Seven*	7
●●●	*Three*	3	●●●●●●●●	*Eight*	8
●●●●	*Four*	4	●●●●●●●●●	*Nine*	9
●●●●●	*Five*	5	●●●●●●●●●●	*Ten*	..

1. The signs 1, 2, 3, 4, 5, 6, 7, 8, 9, are called the ***nine digits;*** because first used to represent a number of fingers.

The word *digit* is sometimes used for the word *finger*.

The following are the written forms of the digits:

1, 2, 3, 4, 5, 6, 7, 8, 9.

Tens and Units.

2. When we count more than nine we begin to count by tens and ones. After *nine* we say *ten,* then *eleven,* which means ten and one, *twelve* (ten and two), *thirteen* (ten and three), *fourteen* (ten and four), etc., to *nineteen* (ten and nine). Then we come to *twenty,* which means two tens, *twenty-one* (two tens and one), etc., after twenty-nine we have thirty, forty, fifty, etc.

Counting.—Count the balls of the numeral frame or other objects from ten to ninety-nine, as follows:

ten and one	**two tens and one**	**three tens and one**
ten and two	**two tens and two**	**three tens and two**
ten and three	**two tens and three**	**three tens and three**
etc., to	**etc., to**	**etc., to**
ten and nine	**two tens and nine**	**three tens and nine**
two tens	**three tens**	**four tens, etc.**

Writing.—We may write these numbers by using the digits 1, 2, 3, etc., instead of the words one, two, three; thus:

1 ten and 1	**2 tens and 1**	**3 tens and 1**
1 ten and 2	**2 tens and 2**	**3 tens and 2**
1 ten and 3	**2 tens and 3**	**3 tens and 3**
etc., to	**etc., to**	**etc., to**
1 ten and 9	**2 tens and 9**	**3 tens and 9**
2 tens	**3 tens**	**4 tens**

and so on to 9 *tens and* 9.

3. But the writing of numbers is still further shortened by omitting the words *ten and,* or *tens and;* thus, for *1 ten and 1* we write 11; for *1 ten and 2* we write 12; for *2 tens and 1* we write 21, and so on up to 99 (9 tens and 9). Thus we can tell whether a digit stands for *ones* or tens by the place it occupies. If it represents *ones,* it has the first place at the right; if *tens,* it occupies the next on the left.

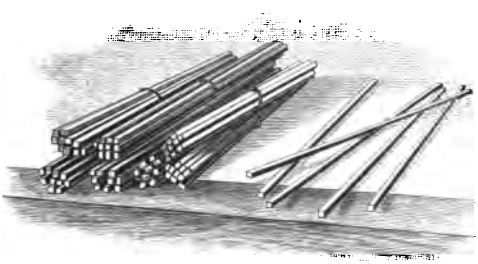
4. The right-hand place is called the ***units'*** or ***ones' place.*** The next place to the left of the units is the ***tens'*** place.

Note.—We use the word *unit* for the word *one*, and *units* for *ones*. See Art. 12.

If, as in 1 ten, 2 tens, etc., there are no *ones* or *units* to be expressed, we write the figure 0 in the *units'* place; thus, 10, 20.

5. The figure 0 does not express any number, but it is used to fill vacant places, as in the case above. It is called ***cipher*** or ***zero.*** It is a figure, but not a digit.

6. This is the ***decimal*** or ***tens' system*** of counting and of writing numbers.



Note.—For a simple illustration of this system, imagine a boy counting sticks. He counts ten and ties them together; then ten more, and so on till he has seven bundles and five sticks over. Then, writing 75, he shows that he has seventy-five sticks, the 7 being in *tens*, and the 5 in *units'* place.

7. Between twelve and twenty we call the *ten "teen,"* which means *"and ten"*; as, *fourteen,* that is *four* and *ten.* From nineteen to ninety-nine we call the *ten "ty,"* which means *"times ten"*; as, *sixty,* or *six times ten; seventy-five,* or *seven times ten and five units.*

EXERCISES IN WRITING AND READING NUMBERS.

Note.—Pupils should follow the forms of the digits given at the top of the preceding page, or other good copy. Let him here lay the foundation of neatness and accuracy in the writing and use of figures.

1. Write very neatly, in figures, the numbers from one to nine; from thirty to thirty-nine; forty to forty-nine; nineteen to ten; sixty to sixty-nine; ninety to ninety-nine; eighty-nine to seventy; fifty-nine to fifty; thirty-four to fifty-six.

2. Write in words: 75, 43, 51, 98, 29, 83, 11, 32, 64, 49, 17, 56, 19, 68, 31, 77, 99, 10, 24, 48.

Note.—The pupils may be required to show a number of jack-straws or of other objects equal to the numbers expressed. They should be arranged appropriately in tens and units.

3. How many tens are in twenty-eight? In sixty-two, fifty-six, ninety-five, eighty, seventy-one, forty-eight, sixty-five, thirty-three, fifty-one, etc.?

4. Write the following numbers in a column, and opposite each one express the same value in words: 92, 65, 38, 71, 40, 83, 14, 54, 17, 26, 90, 12, 83, 24, 75, 36.

5. The number 39 is expressed by two digits. What does the 9 stand for? The three? Which one, as it stands here, expresses the greater value? Why?

6. Answer the same questions in regard to 89, 48, 56, 35, 67, 22. Illustrate by objects.

Note.—In English the digit expressing the tens is generally read first; as, forty-eight; but in the German language they say: eight and forty. Sometimes we hear the same in English.

7. Read the following numbers in the German way: 27 (seven and twenty), 36, 58, 67, 89, 38, 45. If the places of these digits were exchanged, would the numbers thus expressed be larger or smaller? Why? Illustrate by objects.

8. What is the greatest number that can be expressed by two figures? What is the smallest?

9. Write in figures: Seventeen, thirteen, fifteen, twenty-eight, ninety-five, forty-two, eighty-three, thirty-four, sixty-nine, seventy-seven, eighteen, fifty-one, sixty-seven, forty-eight, ninety-five, eighty-eight, thirty, sixty.

10. Read the following. (May be copied, or written at dictation, and then read.) 10, 19, 13, 18, 12, 17, 11, 16, 14, 20, 23, 21, 25, 27, 29, 22, 24, 26, 28, 32, 36, 31, 35, 33, 38, 37, 49, 44, 48, 41, 47, 50, 56, 52, 63, 75, 84, 95, 58, 67, 78, 89, 91, 65, 85, 96, 72, 15, 30, 35, 39, 42, 55.

11. The pupil may copy the following numbers:

56	34	67	23	11	89	78	45	98	10	87	54	32	43	21
65	76	17	26	49	61	88	39	58	72	99	30	27	83	57

Note.—Care should be taken that each number be recognized as a whole, that the pupil may not copy figures merely. He should recognize *56* as *fifty-six*, and not as the figures 5 and 6.

12. Take in at a glance as many numbers as you can, and repeat them, looking off the book:

16	23	62	43	50	84	31	74	39	20
78	47	39	91	98	67	17	58	46	55

13. The teacher may dictate two or more numbers at a time from exercises 10 and 11.

14. Write the number of jack-straws represented in each group below. The bundles are of ten each.

Suggestion.—Let the pupils make original notation exercises similar to the above, arranging the objects in groups, and noting the number both in words and figures.

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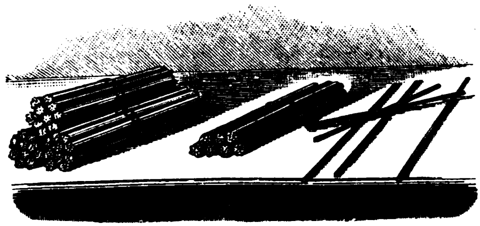
8. Hundreds.—If we count one more than ninety-nine we shall have nine tens and ten ones, or *ten tens.* Ten tens make *one hundred.* To express one hundred in figures we write *1* in the third place, thus, *100,* filling the places of tens and units with ciphers. The *1* now stands for *one hundred.* A digit in the third place from the right stands for *hundreds,* and hence we write:

100 (one hundred),	400 (four hundred),	700 (seven hundred),
200 (two hundred),	500 (five hundred),	800 (eight hundred),
300 (three hundred),	600 (six hundred),	900 (nine hundred).

If with the hundreds we have to write any number of tens, as three hundreds and seven tens, we place the digit representing

the tens in the tens' place ; thus, 370, read (3 hundreds 7 tens), three hundred seventy.

Again, if with the hundreds we have to write any number of ones, as three hundreds and five ones, we place the figure representing the ones in the ones' place ; thus, 305.



Three hundreds, seven tens and five ones here represented are written thus : 375, and read *three hundred seventy-five.*

We have learned, 1st, that *ten ones* make *one ten,* and *ten tens* make *one hundred ;* 2d, that in writing numbers the place on the right is the ***ones' place;*** the next, the ***tens' place;*** and the next, the ***hundreds' place.***

EXERCISES IN WRITING AND READING NUMBERS.

1. Express in figures : One hundred, six hundred, nine hundred, seven hundred, four hundred, two hundred, etc.

2. Also, one hundred thirty, six hundred twenty, five hundred eighty, three hundred fifty, two hundred seventy, etc.

3. Also, one hundred sixty-five, three hundred eighty-four, nine hundred seventy-one, four hundred thirty-three, etc.

4. How many hundreds in 481 ? How many tens ? How many ones ? How many of each in 385, 610, 974, 572, 137, 448 ?

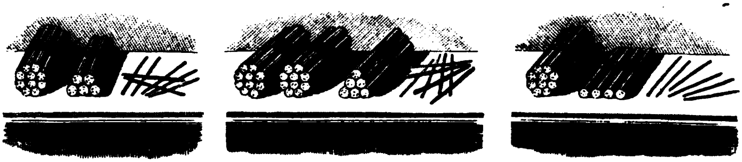
5. Write the following numbers in a column, and opposite each the same number in words : 218, 117, 916, 675, 854, 370, 523, 388, 446, 770, 978, 101, 340, 620, 304.

6. Read, taking in at one glance as many numbers as possible :

100	201	310	404	500	691	700	800	909
102	204	320	440	572	673	719	808	910

Note.—The foregoing numbers may be read in lines or columns, forward or backward, as the teacher may direct.

7. Write in figures the number of jack-straws represented in each of these groups.



8. Read 792. What does the 2 stand for? The 9? The 7?—Show what each figure stands for in the following numbers: 439, 562, 101, 760, 875, 460, 140, 104, 583, 61.

9. In a number of three places, which figure is read first? Which represents the highest order? How would you write three hundred nine, having no tens? How would you write 7 hundred twenty, having no ones? Will it do to leave the place of the ones or tens vacant? Why?

10. What is the largest number that can be represented by three figures? What is the smallest whole number?

11. Write in figures: Three hundred fifty, six hundred eighty, two hundred seventy, eight hundred fifteen, four hundred twenty-eight, nine hundred nine, one hundred ninety-six.

12. Copy the following, glancing at each number but once: (Think of the numbers represented, not merely of the figures to be written.)

107	400	212	560	309	653	356	365	635	536
801	118	180	870	357	429	560	608	742	897
215	419	711	999	233	100	677	822	301	405
103	205	340	409	583	655	728	846	979	893

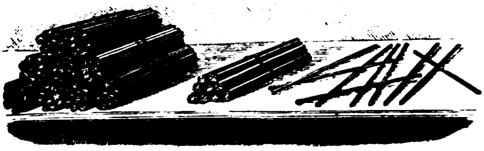
13. **Note.**—The teacher may also dictate two or more of the foregoing numbers at once, thus quickening the attention of the pupils.

14. Write in regular order the numbers from 150 to 199; from 260 to 299; from 307 to 328; from 480 to 499; from 585 to 602; from 687 to 706; from 791 to 809.

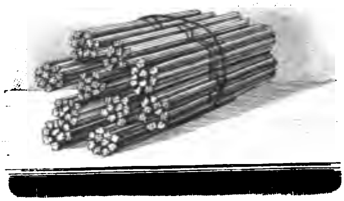
15. Write in words the numbers from 337 to 345; also from

883 to 890; from 555 to 563; from 98 to 104; from 872 to 883; from 190 to 205.

9. Thousands.—The greatest number we have written thus far is 999, or 9 hundreds, 9 tens, 9 ones. If we count one more, the nine ones at the right hand will become *one ten;* and putting this with the nine tens, we have *ten tens,* or *one hundred.*



Putting this one hundred with the nine hundreds, we have *ten hundreds;* and, as we made *one ten* out of ten ones, and *one hundred* out of ten tens, so we make *one thousand* out of ten hundreds. Thus, after adding one stick to the nine hundred and ninety-nine shown on the table above, the result would be as represented in this picture.



To express one thousand in figures, we write *1* in the fourth place; thus, *1000,* filling the places of hundreds, tens, and units with 0's. The *1* now stands for one *thousand.* A digit in the fourth place stands for thousands, hence we have 2000 (two thousand), etc. Hundreds, tens, and units, if any, fill their proper places.

EXERCISES IN READING AND WRITING NUMBERS.

1. Read 77, 15, 93, 106, 601, 810, 7080, 9107, 5006, 561, 3091.

2. *Write at dictation and read:*

5783	2100	9009	1706	5430	8071	2360	3902	1003	5701
6702	4000	3201	4300	5701	7010	8090	9100	1901	7707

3. Write ten such numbers as you please, and read them.

4. In the following numbers, how many units, tens, and hundreds are expressed by the figures in those orders ?

2375	1318	2380	3689	7401	8120	9000	7895
3624	5074	6138	8376	2380	8016	7980	1234

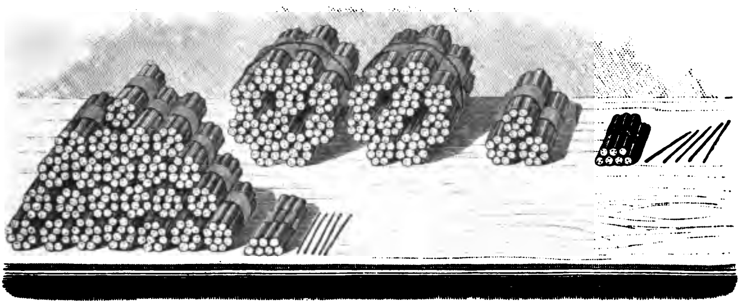
5. What is the smallest whole number that can be expressed by four figures ? What is the greatest ?

6. *Read :*

6789	9867	5432	4756	8912	3129	7891	4321
6000	6600	6660	6666	7654	6035	8765	8003
1098	2076	3054	4032	5010	7123	7009	5273
9002	9387	8793	1100	4002	7628	9347	6102

7. Read the foregoing columns downward and upward, and the lines from right to left and left to right. They may also be written at dictation and read.

8. Read the numbers of Exercise 4, reading the thousands and hundreds together as hundreds. Thus, 2375=twenty-three hundred seventy-five.



9. Copy the numbers of Exercise 6. No copying of single figures should be allowed ; the number should be recognized and written as a whole.

10. Which figure in a number of four places is read first ? Which represents the highest order ? Which the lowest ?

11. How many ciphers are needed in 4 thousand 17 ? Why ?

What difference is there between the written forms 468 and 4608? Between 375 and 3705?

12. Is there any difference between the numbers indicated by 46 in 468 and in 4608? Is the value of 8 in one number different from its value in the other? Why?

13. What number is expressed by the figure 9 in 7009? In 7900? In 7090?

Review Exercises.

1. Write in columns the figures which express the following numbers:

four	forty	four hundred	four thousand
one	ten	one hundred	one thousand
five	fifty	five hundred	five thousand
two	twenty	two hundred	two thousand
nine	ninety	nine hundred	nine thousand
six	sixty	six hundred	six thousand
three	thirty	three hundred	three thousand
eight	eighty	eight hundred	eight thousand

2. How can you make the digits in your first column express tens? (Answer: By annexing a cipher.) How hundreds? How can you make the digits in the second column express units? How hundreds? How can you make the digits in the third column express tens? How ones? (Answer: By erasing two ciphers.) Make the digits of the fourth column express hundreds; also tens. Will it change the value of the digits to place a cipher at their left?

3. *Express in figures:*

sixteen	one hun. seven	three thou. seven hun. eight
twenty-nine	three hun. eighteen	six thou. one hun. twelve
fifty-two	five hun. twenty-six	one thou. six hun. thirteen
thirty-six	seven hun. sixty-four	eight thou. two hun. twenty
seventy-eight	eight hun. fifty-six	three thou. four hun. fourteen
forty-five	nine hun. thirty-eight	four thou. nine hun. ninety
ninety-four	two hun. eighty-nine	two thou. six hun. ten
eighty-three	four hun. forty-five	five thou. three hun. thirty

4. Make 60, 40, 80, 10, 30, 50, 90, 70, 20 larger by 100; by 300; by 500; by 400.

5. Make 61, 42, 53, 74, 85, 26, 37, 18 larger by 400; by 200; by 700.

6. Would it alter the value of 8 in 81 if you were to place a cipher on the right of the 1? Answer similar questions in regard to the first figures in 29, 36, 45, 19, 58, 67, 71.

7. How many thousands, hundreds, tens, and units are expressed in each of the following numbers: 3624, 5781, 9010, 8107?

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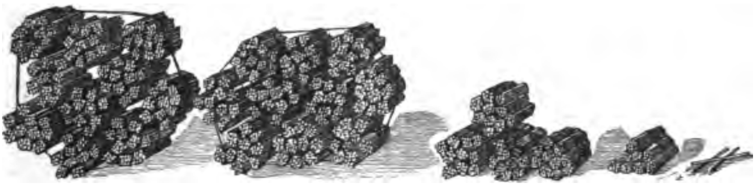
8. Express by figures the number of sticks represented in the 1st or units' group; also in

the 3d	the 4th and 3d	the 2d and 1st	the 3d, 2d, and 1st
the 2d	the 3d and 1st	the 4th and 1st	the 4th, 3d, and 1st
the 4th	the 3d and 2d	the 4th and 2d	the 4th, 2d, and 1st

What number is represented in all the groups together?



Note.—Pupils should prepare suitable objects for such illustrations. Bundles of tens, hundreds, etc., with single objects, should often be arranged promiscuously, and the learner be required to write the number in figures. Let him observe that we may estimate the value of the bundle by its size, but whether a digit represents tens or thousands depends on the place it occupies.

9. Write in columns, of ten each, all the numbers from one hundred to one hundred fifty-nine. Also, from two hundred fifty-one to three hundred. Also, from seven hundred eighty-three to eight hundred thirty-two. Also, from six hundred twenty-two to seven hundred one.

10. Write all the numbers from one thousand six hundred seventy-three to one thousand seven hundred two. Write in columns of ten numbers each.

10. After Thousands.—1. We have thus learned, *First,* the names of the orders to thousands; *Second,* that ten of any order make one of the next higher; and, *Third,* that the order of a figure—that is, whether it represents units, tens, hundreds, or thousands—is known by the place which it occupies, numbered from the right.

2. When we reach a thousand we begin to count the thousands as we did the units or ones: that is, we count 1 thousand, 2 thousand, up to 999 thousand, and when we have a thousand thousand we call the number one million. Millions we count in the same way: that is, 1 million, 2 millions, etc., up to 999 millions. When we reach a thousand millions, we call the number a billion. Billions we count in the same way, so trillions, quadrillions, etc.

3. In writing these numbers we might write the number of thousands as we do the numbers up to a thousand, and attach the word thousand to each number; thus, 8 thousand, 76 thousand, 999 thousand, etc., etc. But just as we avoid writing the words units, tens and hundreds, by giving to each order its place, so we avoid writing the word thousand by giving thousands the three places to the left of hundreds. In the same manner we give millions the three places to the left of thousands.

4. In this way it comes that, when more than three figures are employed to express any whole number, they are divided into groups, the first of which, numbering from the right, is used to denote any number from 1 to 999 units; the second, from 1 to 999 thousand; the third, from 1 to 999 million, etc. These groups are called ***Periods,*** and, for convenience in reading, are sometimes separated from each other by commas.

5. Thus, beginning at the right, we have the *first period,* consisting of ones, tens and hundreds *of units;* the *second period,* ones, tens and hundreds *of thousands;* the *third period,* ones, tens and hundreds *of millions.* The *fourth period* is that of *billions;* the *fifth, trillions;* the *sixth, quadrillions;* each period containing ones, tens, and hundreds of that period.

11. Numeration Table.

Hundreds of	Tens of	Ones of	Hundreds of	Tens of	Ones of	Hundreds of	Tens of	Ones of	Hundreds of	Tens of	Ones of	Hundreds of	Tens of	Ones of	Hundreds of	Tens of	Ones of
Quadrillions			**Trillions**			**Billions**			**Millions**			**Thousands**			**Ones**		
SIXTH			FIFTH			FOURTH			THIRD			SECOND			FIRST		
7	4	1	8	5	2	9	6	3	0	7	4	1	9	7	5	8	1
		7	2	9	9	9	7	7	8	0	2	8	1	4	3	5	6
			8	6	5	2	4	3	5	7	6	0	0	6	0	5	0
1	8	2	6	5	3	5	7	8	0	0	0	7	6	9	3	7	8
				3	1	8	4	0	9	9	9	0	0	0	6	4	2
	1	0	0	0	0	0	0	0	9	9	6	3	4	2	8	7	6
8	7	5	7	6	3	9	5	4	1	2	3	5	0	8	6	2	7

EXERCISES IN READING AND WRITING NUMBERS.

Read the foregoing numbers, consulting the headings, till you get accustomed to the names of the periods.

Read also as follows, or as may be directed by the teacher.

a. Read the tens' and ones' columns in the first period on the right.

b. Read all the numbers of the ones' period.

c. Read the right-hand column in the thousands' period.

d. Read the tens' and ones' columns in the thousands' period.

e. Read the two right-hand periods.

f. Read the right-hand column of the millions' period with the left of thousands, as hundred thousands.

g. Read the millions' and ones' periods, omitting the thousands' period as if filled with ciphers.

Note.—These exercises may be varied to almost any extent.

1. *Read these numbers:*

10,000	83,000	60,000	75,000	150,000	756,000
23,600	14,900	46,300	65,100	294,000	632,480
47,225	83,720	85,493	62,340	392,500	290,405
80,027	90,008	84,003	60,050	576,168	161,002

2. *Write and read:*

800,000	450,000	790,000	206,960	410,000
743,000	200,000	359,200	941,000	562,387
875,670	237,090	684,210	678,800	143,576
135,791	987,654	500,007	246,890	635,794

3. Write and read 2,000,000; 5,000,000; 7,000,000; 4,000,000. Fill the places occupied by ciphers with any digits you choose, and then read the numbers thus formed. Do this in various ways.

4. Copy the following numbers, and read them; then erase the digit at the right hand, and arrange the periods anew, by placing the commas where they should be, and read:

1,635,987	416,429,863	134,764,211	7,763,664	29,876,354

Continue this exercise by erasing the digits one by one, and pointing off the periods correctly.

5. How many tens, hundreds, thousands, ten-thousands, hundred-thousands and millions are expressed in those places respectively, in the numbers of Exercise 4?

Review Exercises.

Hundreds.—1. Write 300 and 20 and 7 as one number, expressed by three figures. In the same way write:

400 and 50 and 3; 100 and 70 and 6; 200 and 80 and 2;
300 and 60 and 6; 600 and 90 and 4; 900 and 10 and 1.

2. Write:

1	hundred	4	tens	6	ones;	2	hundreds	6	tens	3	ones;
1	"	8	"	7	"	4	"	7	"	5	"
1	"	9	"	2	"	8	"	0	"	0	"

3. *Read:*

527	723	168	365	134	524	340
729	792	843	290	209	902	299
313	901	910	109	953	646	728

Thousands.—4. Make the following numbers larger by one thousand: 328, 456, 508. By three thousand. By five thousand.

5. What number is next greater than 1599, 3019, 4091, 8400, 6379, 4599, 9999, 8765, 9109, 3099, 4098?

6. What number comes next before 2000, 7000, 4600, 5060, 3010, 2790, 8970, 1000, 1010, 7801 ?

7. Count and write from 996 to 1006; from 3189 to 3200; from 7990 to 8012 ; from 3001 back to 2989 ; etc.

8. How many hundreds and tens in 43 tens ? In 68, 37, 56, 27, 49, 168, 434 tens ? How many in 386 units ? In 468, 125, 632 units ? In 354, 538, 624 tens ?

9. How many thousands and hundreds are in 25 hundreds ? In 61, 52, 47, 56 hundreds ? How many in 250 tens ? In 310, 161, 289, 364, 543 tens ? How many in 6987 units ?

Tens and Hundreds of Thousands.—10. Prefix first twenty, then sixty, then forty thousand to 438, 132, 596, 100.

11. What number next greater than 25,999 ? 130,109 ? 199,999 ? 888,889 ? 986,290 ? 18,400 ? 689,999 ?

12. What number next less than 300,001 ? 700,000 ? 147,000 ? 354,989 ? 500,790 ? 100,000 ? 600,999 ? 489,123 ? 500,000 ?

Definitions.

12. A ***unit*** is one of any order or kind.

13. A ***number*** is a unit or collection of units.

14. ***Notation*** is the expression of number by *figures* or *letters*.

15. ***Numeration*** is the reading of numbers written in letters or figures.

16. All the digits have a ***Simple Value*** and a ***Local Value.*** A simple value, when they represent units or ones ; a local value, when used to express tens, hundreds, etc. This value is called local because it depends on *the place which the digit occupies* (its locality).

17. The nine digits are signs of number, hence they are called ***Significant Figures.*** In this sense, the cipher "0" is not a significant figure.

Roman Notation.

18. The following table gives a complete view of a method of representing numbers by letters. This is called the ***Roman*** method because first used by the Roman people.

Table.

Thousands.	Hundreds.	Tens.	Units.
M	C	X	I
MM	CC	XX	II
MMM	CCC	XXX	III
$\overline{\text{IV}}$	CD	XL	IV
$\overline{\text{V}}$	D	L	V
$\overline{\text{VI}}$	DC	LX	VI
$\overline{\text{VII}}$	DCC	LXX	VII
$\overline{\text{VIII}}$	DCCC	LXXX	VIII
$\overline{\text{IX}}$	CM	XC	IX

$\overline{\text{X}}$—10000

Note 1.—It will be noticed that in writing four and nine of each order, a letter of less value is placed before one of greater value. In this case the less value is deducted from the greater. Thus, XL (ten less than fifty) is written for XXXX. CD (one hundred less than five hundred) is written for CCCC, etc. This mode of abbreviation is common, not universal.

Note 2.—A bar over a letter, or combination of letters, increases its value a thousand times.

For writing numbers in Roman numerals, we have the following

19. ***Rule.*—Write the several terms in order as given in the table.**

EXERCISES IN THE ROMAN NOTATION.

1. Read XXIV, IX, XIX, XV, XIV, LX, XLIV, LXXXIX, XC, XCIX, CCI, CCCXCIX, CD, CDLVIII, CDLIX.

2. DV, CDXCIX, DXLVI, $\overline{\text{DCCCIX}}$, CMXCIX, MD, MIX, LIX, DIX, MDCCCLXXXIV, $\overline{\text{MCDLXX}}$.

---

3. Write in Roman numerals, 54, 72, 83, 59, 119, 72, 38, 49, 63, 98, 75, 69, 43, 91, 108, 319, 444, 333, 991, 3847, 2563, 3482.

CHAPTER II.

ADDITION.

Adding Ones and Tens.

Examples.—1. A hunter shot 6 rabbits on Monday, 7 on Tuesday, 8 on Wednesday, but only one on Thursday. How many rabbits did he shoot ?

2. Charles is 9 years old. How old will he be in 6 years ? In 5 years ? In 8 years ?

3. Fred had 8 dollars in his bank ; he received 7 more on his birthday, and 4 at Christmas. How much had he then ?

4. Grandfather was 53 years old when his grandchild was born. How old is he now that his grandchild is 9 years old ?

5. The sun rose at 6 o'clock this morning ; that was 3 hours ago. What o'clock is it now ? What o'clock 5 hours after sunrise ? 4 hours ? 6 hours ?

6. William read 7 pages in the morning, 3 in the afternoon, and 4 in the evening. How many pages did he read that day ?

7. Sarah goes up and down stairs 8 times in the morning, and 5 times in the afternoon. How many times in the day ?

---

8. Count to one hundred.—Count by twos to 100.—Count by threes to 99. Count by fours to 100.—Count by fives to 100.—Count by sixes to 96.—Count by sevens to 98.—Count by eights to 96.—Count by nines to 99.

9. How many units in 10 twos ? (Count by 2's till you find out.) How many in 10 threes ? In 10 fours ? In 10 fives ? etc.

10. Draw lines upon your slate, so as to divide it like a checker-board, but make ten squares instead of eight in each row, and as you count by 1's, write the results in the first line of squares from left to right; as you count by 2's, write the results in the second line of squares; as you count by 3's, write the results in the third line, and so on.

Definitions.

20. ***Addition*** in arithmetic is a process of finding the sum of two or more numbers.

21. Signs.—1. The sign + is read *plus*, and indicates addition; thus, 5 + 3 means 5 and 3 more.

2. The sign = is read *equals*, or *is equal to;* thus, 5 + 3 = 8 is read, 5 plus 3 equals or is equal to 8.

ORAL EXERCISES.

Write on slate or paper these two lines of figures.

4, 7, 2, 8, 8, 5, 6, 3, 9, 2, 6, 6, 8, 7, 9, 5, 3, 3, 4, 2, 5,
5, 3, 7, 7, 5, 5, 4, 6, 9, 9, 8, 4, 4, 2, 3, 8, 5, 4, 9, 6, 7.

11. To each number represented add 2, add 4, add 6, add 8.

Caution.—Do not say 4 and 2 are 6, but speak only the results, as *6*, *9*, etc. In *13* (below) give results directly, as *11*, *9*, *10*, *16*, etc.

12. Add 3, add 5, add 7, add 9, to each one.

13. Add the first to the second; add the second to the third, etc., beginning at the left—beginning at the right.

14. Add each number in the lower line to the one above it, proceeding first from left to right, and then from right to left.

---

15. 4+5+6=	**20.** 2+3+4+5=	**25.** 4+4+4+4=
16. 5+6+7=	**21.** 3+1+4+8=	**26.** 6+3+6+3=
17. 6+2+8=	**22.** 4+7+6+3=	**27.** 5+5+5+5=
18. 4+9+3=	**23.** 9+3+2+3=	**28.** 7+2+7+2=
19. 7+4+9=	**24.** 7+4+6+3=	**29.** 8+4+5+4=

30.	31.	32.
6+5+3=	5+4+8+2=	5+3+4+2=
8+4+2=	2+4+6+8=	9+7+1+2=
9+7+4=	3+5+7+4=	6+4+4+3=
5+8+3=	3+6+3+7=	9+4+4+3=
7+4+7=	8+1+9+2=	4+9+3+4=

33.	34.	35.	36.
10+6=	20+9=	90+2=	20+2+4=
30+5=	50+7=	80+4=	40+3+2=
40+4=	70+5=	70+6=	60+4+5=
50+3=	90+3=	60+3=	80+5+2=
60+2=	30+2=	50+5=	30+1+7=
80+7=	60+8=	40+9=	70+2+6=

37–117. Add 1, 2, 3, etc., up to 9, separately to each number in each line. Observe the units of the results.

1	11	21	31	41	51	61	71	81
2	12	22	32	42	52	62	72	82
3	13	23	33	43	53	63	73	83
4	14	24	34	44	54	64	74	84
5	15	25	35	45	55	65	75	85
6	16	26	36	46	56	66	76	86
7	17	27	37	47	57	67	77	87
8	18	28	38	48	58	68	78	88
9	19	29	39	49	59	69	79	89

This may be done orally, or on the slate, thus:

(37.)	(117.)
1+1=	9+9=
11+1=	19+9=
21+1=	29+9=
31+1=	39+9=
41+1=	49+9=
51+1=	59+9=
etc.	etc.

118.	119.	120.	121.
32+9=	33+9=	87+6=	56+8=
43+8=	44+8=	78+5=	65+6=
54+7=	55+7=	45+8=	29+9=
65+6=	66+6=	54+7=	92+5=
76+5=	77+5=	65+9=	87+7=
87+4=	88+4=	56+4=	78+6=
91+8=	22+7=	37+8=	47+6=

122–127. Add 6 to 21, 18, 36, 48, 54, 63, 17, 82, 88. Add also 8; 4; 5; 7; 9.

128–131. Increase the numbers 14, 19, 23, 25, 48, 84, 56, 37, 64, 83, 52, 38, 90, 87, 75, 61, 47, 79, 39, 59, 27, 69, 89, by 4; by 6; by 8; by 10.

133–135. Increase each of the numbers 23, 35, 48, 64, 56, 37, 41, 90, 82, 52, 61, 73, 84, 59, 47, 36, by 3; by 5; by 7; by 9.

Direction.—The foregoing exercises should be so thoroughly practiced, both orally and in writing, that the pupil can announce the sum of any two numbers expressed by single digits as readily as he can read them.—If he has to count his fingers in addition, he can proceed but slowly. He might as well spell every word as he reads, or crawl on his hands and knees instead of walking. Again, if he says "9 and 7 are 16," he uses five words where one, "sixteen," would be better.

Applications.—136. An hour has 60 minutes, and a half-hour has 30. How many minutes are there in one hour and a half?

137. There were hanging on a Christmas-tree 10 oranges, 20 apples, 30 nuts, 20 sugar-plums. How many gifts in all?

138. There are at work in a factory 40 men on the ground-floor, 30 on the second floor, 20 on the third floor, and 7 in the office. How many men are at work in the factory?

139. Grandmamma is 60 years old, mamma 30, and I am 7 years old. What is the sum of our ages?

140. An overcoat costs 30 dollars, a coat 20 dollars, a vest 4, and a pair of trousers 8 dollars. How much does the whole suit cost?

141. A fisherman caught in his net 36 pike, 30 bass, and 10 trout. Can you tell how many fish he caught?

142. A butcher bought two calves; one weighed 53 pounds, the other 47. How much did they weigh together?

ORAL EXERCISES.

Direction.—In adding, do not say (see 1st example) 4 *and* 5 *are* 9 *and* 6 *are* 15, etc., but give results at once; thus, *4*, *9*, *15*, *23*, etc.

143–168. *Add by columns and lines.*

4+5+6+8+9+4+3+7=	9+6+8+2+7+5+9+3=
3+2+1+9+7+6+2+8=	6+7+4+9+3+1+7+5=
8+4+3+9+5+7+8+6=	8+5+3+6+2+8+4+7=
6+6+8+3+7+4+4+5=	3+5+4+9+7+2+8+2=
7+8+2+7+6+9+6+5=	3+4+2+5+3+1+6+2=

169.	170.	171.	172.	173.
10+40=	90+10=	20+70=	40+15=	56+30=
20+50=	80+10=	30+60=	30+26=	44+20=
30+40=	60+30=	40+50=	60+38=	38+60=
40+50=	70+20=	50+30=	50+47=	29+50=
50+30=	50+40=	60+15=	70+28=	14+80=
60+40=	40+40=	70+26=	40+48=	63+30=
70+20=	80+20=	80+17=	60+37=	74+20=
90+30=	20+60=	20+45=	70+46=	88+40=
50+50=	30+30=	40+41=	80+11=	30+53=

174. 30+20+10+30=

175. 20+10+30+20=

176. 10+30+20+30=

177. 40+20+30+10=

178. 30+15+20+15+10+3+ 7=

179. 10+25+30+10+20+2+ 2=

180. 20+25+10+15+10+3+14=

181. 15+25+30+10+12+4+ 4=

Suggestions.—If, from this point to the rule on page 28, the examples seem too difficult, they may be omitted, to be taken up under the rule, but let the oral work be carried as far as possible. The learner who is left to himself to work out all his exercises on the slate is apt to form habits fatal to accuracy.

Applications.—182. There are 25 girls and 23 boys in a schoolroom. How many pupils in all ?

183. On one side of Blair Street there are 34 houses, on the other side, 59. How many on both sides ?

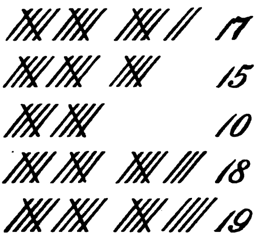
184. Mr. H. bought a horse for 73 dollars ; he sold it and gained 19 dollars. For how much did he sell it ?

185. Our house has 17 windows, that of one neighbor has 26, and that of another neighbor has 13. How many windows are there in the three houses ?

Exercises.—186–206. Add rapidly, by columns and lines, giving results only :

4+6+4+8+7+4+9+4+3+5+2+3+1+8+9=
8+5+3+9+6+8+5+3+6+7+9+7+6+9+1=
4+9+5+6+9+7+6+2+1+8+3+4+2+3+9=
7+6+5+4+3+2+1+9+7+5+6+4+9+4+8=
4+3+2+1+2+3+4+5+6+7+8+9+7+6+7=
8+4+6+9+1+7+3+5+7+9+6+8+7+5+6=

Note.—Let the pupil illustrate examples 207 to 226 by the use of buttons, acorns, or other objects which he can tie into bundles or string together in collections of ten; or let him make marks upon the slate such as these at the right, designed to illustrate example 207.



Thus, understanding well the nature of the thing to be done, he will need no rule for the simple operations here required. Let him first add the tens, and to the sum let him add the numbers in units' place.

207. 17+15+10+18+19=
208. 14+16+18+20+13=
209. 12+13+20+14+16=
210. 20+19+11+18+13=
211. 18+15+14+17+16=
212. 12+14+16+13+15=
213. 15+17+19+16+14=
214. 19+12+13+17+16=
215. 17+18+14+17+18=
216. 20+19+19+12+13=
217. 12+15+ 9+10+13=
218. 18+10+17+ 9+18=
219. 19+17+15+13+11=
220. 13+20+ 7+16+12=
221. 23+17+18+ 2+15=
222. 8+12+13+19+21=
223. 11+17+28+ 6+18=
224. 27+19+ 8+20+16=
225. 12+24+18+ 6+17=
226. 20+13+16+10+25=

Suggestion.—Exercises in numeration should precede the following examples.

Applications.—227. There were at a party 50 gentlemen, 60 ladies, and 70 children. How many people were there ?

228. A farmer raised 80 bushels of wheat, 39 bushels of oats, and 10 bushels of barley. How many bushels in all ?

229. There are in an orchard 63 plum-trees, 75 apple-trees, and 11 peach-trees. How many trees in all ?

230. A book-case has on the first shelf 48 books, on the second 57, and on the third 75. How many on the 3 shelves ?

231. There are 68 boys in one room of a school-house, 73 girls in another, and 87 girls and boys in a third. How many pupils are there in the school ?

232. The first book of Moses has 50 chapters, the second 40, the third 27, the fourth 36, and the fifth 34. How many in the 5 books ?

Add without the use of the slate:

233.	50+60=	**234.**	40+60+30=	**235.**	80+38=	**236.**	64+50=
	60+80=		50+30+50=		70+59=		76+60=
	70+40=		60+90+70=		90+67=		85+90=
	80+30=		70+40+90=		50+74=		59+70=
	90+50=		80+70+40=		60+83=		68+80=
	40+90=		90+50+60=		70+92=		75+40=

Note.—The examples in 235 and 236 require only one oral step, that is, the direct announcement of the result; as, for instance, in adding 80 and 38, think 80 and 30 (=110) and 8, but *say* at once ***118.*** In examples 237 to 241, two steps are enough; thus, in adding 59 and 32, first think 59 and 30, and say ***89,*** then 89 and 2, and say ***91.*** In examples 242 to 244, four steps may be necessary for the learner; thus, in adding 25, 38, and 49, say ***55, 63, 103, 112.***

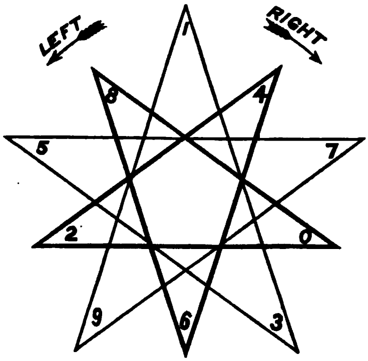
237.	**238.**	**239.**	**240.**	**241.**
59+32=	36+42=	25+22=	57+28=	26+34=
44+27=	47+33=	24+35=	28+39=	24+16=
36+24=	38+24=	33+19=	45+37=	74+24=
47+25=	58+25=	42+35=	36+38=	25+33=
88+26=	64+26=	53+43=	25+47=	47+29=
45+37=	47+27=	65+26=	34+39=	58+33=
62+25=	29+34=	35+36=	27+48=	34+28=

242.	**243.**	**244.**
65+36+3=	85+47+5=	25+38+49=
85+74+5=	83+68+6=	97+23+65=
67+63+7=	94+42+9=	56+44+68=
58+54+6=	86+29+3=	83+28+25=
73+72+9=	92+56+5=	62+37+27=
68+45+4=	75+65+6=	56+48+39=

245–259. *Add by columns and lines:*

40+60+30+70+80+90+60+80+80+30
50+80+90+80+40+50+90+70+20+50
60+40+70+90+20+70+70+60+70+30
70+90+20+60+30+30+40+50+90+70
80+70+60+50+40+30+20+40+30+50

Suggestions for Blackboard Exercises.



22. In drill exercises, the double star affords some advantages over the circle, and at the same time facilitates the learning of the several series arising from successive additions of 2's, 3's, 4's, etc.

Direction. — Beginning at the unit figure of any given number, the unit figures of the successive sums will be found as follows:

1. In adding 3's, at the *next* point to the right, and so on; in adding 7's, at the *next* to the left.
2. In adding 6's, at the *second* point to the right, and thus on, from point to point, of the same star. In adding 4's, at the *second* point to the left, and so on.
3. In adding 9's, at the *third* point to the right.
4. In adding 2's, at the fourth point to the right, and thus on (following the line at the right of the last unit figure). In adding 8's, at the fourth point to the left (following the line at the left of the last unit figure).
5. In adding 5's, at the point directly opposite the unit figure of the given number, and thus to and fro.

23. Other Uses of the Figure.—A suitable number being written at the center, the numbers at the points can be combined with it, in addition, subtraction, multiplication, or division, as may be desired. The number at the center being changed from time to time, there is no end to the variety of exercises that may thus be had at little expense of time or labor on the part of the teacher. Exercises in common and decimal fractions may be given in the same way.

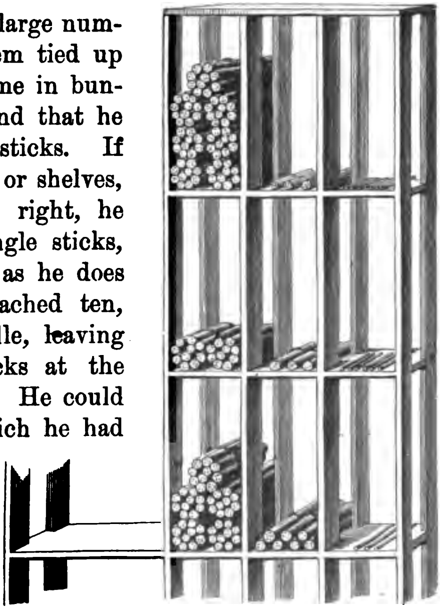
Addition of Higher Orders.

24. Note.—The illustrations of this book are not intended to be merely observed and read about, but they are designed to picture to the eye, as far as possible, the actual work which it is intended shall be done by the pupils with objects. These objects should be supplied by the school authorities, or, with slight suggestions by the teacher as to what is best or most available, according to the circumstances of the school, they may be brought in by the pupils. They should be as large as possible, and yet not inconvenient to handle in great numbers.

SLATE WORK.

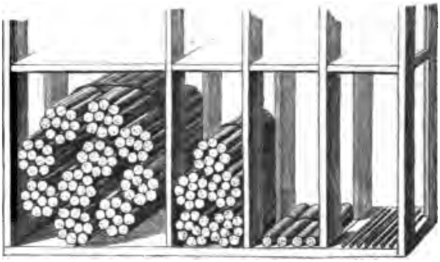
Example.—Find the sum of 738, 236 and 573.

One who knows nothing more of arithmetic than how to count to ten might find the sum of these numbers by some such means as the following:



Suppose that he has a large number of sticks, some of them tied up in bundles of ten, and some in bundles of a hundred each, and that he has, besides, some single sticks. If these were placed in rows or shelves, as in the picture at the right, he might count first the single sticks, taking them in his hand as he does so, and when he has reached ten, tie them in a small bundle, leaving the remaining single sticks at the right on the shelf below. He could then count the bundle which he had just made, with the tens' bundles on the shelves, and tie each ten of these bundles into larger bundles of a hundred each, and leaving the odd bundles of tens on the shelf below where they formerly were, he could

count all the bundles of hundreds together. And, again, tying ten of these larger bundles into one, he would have the sticks arranged as here represented; that is, one bundle containing a thousand, five of a hundred each, four of ten each, and seven single sticks.



25. The foregoing method is the same as that which is indicated in the following arithmetical process:

Thous.	Hund.	Tens.	Units.
	7	3	8
	2	3	6
	5	7	3
(1)	4 + (1)	3 + (1)	7

or

1	4+1	3+1	7
1	5	4	7

$\underbrace{1 \quad 5 \quad 4 \quad 7}_{1,547}$

or;

Thous.	Hund.	Tens.	Units
	7	3	8
	2	3	6
	5	7	3
1	5	4	7

$\underbrace{1 \quad 5 \quad 4 \quad 7}_{1,547}$

Having seen how it is, that this process really produces a number equal to the sum of the numbers added, the pupil is prepared for the rule for addition.

26. ***Rule.*—1. Arrange the numbers to be added so that the figures of the same order shall stand in the same column, units under units, tens under tens, and so on.**

2. Begin at the lowest order, and add each column separately. If the sum of any column is less than 10, write it underneath. If it is equal to or greater than 10, place the right-hand figure of the sum under the column added, and unite the left-hand term or terms with the next column.

***Proof.*—In order to be quite sure that the addition is correct, add each column both upward and downward. If the two results are the same, there is little danger of error.**

SLATE EXERCISES.

Examples 1–11. *Find the sum of*

27	37	29	68	68	92	53	27	57	91	84
58	28	36	43	79	72	67	72	46	24	37
90	64	39	29	25	68	85	28	90	56	62
56	47	76	46	97	73	39	57	51	45	78

12–22. *Find the sum of*

48	83	69	21	43	96	81	49	36	85	20
47	59	56	53	28	45	47	57	17	24	38
56	62	67	38	57	51	63	42	58	66	47
29	48	58	73	46	23	86	56	32	77	93

23–77. *Add 345 to each.*

639	542	894	457	837	910	735	628	246	802	135
429	900	312	163	524	254	736	547	749	630	757
372	713	457	298	337	698	507	192	293	394	495
345	298	110	206	387	471	289	509	135	398	211
213	361	425	357	862	135	779	387	600	731	877

78–88. Add together the numbers in each column.

89–93. Arrange each line of numbers in column and add.

94.	**95.**	**96.**	**97.**	**98.**	**99.**	**100.**
Apples.	**Nuts.**	**Oranges.**	**Peaches.**	**Lemons.**	**Plums.**	**Books.**
136	268	204	268	301	718	593
241	194	237	473	275	629	868
217	187	168	118	478	446	687
153	253	352	323	262	537	774
302	145	249	248	164	855	956

Add the following numbers, first arranging them in columns:

101. 125, 126, 138, 139, 140.

102. 87, 9, 55, 394, 225, 194.

103. 83, 194, 56, 168, 473.

104. 336, 195, 987, 9, 11.

105–114. *Add by columns. Also by lines.*

2123 + 2364 + 7025 + 428 + 20 + 2103 =
6354 + 2559 + 843 + 1125 + 359 + 23 =
698 + 1994 + 1427 + 2496 + 2478 + 437 =
1927 + 49 + 7917 + 6579 + 1000 + 3706 =

115–124. 3254 + 4015 + 7848 + 1570 + 439 + 7986 =
968 + 916 + 3407 + 4630 + 1690 + 375 =
725 + 1207 + 197 + 1820 + 420 + 9 =
4302 + 885 + 8329 + 7 + 7756 + 8975 =

The following may be solved first without the use of the slate. Only results should be pronounced. (In the last line, Ex. 127, for instance, say ***892, 952, 956.***)

125. 300 + 600 =	**126.** 600 + 70 + 38 =	**127.** 300 + 56 + 84 =
400 + 900 =	700 + 50 + 49 =	400 + 72 + 65 =
500 + 100 =	800 + 80 + 76 =	500 + 83 + 49 =
600 + 400 =	900 + 90 + 47 =	600 + 64 + 57 =
700 + 700 =	300 + 60 + 83 =	700 + 53 + 88 =
800 + 200 =	200 + 80 + 72 =	800 + 92 + 64 =

128–148. *Add by columns and by lines.*

568 + 487 + 2000 + 5872 =	4769 + 634 + 2465 =
435 + 675 + 6060 + 9321 =	1250 + 4 + 1975 =
357 + 894 + 7009 + 7234 =	3456 + 27 + 888 =
479 + 383 + 5800 + 5321 =	5861 + 5 + 935 =
692 + 596 + 3750 + 6947 =	9642 + 95 + 576 =
824 + 776 + 4680 + 3579 =	8347 + 7 + 6491 =
546 + 484 + 3541 + 2468 =	4936 + 183 + 587 =

Find the sum of

149.	**150.**	**151.**	**152.**
345,271	4,391,002	4,622,715	5,324,681
65,382	686,975	9,874,963	1,964,735
7,491	68	8,472,465	28,497
83,257	4,937	8,010,706	68
496,350	53,286	4,506,080	5,834
1,849	487,659	432,741	4,793
65,472	7,321,445	98,653	56,689

153. Add 768, 5,643, 12,354, 678,901, 5,847, 2,146,353, 975,321, 64,387,510.

154. Add nineteen, ninety, seventy thousand four hundred eight, 87, 1,625,847, 269,751, 3,894, twelve hundred sixty-one, 5,050,050, six hundred thousand six.

155. Add 198,725, 918,273, 1,928,370, 4,354,651, 34,234,534, 6,712,893, 647, 19, 1,345, 67,351.

156. Add 283,857, two thousand twenty, 998,722, five millions fifty thousand fifty, eight hundred thousand seven hundred twelve, 27, 192,875, 909,090, six hundred eight thousand four hundred ten, 34,827, fourteen hundred fourteen.

157.	**158.**	**159.**	**160.**
1,783	4,328	42,235	9,999,999
19,456	369,300	10,305,236	8,000,000
5,788	53,528	84,165,352	4,730,876
94,374	51,279	1,236,536	12,359,776
100,855	13,975	163,021	76,305,864
456,788	34,975	29,363,987	32,467,209
872,543	124,900	37,903,210	57,264,902
321,354	1,243,651	16,988,710	98,537,873

Direction.—In adding these columns, do not say (see Ex. 157) 4 and 3 are 7, and 8 are 15, and 5 are 20, and 4 are 24, etc., but simply speak results; thus: *4, 7, 15, 20, 24, 32, 38, 41.* The repetition of the numbers to be added increases liability to error.

Some can learn to add mentally numbers of even three places. Treating the tens and hundreds as units and tens (see note, p. 25), they would say, in Ex. 161, *27, 57, 65* tens = *650, 655, 661,* and set down the answer at once.

27. Adding two or more columns of figures at once is valuable practice in "mental arithmetic." It should be carried as far as time and the ability of the pupil will permit.

161–169.								
275	369	876	629	483	987	797	519	357
386	625	529	875	759	654	686	982	678

Examples for Practice and Review.

Applications.—**1.** I gave 83 marbles to Lewis, 34 to William, and 97 to Charles. How many did I give away?

2. In one book there are 89 pages, in another 246, and in a third 387. How many pages in all?

3. A certain tract of land was divided into four farms, one containing 113 acres, another 237, a third 180, and the fourth 320 acres. How many acres did the original tract contain?

4. There was a large number of cents in a bag. I took out of it first 289 cents, then 397, then 478, then 693, and then I found 134 cents left in the bag. How many cents did it contain at first?

5. Our school-house contains 6 rooms. In room No. 1 there are 25 boys and 31 girls; in No. 2 there are 18 boys and 29 girls; in No. 3, 21 boys and 37 girls; in No. 4, 29 boys and 19 girls; in No. 5, 45 boys; in No. 6, 53 girls. How many boys in our school? How many girls? How many children in all?

6. In a certain township there are six farmers. The first has 5 horses, 12 cows, 35 sheep, and 20 hogs. The 2d has 5 horses, 10 cows, 18 sheep, and 12 hogs. The 3d has 3 horses, 6 cows, 27 sheep, and 9 hogs. The 4th has 4 horses, 8 cows, 25 sheep, and 14 hogs. The 5th has 1 horse, 2 cows, and 6 hogs. The 6th has 8 horses, 17 cows, 45 sheep, and 27 hogs. (1) How many horses have the 6 farmers? (2) How many cows? (3) How many sheep? (4) How many hogs? (5) How many head of live stock has the first, the second, the third, the fourth, the fifth, the sixth? (6) How many head of live stock on the 6 farms?

For the solution of this and similar examples, follow the arrangement given here. It is called "Tabulating," or arranging in tables.

FARMERS	HORSES	COWS	SHEEP	HOGS	NO. HEAD
The First	5	12	35	20	
The Second	5	10	18	12	
The Third	3	6	27	9	
The Fourth	4	8	25	14	
The Fifth	1	2	—	6	
The Sixth	8	17	45	27	
Total					

Note.—If the sum of the footings in the last line is not equal to the sum of the extensions in the last column, the work is incorrect. Why?

7. Three farmers have fruit-trees as follows: The first, 72 apple-trees, 108 peach-trees, 18 quince-trees, 16 plum-trees, 19 cherry-trees. The second, 38 apple-trees, 219 peach-trees, 9 quince-trees, 27 pear-trees, 38 plum-trees, 3 cherry-trees. The third, 19 apple-trees, and 43 peach-trees. (1) How many trees of each kind? (2) How many fruit-trees has each farmer? (3) How many trees have all?

8. In the year 1880, Cincinnati had 255,139 inhabitants; Cleveland 160,146; Toledo 50,137; Columbus 51,647; Dayton 38,678; Sandusky 15,838; Springfield 20,730; Hamilton 12,122; Portsmouth 11,321. How many inhabitants in these 9 cities of Ohio?

9. Three farmers, last year, sold fruit as follows: The first, 79 bushels of apples, 391 bu. of peaches, 8 bu. of quinces, 39 bu. of pears, and 8 bu. of cherries. The second, 43 bu. of apples, 539 bu. of peaches, 19 bu. of quinces, 37 bu. of pears, 47 bu. of plums, and 6 bu. of cherries. The third, 27 bu. of apples, and 87 bu. of peaches. (1) How many bushels of each kind of fruit were sold? (2) How many bushels of fruit did each farmer sell? (3) How many bushels did all of them sell?

10. North America is inhabited by 54,566,936 people; the West Indies by 4,316,718; South America by 26,913,531; Europe by 311,694,029; Asia by 791,031,473; Africa by 199,921,600; Oceanica by 38,318,771. Find how many in all. (Statistics 1885.)

11. Three farmers divided their land as follows: The first had 6 acres in rye, 20 acres in wheat, 69 acres in corn, 2 acres in potatoes, 26 acres in meadow land, 19 acres were lying fallow, and 3 acres were in grapes. The second had 12, 39, 136, 5, 75, 26, 2 acres (take them in the same order). The third had 4, 19, 53, 19, 5, 4, 9. (1) Find, how many acres of each kind; (2) how many acres to each farmer; (3) How many acres in all.

12. Lake Superior has an area of 32,000 square miles; Lake Michigan 24,000, Lake Huron 20,400, Lake Erie 9,600, Lake Ontario 6,300. What is the total area of the five great lakes?

Original Problems.

28. **Note.**—Problems such as the first may be made up under the direction of the teacher with the aid of all the pupils of the class, due notice having been given of the kind of contributions desired. One problem per day, if possible, should be required of each pupil.

1. Find the number of pages read by all the pupils in books not used in school. (Each reports for himself, all set down the items and find the sum.)

2. Find how many examples all have solved within any given time; how many lines all have read; how many words all have spelled or missed; how many chestnuts, walnuts, hickory nuts, acorns, etc., all have gathered.

3. In a rural district an enumeration may be made of the number of horses, cows, sheep, hogs, chickens, etc.; of the apple, peach, cherry trees, etc.; of the eggs gathered, etc., etc., in the district, or on all the farms from which the children come. Let them report in writing, so that no sensitive child or parent be offended.

Note.—Such problems as the following should be written out by the pupils separately, and without the aid of the teacher. Paper, cut to uniform size, should be used for the purpose, and the exercises corrected, as in other language lessons.

4. Each pupil may imagine himself to be a store-keeper in any line of trade he pleases, and make problems in regard to it. He should write out each problem with the utmost care, and be sure that he knows the answer before he gives it to the class.

5. Questions may be prepared with the aid of a text-book in geography; as, for instance: What is the population of the —— largest cities of —— ? (the pupil's own state, the United States, any foreign country, or the world).

6. The teacher can at sight judge of the correctness of answers to such questions as the following: If there are ten wagons, and each one contains 23 bushels of apples and 27 bushels of potatoes, how many bushels of potatoes and how many bushels of apples in them all?

CHAPTER III.

SUBTRACTION.

Units and Tens.

Examples.—1. Fred had 13 cents, and bought a top costing 8¢. How many cents had he left? How many would he have had left if the top had cost 7¢? 6¢? 4¢? 5¢? 9¢? 3¢? (¢ is a sign for cents.)

2. In a class of 16 pupils there are 9 girls. How many boys are in the class?

3. Emily is 18 years old, and her brother is 9 years younger. How old is he? Their sister is 11 years younger than Emily. How old is the sister?

4. A little boy had 9 marbles, and bought enough more to make 15. How many did he buy?

5. What number must we add to 7 to make 14? 16? 19?

6. Beginning with 100 take away twos, till nothing is left. From 99 take threes, till nothing is left; from 100 take fours; from 100 take fives; from 96 take sixes; from 98 take sevens; from 96 take eights; from 99 take nines.

Definitions.

29. ***Subtraction*** is a process of finding what is left of a number when a part of it is taken away; also, of finding the difference between two numbers.

30. Terms used.—The number from which a subtraction is made is called the ***Minuend.*** The number which is subtracted is

called the ***Subtrahend.*** What is left of a number after taking away a part of it is called the ***Remainder.*** The remainder is the ***Difference*** between the minuend and subtrahend.

Note.—*Minuend* means *to be diminished*, and *Subtrahend*, *to be subtracted.* Numbers added are sometimes called *Addends*, or, more properly, *Addenda.*

31. Signs.—The sign — (minus) placed between two numbers indicates that the number on the right is to be taken from the one on the left; as, 7—2=5, which is read, 7 minus 2 equals 5. ***Minus*** means ***less.***

ORAL EXERCISES. IN TENS AND UNITS.

8.	9.	10.	11.	12.	13.
6—2=	8—3=	7—4=	11—2=	12—5=	13—7=
26—4=	14—3=	35—2=	41—4=	42—3=	25—6=
54—2=	46—5=	27—6=	53—6=	76—7=	57—8=
78—6=	68—7=	59—8=	65—8=	64—5=	63—9=
88—8=	58—3=	63—2=	71—2=	52—3=	35—7=
92—2=	86—5=	75—4=	83—4=	26—9=	41—5=

14–103. From each of the following numbers subtract 3 as many times as you can. Subtract also 5, 7, 9, 2, 4, and 8.

10	20	30	40	50	60	70	80	90
11	21	31	41	51	61	71	81	91
12	22	32	42	52	62	72	82	92
13	23	33	43	53	63	73	83	93
14	24	34	44	54	64	74	84	94
15	25	35	45	55	65	75	85	95
16	26	36	46	56	66	76	86	96
17	27	37	47	57	67	77	87	97
18	28	38	48	58	68	78	88	98
19	29	39	49	59	69	79	89	99

104. Tell how many must be added to 4 (see line of numbers below) to make 7, how many to 7 to make 13 (the next higher number ending in the next figure), and so on.

4, 7, 3, 8, 7, 9, 6, 7, 4, 3, 0, 8, 6, 9, 5, 4, 5, 8, 6, 5.

Note.—Say 4 and *3* (=7), 7 and *6* (=13), 3 and *5* (=8), 8 and *9* (=17). Omit numbers in parenthesis.

105. From each number following subtract the sum of its digits; thus,

5 + 6 = ***11***, 56 − 11 = ***45***. **Announce only results, *11, 45*.**

56, 44, 32, 53, 65, 87, 48, 29, 51, 73, 18, 92, 47, 64, 20, 70, 98, 85.

Applications.—106. Mr. Smith is 70 years old, his son is 40. How old was Mr. S. when his son was born?

How much greater is 7 tens than 4 tens.

107. Sarah had 30 cents, and her sister 50. How many more did the sister have than Sarah?

108. John has 20 marbles, William 60. How many more has William than John?

109. There are 52 houses on the east side of Linden St., and only 20 on the west. How many more on the east than on the west side?

110. A farmer sold 32 of his 92 sheep. How many had he left? How many would he have had left if he had sold 12 more?

111. One book has 84 pages, another 72. How many more pages in the first than in the second?

112. If you had 55¢ in your bank, how many would be left if you were to take out 15¢?

ORAL EXERCISES.

Speak only the remainders, thus, ***97, 93, 88,*** etc.

113. 100—3—4—5—7—9—3—5—5—7—2—7—5—8—3=

114. 99—5—2—3—8—6—5—7—8—2—3—5—6—9—5=

115. 95—2—8—3—2—3—1—4—1—3—5—7—6—5—4=

116. 87—3—5—7—9—8—6—4—2—3—1—4—7—6—8=

117. 98—7—3—4—7—2—1—2—5—2—8—6—9—4—6=

118–124. From 90, 80, 70, 60, 50, 40, 30 take 10, take 20, take 30.

125–131. From 67, 83, 42, 56, 95, 72, 61 take 20, take 30, take 40.

132–138. From 90, 80, 70, 60, 50, 40, 30 take 16, take 27, take 23.

139–145. From 67, 87, 47, 57, 97, 77, 67 take 37, take 17, take 27.

In Exercises 146–150 subtract first tens, then units. Speak only results.

146.	147.	148.	149.	150.
45—25=	61—21=	65—35=	64—54=	97—57=
54—34=	72—32=	76—46=	75—55=	55—35=
29—19=	83—53=	42—22=	86—46=	58—28=
67—37=	94—54=	37—17=	97—57=	63—33=
73—23=	45—15=	84—34=	58—48=	88—48=

In the following examples, subtract first the tens then the units, announcing only two results. Thus, in solving the first, *think* 30 less 10 and *say 20,* then think 20 less 2 and *say 18*.

151.	152.	153.	154.	155.
30—12=	20—18=	31—12=	37—19=	62—28=
40—14=	50—24=	42—16=	48—29=	78—36=
50—16=	40—35=	53—18=	54—28=	84—37=
60—18=	60—43=	64—15=	64—27=	95—39=
70—13=	80—24=	75—19=	76—39=	56—48=
80—15=	90—42=	86—17=	88—49=	68—29=
90—17=	70—21=	97—18=	96—48=	83—27=
100—19=	100—46=	98—19=	69—47=	94—48=

156–160. Subtract 11 from each of the numbers, 21, 31, 41, 51, 61, 71, 81, 91. Subtract also 13, 15, 17, 19.

161–164. Similarly from 24, 34, 44, 54, 64, 74, 84, 94 subtract 12, 14, 16, 18.

165–168. Similarly from 27, 37, 47, 57, 67, 77, 87, 97 subtract 19, 14, 18, 17.

169–224. Find the difference between each number and the one to the right of it in the same line.

99	82	73	69	58	42	36	24
91	87	75	63	52	38	25	12
93	89	69	55	47	32	21	14
95	81	79	67	53	41	39	21
94	88	71	66	57	44	27	13
99	77	68	56	41	35	24	17
92	88	77	66	55	42	37	19
97	85	73	61	50	45	35	23

225–230. From each one of the numbers 49, 52, 65, 76, 82, 93 take first 12, and from the remainder take 15. In the same manner take 13 and 16; 14 and 17.

231–236. Similarly from 41, 58, 63, 74, 85, 97 take 11, and from the remainder take 18; also 16 and 19; also 12 and 17.

Tens and Hundreds.

237.	**238.**	**239.**	**240.**
200—20=	500—50=	300—80=	700—60=
300—70=	600—40=	400=40=	800—30=
400—50=	700—30=	500—70=	900—50=
500—30=	800—20=	600—20=	300—60=
600—60=	900—10=	700—40=	400—80=

Subtract the hundreds, then the tens.

241.	**242.**	**243.**	**244.**
400—240=	800—420=	500—180=	600—280=
600—130=	500—450=	400—290=	800—370=
800—320=	700—260=	600—370=	900—580=
900—440=	900—310=	700—490=	600—390=
500—330=	300—220=	900—630=	700—210=

Applications.—245. Mr. A has 400 sheep, Mr. B 200. How many has Mr. A more than Mr. B?

246. A clerk receives a salary of $1,200, and pays $290 for rent. How much remains for other expenses?

Note.—The sign $ is used to denote dollars. It is called the ***dollar mark.***

247. Mr. Abel sold his house for $860. He had bought it for $730. How much did he gain?

248. If a boy resides 580 steps from the school-house, but, on going to school, stops on his way after taking 380 steps, how many has he yet to take?

249. A farmer bought a horse and two cows for $195. If one cow cost $49, and the other $50, how much did he pay for the horse?

Units, Tens, and Higher Orders.

32. Case I. When no term of the subtrahend is greater than the term in the same order of the minuend.

Example.—From 796 subtract 354.

Illustration.—Suppose that Mr. Jones has seven sacks of money, each containing one hundred silver dollars, nine rolls of ten dollars each, and six dollars lying loose on his table, as represented in the following picture, and that Mr. Smith calls to collect 354

dollars. The pupil will readily understand that Mr. Jones has only to give Mr. Smith *four* of the single pieces, *five* of the ten dollar rolls, and *three* of the sacks containing one hundred dollars each, and that he will then have 442 dollars remaining.

Note.—The learner should practice himself in such illustrations till he is familiar with them. He will thus surely learn the significance of the processes of arithmetic.

Process of subtraction for slate work.

	Dollars.		Dollars.
Mr. Jones had	7 hundreds 9 tens 6 units.	. Or,	796
Of this he paid	3 hundreds 5 tens 4 units.		354
He had left	4 hundreds 4 tens 2 units.		442

In the same way, tell how you would take 325 from 697 buttons, supposing that you had 6 cards having 100 buttons sewed on to each, 9 cards with 10 buttons on each, and 7 loose buttons. Show how the remainder would be found by work on the slate.

SLATE EXERCISES.

1.	2.	3.	4.	5.	6.	7.	8.	9.
495	589	686	798	894	995	789	795	325
182	273	325	496	472	572	364	682	124

10.	11.	12.	13.	14.	15.	16.	17.	18.
578	496	758	983	492	854	786	854	750
235	314	245	372	271	622	548	503	600

19–32. From 879 take 213, 425, 263, 34, 728, 658, 870, 457, 23, 43, 654, 222, 333, 400.

33–46. Take 432 from each of the following numbers: 543, 733, 645, 987, 655, 438, 679, 542, 632, 777, 989, 656, 686, 567.

Note.—In the process of subtraction there are two modes of reckoning. For instance, in subtracting 5 from 9, some say, "5 from 9 leaves ***4***"; others, "5 and ***4*** are 9"; both writing the ***4*** as it is spoken, or better, as it comes into the mind. The results are the same, but the latter wording is recommended in practice for many reasons, one of which is that it is less liable to error. It should not be introduced, however, till the former is well understood by the learner.

Example.—From 789 subtract 435.

789
435
354

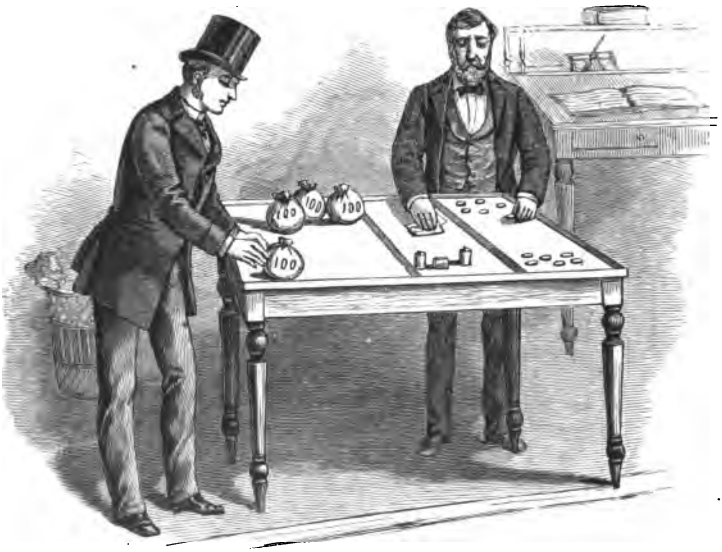
Wording.—5 and ***4*** are 9, 3 and ***5*** are 8, 4 and ***3*** are 7. Only the results printed in heavy type should be spoken, and these should be written as uttered.

This is called the "making up method." Besides being less liable to error, other advantages will be seen in Case II. and further on.

33. Case II. When any term of the subtrahend is greater than the term in the same order of the minuend.

Example.—From 442 subtract 136.

Illustration.—After Mr. Jones had paid Mr. Smith, he had $442 left, out of which he is paying Mr. Brown $136. But since he has only two loose dollars, he is here represented as having



taken ten dollars from one of the rolls, and put them with the two, thus making twelve single pieces. From these he has put forward *six* pieces. He has shoved forward also the *three* remaining ten dollar rolls, and *one* sack containing a hundred dollars. Thus he has —— dollars left.

The process is represented in figures as follows :

	Dollars.		Dollars.
Mr. Jones had	4 hundreds 4 tens 2 units.	Or,	442
Mr. Brown was paid	1 hundred 3 tens 6 units.		136
Mr. Jones had left	3 hundreds 0 tens 6 units.		306

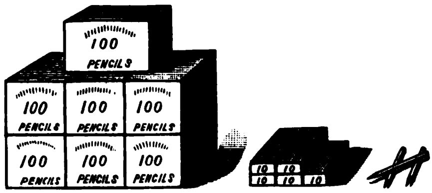
SLATE EXERCISES.

Subtract

47.	48.	49.	50.	51.	52.	53.	54.	55.	56.
324	543	384	792	325	986	847	761	568	738
116	327	259	368	217	509	719	234	139	309

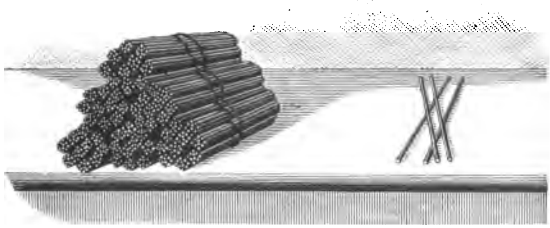
57.	58.	59.	60.	61.	62.	63.	64.	65.	66.
382	44	631	52	166	240	874	760	293	871
279	25	408	39	39	19	65	347	175	257

Note.—The pupil does not understand the foregoing illustrations if he can not go farther and explain the case, where he has to "borrow," both from the tens and from the hundreds; or, where there are no tens, perhaps also no units.



67. Explain how you could most conveniently take 585 pencils from the number of pencils represented above.

68. In the same way explain how you would proceed to take 378 match-sticks from the number represented below.



Arithmetical process:

There are	6 hundreds, 0 tens, 4 units.	Or,	604
We take	3 hundreds, 7 tens, 8 units.		378
There will be left	2 hundreds, 2 tens, 6 units.		226

According to the "making up method" mentioned in note at foot of p. 41, the wording in the process of subtracting 378 from 604 would be as follows:

604 378 226	Eight and ***6*** are 14, carry one to 7, 8 and ***2*** are 10, carry 1 to 3, 4 and ***2*** are 6, the numbers represented in heavy type being the only ones spoken aloud. These should be written while they are being pronounced.

69–78. Subtract 328 from each of the following numbers: 442, 560, 643, 751, 962, 876, 777, 691, 564, 886.

79–86. Find the difference between 435 and each of the following numbers: 561, 872, 960, 253, 864, 950, 762, 341.

Subtract

87.	**88.**	**89.**	**90.**	**91.**	**92.**	**93.**	**94.**	**95.**	**96.**
624	733	521	635	742	815	740	490	600	586
156	56	68	469	387	466	295	399	579	297

97.	**98.**	**99.**	**100.**	**101.**	**102.**	**103.**	**104.**	**105.**	**106.**
350	440	560	690	720	870	980	550	360	570
123	145	467	498	445	456	379	258	177	276

Try to solve these examples orally before using the slate.

107. 255—15=	**115.** 256—150=	**123.** 378—108=	**131.** 457—259=
108. 836—17=	**116.** 743—240=	**124.** 643—207=	**132.** 756—257=
109. 952—19=	**117.** 428—320=	**125.** 418—309=	**133.** 321—134=
110. 115—14=	**118.** 549—540=	**126.** 237—108=	**134.** 432—246=
111. 303—13=	**119.** 387—180=	**127.** 276—208=	**135.** 527—358=
112. 517—19=	**120.** 613—410=	**128.** 517—309=	**136.** 621—247=
113. 206—16=	**121.** 309—200=	**129.** 226—109=	**137.** 732—544=
114. 614—18=	**122.** 487—280=	**130.** 375—207=	**138.** 425—247=

Find the differences

139.	**140.**	**141.**	**142.**	**143.**	**144.**	**145.**
7834	9425	6453	7542	28769	52639	43892
4915	3568	2575	4634	3454	4758	5893

146.	**147.**	**148.**	**149.**	**150.**	**151.**
628234	23456	180020	70000	60000001	3845
149365	15897	147835	39876	12345678	1578

34. *Rule.*—**1. Write the subtrahend under the minuend, so that units may stand under units, tens under tens, hundreds under hundreds, etc.**

2. Commence at the right hand, and if possible subtract each term of the subtrahend from the one above it, and write the remainder below in the same order.

3. If any term of the minuend is less than the term to be subtracted, add ten to it, keeping in mind that the next higher term of the minuend must then be diminished by one.

Proof.—**Add the remainder to the subtrahend. If the sum is equal to the minuend the work is correct.**

Examples for Practice and Review.

1. How many years have passed since the discovery of America in 1492? Since the discovery of the Mississippi river in 1541?

2. Benjamin Franklin died in 1790, aged 84 years. In what year was he born? How old is a person now who was born in the year in which Franklin died?

3. A store, valued at $9,050, was destroyed by fire. What was the owner's loss, there being an insurance of $6,000?

4. There were 12,426 soldiers in a fortress, of whom 5,855 were discharged. How many remained?

5. A merchant began business with goods valued at $16,810. After two years he found his goods worth $38,430. How much had their value increased?

---

6. The State of New York had 5,082,871 inhabitants in 1880; the State of Illinois had 3,077,871. What was the difference in the number of inhabitants of these two States?

7. Asia is supposed to have 797,000,000 inhabitants, and Europe 313,834,000. How many more people are supposed to live in Asia than in Europe?

8. In 1880 there were 105,541 children attending the public schools of Philadelphia, 59,768 in Boston, and 270,176 in New York. How many more in Philadelphia than in Boston? How many more in New York than in Philadelphia?

9. Mt. Everest, in Asia, which is 29,002 ft. high, is 22,709 ft. higher than Mt. Washington, in New Hampshire. How high is Mt. Washington?

EXERCISES FOR SLATE WORK.

10. From 750 subtract 75, from the remainder subtract 75, from the second remainder subtract 75, and continue this process of subtraction till you have subtracted 75 ten times.

11. From 4,590 take 459 ten times, as in the preceding example.

12. From 6,380 take 638 nine times. Before doing the work, say what the last remainder will be.

13. Write any number expressed by three figures, annex a cipher, and from the number thus formed take the first one ten times. Why is the answer the same as in 10–11?

14. Copy and solve example 237 (page 25); add the first, second, and third columns separately; subtract the sum of the second column from the sum of the third. Why should the remainder be equal to the sum of the first column?

Suggestion.—There will be no difficulty in stating *why*, if you lay out before you numbers of objects as represented in any one of the examples.

15. Copy and perform examples 118–121 (page 21), and subtract the sum of the first column from the sum of the third. Why should the difference be equal to the sum of the second?

Find the differences:

16.	7432	**17.**	8397	**18.**	9465	**19.**	7546	**20.**	4932	**21.**	5432	**22.**	6420
	2345		4567		7656		1667		2784		3765		2574

23.	4923	**24.**	6843	**25.**	7110	**26.**	8435	**27.**	9425	**28.**	4620	**29.**	7005
	4486		1876		3465		3583		2754		1629		1967

30.	9000	**31.**	50,000	**32.**	60,000	**33.**	90,000	**34.**	80,000	**35.**	1,000,000
	5793		14,312		24,635		45,678		39,876		493,624

36.	200,000	**37.**	700,000	**38.**	8,000,000	**39.**	1,000,000	**40.**	654,321
	146,231		102,009		4,563,921		912,345		200,000

41. 423,021	**42.** 524,632	**43.** 635,124	**44.** 543,210	**45.** 74,321,000
156,798	243,738	78,987	244,567	56,543,289

46. 5,246,312	**47.** 342,151	**48.** 624,001	**49.** 632,031	**50.** 73,500,493
1,472,536	147,367	175,548	234,567	12,345,678

51. 43,821	**52.** 54,312	**53.** 64,213	**54.** 75,314	**55.** 86,753	**56.** 840,170
34,547	34,343	23,456	62,345	23,542	654,398

57. 724,314	**58.** 842,531	**59.** 904,030	**60.** 800,012	**61.** 60,012,345
342,675	123,654	654,321	187,654	45,678,987

62. 8,421,303	**63.** 4,621,621	**64.** 725,321	**65.** 372,100	**66.** 743,628
3,544,534	26,562	46,845	193,876	100,000

67. 7,432,100	**68.** 85,731,465	**69.** 74,000,321	**70.** 9,333,122,210
2,876,201	74,635,679	49,898,767	7,457,935,767

Miscellaneous Examples.

Addition and Subtraction.

1. If the sum of two numbers is 36,251, and the greater one is 26,659, what is the smaller number?

2. From the sum of 3,742 and 89,331, take the sum of 1,137, 2,065, and 3,820.

3. Add 74,321, 85,746, 25,100, 321,098; subtract 26,304, and from the remainder take 54,876.

4. Add the difference between 4,321 and 3,571 to the difference between 52,312 and 19,936.

5. From the difference between 533,016 and 154,693, subtract the difference between 19,876 and 17,987.

6. To what number must 893 be added four times to make 3,804?

7. From what number must 309 be subtracted five times to leave 173?

8. From what number must you take 8,763 to leave 3,849?

9. To what number must you add 89,650 to make 108,731?

10. How many times must 638 be taken from 7,280 to leave 900? to leave 262?

---

11. Fred. had 143 marbles. How many had he left after giving 19 to you, 25 to me, 38 to Paul, 49 to Edward, and losing 3?

12. A congregation had raised $78,596 for the erection of a building which was to cost $125,000. How much yet remained to be raised?

13. A florist had 3,746 tuberose bulbs. How many were left after selling 815, 150, 387, 479, and 1,091?

14. After a robbery a banker finds $1,657 in his safe. The evening before he had left in it $9,336. How much had been stolen?

15. In the year 1880 Chicago had 503,185 inhabitants, Cincinnati 225,139, St. Louis 350,518. 1. Find the sum. 2. How many had Chicago more than St. Louis? 3. More than Cincinnati? 4. How many had St. Louis more than Cincinnati?

The distance by rail from New York

To Albany is	142	miles.	To Chicago,	977	miles.
" Buffalo,	439	"	" Bloomington,	1,104	"
" Cleveland,	622	"	" Jacksonville,	1,193	"
" Toledo,	735	"	" St. Louis,	1,285	"

16. By the aid of this table reckon the distance of Albany from each place named after it.

17. Also reckon the distance from Cleveland to Buffalo; to Chicago; to St. Louis. Also from St. Louis to Chicago; from Chicago to Buffalo.

Suggestion.—Write out a table, showing the distance from each city named in the foregoing list to the next. Make other problems from this or other tables of the kind.

18. A farmer had in his yard 31 chickens, 17 geese, 24 turkeys, and these, with his ducks, made up the entire number of his poultry, which was 97. How many ducks had he?

19. How many times can 93 yds. be cut from a piece of twine 385 yds. long? How much will be left?

20. A train started with 374 passengers. At the first station 16 left and 9 got on; at the second 11 left and 25 got on; at the third 3 left. How many passengers were on the train as it entered the fourth station?

21. In the six working days of a week a newsboy bought 76 papers a day, except Friday and Saturday, when he bought 10 more each day. He sold all but 3 on Monday, and 4 on Saturday. How many did he sell that week?

22. Bought a pair of ponies for $158, and sold them so as to gain $47. What did I sell them for?

23. Bought one pony for $100, and another for $76; paid $2 for shoeing each of them, and sold the pair for $210. How much did I gain?

24. A farmer has in one lot 53 beech, 87 maple, 18 hickory, 54 walnut, 28 poplar, and 327 oak trees. He sells all the walnut, 13 hickory, 78 maple, and 15 poplar trees for lumber. He cuts 281 oaks into railroad ties, and all the remaining oak and other trees for firewood. How many does he cut for firewood?

25. Bought a horse and carriage for $428, and in selling them shortly afterward lost $35 on the carriage, but gained $16 on the horse. How much did I sell them for?

26. On Monday Robert finds 43 eggs, 25 each on Tuesday and Wednesday, 26 on Thursday, 22 on Friday, 26 on Saturday. The next week he finds as many and 17 more. He sells 96 to a neighbor, and the cook uses 58. How many has he to send to market at the end of the two weeks?

27. The number of days in each month of the year is:

January,	31.	April,	30.	July,	31.	October,	31.
February,	28.	May,	31.	August,	31.	November,	30.
March,	31.	June,	30.	September,	30.	December,	31.

How many more days in the last 6 than in the first 6 months?

28. How many days in all the months which have *a* for the second letter of their names? In all the rest of the year?

29. How many days in all the months which have the letter *c* in their names? In all the rest of the year?

Original Problems.

35. Write problems for yourself and classmates.

Note.—The following skeleton problems may be used at first, if thought best.

1. —— goes to the store, buys — for —¢, and — for —¢. How much change does —— bring home out of —¢?

2. —— has —¢, buys —¢ worth of —, and — yards of tape, but forgets what she paid for it. She has —¢ left, and has lost nothing. What did she pay for the tape?

3. —— wished to buy —, which would cost —¢; had saved up —¢, and uncle would give —¢. How much more was still needed?

4. ——, ——, —— (three ladies), buy — bolts of muslin, each containing 39 yards. Mrs. —— takes — yards, Mrs. —— — yards, Mrs. —— — yards. They send the rest to a poor neighbor. How many yards does she receive?

5. —— has — inhabitants; —— has not so many by —. How many has the latter?

Hints.—Boys raise money for a foot ball; make contributions for buying an overcoat or pair of shoes for a poor schoolmate; cut ties for a railroad; buy and sell newspapers. The girls make squares for a quilt; cakes for a picnic. Father gets and pays out money. Compare the heights of mountains, distances of cities from each other, of places from the school-house, of the weight of a dozen boys with that of as many girls of about the same age, each giving his or her own weight. Ask parents for problems.

CHAPTER IV.

MULTIPLICATION.

1. How many marks are there here? (Count by 3's.)

/// /// /// /// /// /// /// /// ///

2. How much will 6 tops cost at 4¢ a piece?

Suggestion.—If you had learned nothing more of arithmetic than how to count by 4's, you could take one top and pay for it, and then another and pay for that, and so on till you had six tops, and had put down six piles of 4¢ each to pay for them. Then, counting by fours, you would find 24¢ to be the cost of the six tops.

3. In like manner with pebbles, acorns, grains of wheat, match-sticks, marks upon the slate, or other convenient objects, for counters, find the cost of 4 oranges at 7¢ a piece; also, separately, of 6, 8, and 9 oranges at 5¢ a piece.

4. In the same way illustrate how you might find the cost of 9 pounds of sugar at 5¢, at 7¢, at 10¢, at 8¢, at 6¢, at 9¢ a pound.

36. The process of counting in this way would become, in a short time, very tiresome. It would certainly be more convenient to learn, once for all, the sum of five 8's, than to have to find the sum every time we need to know it.

Finding the sum of $8+8+8+8+8$, as we have already learned, is called ***Addition,*** but taking 40 for 5 times 8, without at the same time making the addition, is ***Multiplication.***

Having constructed a table which shows the sum of from one to ten 1's, 2's, 3's, 4's, etc. (see Ex. 10, p. 20), the pupil should now commit it thoroughly to memory.

For convenience it is here given in full, with the addition of the 11's and 12's.

Multiplication Table.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

Directions for learning the Multiplication Table.—1. Multiply *2* (see figure at the head of the second column) by the numbers in the first column, repeating the two numbers and their product, thus: 2 times 2 are four, 3 times 2 are 6, 4 times 2 are 8, 5 times 2 are 10, etc. After learning a column in this manner, it will be well to reverse the order, and learn 2 times 2 are 4, 2 times 3 are 6, 2 times 4 are 8, etc.

2. The shading indicates the parts which are learned most easily. The real difficulties of the table are to be found in the unshaded parts. Special attention should therefore be given to them.

Definitions.

37. ***Multiplication*** is a short process of finding the sum of two or more equal numbers.

38. Terms used.—When we say 9 times 3=27, we *multiply* 3 by 9. The numbers that are multiplied together are called the ***Factors*** (makers) of the result.

Note.—Think of the word *factory*, a *place* where things are *made*.

39. The factor which is multiplied is called the ***Multiplicand.*** The factor which we multiply by is called the ***Multiplier.*** The

result obtained, that is, what is produced by multiplication, is called the ***Product*** (the thing produced).

Note.—Twice a number is double that number, three times is triple, four times is quadruple, etc. Any number of times another number is a multiple of (that is, *many times*) the number. Hence, any product of a number is a multiple of that number. The multiplication table is a table of multiples.

Notice *multi* in *multi*ply, *multi*plicand, *multi*plier, *multi*ple. Multi means *many*, and ply, pli, or ple, means *fold.* Multiplier means *many folder*.

40. Signs.—The sign × is used to show that two numbers are to be multiplied together, as $4 \times 7 = 28$ may be read, 4 multiplied by 7 equals 28, or 4 times $7 = 28$. The latter reading will be preferred in this book.

The product of two factors is the same, whichever is used as the multiplier; hence, in *performing* a multiplication, we generally use the one containing the fewest significant figures, because it is most convenient to do so; but, in *indicating* a multiplication, it is best for the learner to write that term before the sign (×) which properly comes before the word *times* in stating the reason for the multiplication.

SLATE EXERCISES.

5. After learning the table on page 52, complete the following table. Make other tables, changing the order of the factors.

	7	2	8	5	3	9	1	6	10
10	70	20	80	50	30	90	10	60	100
6	42	12							
7									
9									
3									
5									
8									

ORAL EXERCISES.

6. Write in order, ***5, 7, 4, 1, 8, 3, 6, 9, 2, 10,*** and multiply each number by 2, by 3, by 4, etc., to 10.

Caution.—Do not say, 3 times 5 are 15, 3 times 7 are 21, etc., but, pointing at 5, 7, 4, etc., and knowing that you are to multiply each by 3, say *15, 21, 12,* etc.

7. Write the following lines of figures upon slate or paper:

4, 7, 2, 8, 8, 5, 6, 3, 9, 2, 6, 6, 8, 7, 9, 5, 3, 3, 4, 2, 5,
5, 3, 7, 7, 5, 5, 4, 6, 9, 9, 8, 4, 4, 2, 3, 8, 5, 4, 9, 6, 7.

Then, pointing successively between 4 and 7, 7 and 2, 2 and 8, etc., announce their products, thus: ***28, 14, 16,*** etc.

Note.—This affords an excellent exercise upon the multiplication table, within 10 times 10, and when the pupil can give the products almost as rapidly as he can speak, he is ready to go forward, and not till then.

Seven times

7 × five tens are thirty-five tens.

Tens.—8. Multiply 90, 30, 60, 50, 80, 20, 70, 40, by 1; 2; 3; 4; 5; 6; 7; 8; 9.

Note.—Here the pupil should *think* 4 times 9 tens, but should say only, "*36 tens,* or *360*"; "*12 tens,* or *120,*" etc.

Seven times

7 × five hundred are thirty-five hundred.

Hundreds.—9. Multiply 600, 300, 900, 200, 800, 500, 400, by 9; 3; 6; 8; 5; 2; 4; 7, and give the results, thus: ***54 hundred,*** or, ***5 thousand 4 hundred.***

Thousands.—10. Multiply 7000, 5000, 8000, 2000, 9000, 3000, 6000, by 8; 5; 3; 9; 4; 7; 2; 6.

SLATE WORK.

41. Units, Tens, Hundreds, etc.—**11.** Write 4685 nine times, as in the margin, and add.

We add thus: "5, 10, 15, etc., to 45." Then setting down the 5 units, and carrying the 4 tens to the column of tens, we say: "4, 12, 20, etc., to 76," and set down the six tens, carry the 7 hundreds to the next column, and add as before. Or,

Since all the figures in each column are the same, we save time by taking 45 at once for "9 times 5," as learned in the multiplication table. We then set down 5, and carry 4 to 9 times 8, and so on, exactly as if we were finding the sum by addition.

Addition.

```
 4685
 4685
 4685
 4685
 4685
 4685
 4685
 4685
 4685
-----
42165
```

Although 4685 is here written 9 times, as in addition, the last process by which we obtain the result is *Multiplication.*

In multiplying, however, we save more than the time required for counting by 5's, by 8's, etc. We save writing more than once the number to be multiplied, by simply noting the number of times each term is to be taken, as in the margin.

Multiplication.

```
 4685
    9
-----
42165
```

Wording.—Knowing that you are to multiply 5 by 9, do not repeat "9 times 5 are 45," but say at once *"45,"* and write 5 in units' place as the word is pronounced. Then say *72, 76,* not "9 times 8 are 72 and 4 are 76." Use as few words as possible.

Examples.—**12.** What is the value of 9 acres of land at $783 per acre?

13. At $16856 per mile, what is the cost of constructing 8 miles of railway?

14. What will 6 horses cost at $273 each?

15. What is the cost of building 7 locomotives at $13586 apiece?

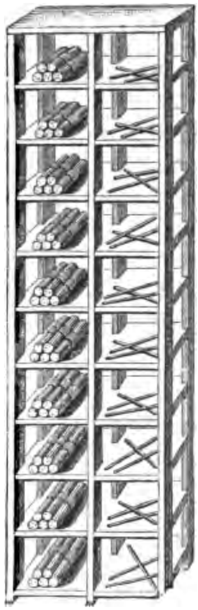
16. What is the value of 5 pianos at $785 each?

17–100. Multiply each of the following numbers by 6; by 9; by 3; by 7; by 4; and by 8:

4759	5678	2134	4157	5132	5426	3627
3846	8679	7986	9768	8979	6978	7896

Multiplying by 10, 100, 1000, etc.

101. Write 367 ten times and add. How many times 367 are there in the sum? How do the figures in the sum differ from the figures in 367?

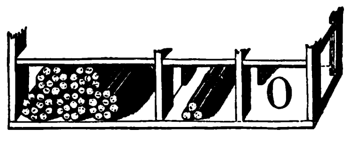


102. Write 3670 ten times and add. How many times 367 in 36700? How do the figures of this sum differ from those of 367?

103. Write 36700 ten times and add. How many times 367 in each 36700? How many times 367 in 367000? How do the figures of this last sum differ from those of 367?

Note.—The results in the last three examples indicate a principle which it is desirable the pupil should discover for himself. Additional examples of the kind may be dictated by the teacher.

42. The solution of the foregoing examples will enable the pupil to see that annexing one 0, thus removing the digits of a number one place to the left, increases the number tenfold; annexing two 0's to a number, thus removing its digits two places to the left, increases, or multiplies, its value *ten times* tenfold, or a hundredfold; that annexing three 0's multiplies the value of a number a thousandfold, etc.



Note on the Illustration.—One stick being taken from each of the right-hand boxes represented above, we have *ten.* These being tied together, are placed in the tens box below. Repeating this operation with the remaining single sticks, and proceeding similarly with the bundles of ten, it becomes evident to the senses that *ten times 53 are equal to 530.*

Hence we have the following

43. ***Rule.***—**To multiply any number by 10, 100, 1000, etc., annex to the number to be multiplied as many 0's as there are 0's in the multiplier.**

ORAL EXERCISES

104. What will be the figures of the results if you increase each of the following numbers tenfold: 18, 92, 37, 802, 460?

105. Multiply 3562, 8921, 7643, 284, 39, 689, 9876, by 10.

106. Multiply 632, 54, 723, 140, 29, 3572, 60, 932, 807, by 100.

107. Multiply 15, 269, 387, 4, 5467, 198, 3287, 6420, by 1,000.

108. Multiply 6, 16, 26, 328, 10, 400, 632, 84730, by 10,000.

109. Multiply 71, 83, 94, 738, 8010, 4283, 738, by 100,000.

Multiplying by any Number of Tens, Hundreds, etc.

44. 110. Copy 卌 // 卌 // ten times, and show that 10 times $2 \times 7 = 20 \times 7$. Copy 卌 // 卌 // 卌 // ten times, and show that 10 times $3 \times 7 = 30 \times 7$.

111. If you were to copy 卌 / 卌 / 卌 / 卌 / one hundred times, you could show in like manner that 100 times $4 \times 6 = 400 \times 6$.

112. How many times *five marks* are represented here? How

卌 卌 卌 卌 卌 卌 卌 卌 卌

many *marks?* How many times *5 marks* in 10 such rows? How many *marks?*

In like manner tell how many times *5 marks* in 100 rows, and how many *marks.*

Suggestion.—Let similar exercises be continued till the pupil becomes so entirely familiar with the results that he can anticipate them with confidence.

---

113. Copy the following, and add by lines and columns.

16+16+16+16+16+16+16+16+16+16
16+16+16+16+16+16+16+16+16+16
16+16+16+16+16+16+16+16+16+16

How many 16's in each column? Three times $16 = ?$ How many columns are there? How many times 3×16? Ten times

$3 \times 16 = ?$ Ten times three 16's are how many 16's? Thirty times 16 then $= ?$

What difference between the figures in the products of 3×16 and 30×16? When you have found the product of 3×16, how can you most readily form the product of 30×16?

---

114. If you were to write three lines, each line containing one hundred 16's, and were to add the columns as before, the sum of each would of course be three times 16, and would $= ?$ How many columns would there be? How many times 3×16? One hundred times $3 \times 16 = ?$ One hundred times 3×16 are how many times 16? Three hundred times $16 = ?$

What difference between the figures of the products of 3×16 and 300 times 16? When you know how many 3×16 are, how can you find 300 times 16 with least labor?

---

115. Copy the following line five times, setting the numbers carefully one under another, and add by lines and columns; then question yourself in the same manner as above.

$$327+327+327+327+327+327+327+327+327+327$$

Caution.—The learner should not neglect to perform all additions as directed.

116. Suppose you were to extend the five lines till you had a hundred 327's in each line?

Note.—You need not write them so often, but you may think of them as if they were written, and ask yourself,

How many 327's in a column? Five times $327 = ?$ How many columns would there be? A hundred times five times $327 = ?$ What is the difference between the figures of the product of 5×327 and of 500×327? Hence we have the following

45. ***Rule.***—**When the multiplier is expressed by a significant figure with ciphers annexed, multiply by the significant figure and annex as many ciphers to the product as there are ciphers at the right of the significant figure.**

ORAL EXERCISES.

117.	118.	119.	120.	121.	122.
20×8=	30×9=	60×3=	80×5=	40×6=	90×4=
20×5=	30×4=	60×7=	80×8=	40×8=	90×9=
20×4=	30×8=	60×5=	80×3=	40×5=	90×3=
20×7=	30×7=	60×6=	80×9=	40×7=	90×7=
20×3=	30×3=	60×4=	80×4=	40×3=	90×5=

123–130. Multiply 5, 9, 1, 8, 2, 7, 6, 4, 3, by 200; 500; 700; 900; 400. Also by 3000; 8000; 6000.

Note.—It may be better that the learner should not attempt the following exercises without the aid of the slate, but if he does, he will find it safest to multiply the tens first, increasing the right-hand term of that product by what he sees at a glance is to be brought up (carried) from the product of the units.

131.	132.	133.	134.	135.	136.
20 times	60 times	40 times	90 times	50 times	70 times
11	32	94	51	39	65
13	45	37	96	64	39
15	68	56	77	58	86
17	93	91	60	84	69
19	74	88	70	57	31

137–156.	8×12=	8×25=	6×33=	30×26=	20×48=
	5×19=	9×34=	8×19=	50×19=	70×13=
	8×24=	5×26=	7×14=	40×18=	50×17=
	9×18=	4×31=	9×12=	20×37=	40×24=

157–216. Multiply each number in the following columns by 3; 4; 5; 2; 6; 7; 8; 9.

217–276. Multiply them by 10; 20; 30; 40; 50; 60; 70; 80.

277–336. Also by 200; 800; 600; 500; and by 3000; 7000; 9000.

52	72	67	35	79	38	89	28	47	99
44	98	24	33	46	78	14	77	51	25
27	29	63	48	23	55	49	42	26	83
85	41	71	34	59	75	10	96	12	87
76	53	73	92	82	88	95	61	62	57
91	18	81	30	40	97	60	70	80	90

SLATE EXERCISES.

337–408. Multiply each number in the columns below by 9; 6; 2; 8; 4; 7; 5; 3.

409–480. Also by 30; 50; 70; 90.

481–552. Also by 300; 600; 800.—Also by 8000; 5000; 4000.

162	171	648	207	984	435	384	613	4212
288	304	621	318	945	119	855	533	2345
335	465	864	424	952	769	564	828	6789
316	568	954	154	238	854	403	261	1234
384	483	848	925	357	968	177	695	5678
266	612	302	193	476	166	324	785	9123
520	132	324	965	587	231	125	674	4567
423	660	216	164	693	219	662	753	8912

Note.—1. With a little practice, such calculations as are required above can be performed without the aid of the slate. Let the *wording* be the simple announcement of results; thus,

2. In multiplying 162 by 9 the announcements should be *9, 144, 1458.*—In multiplying 423 by 90 they would be *36, 378. 3807, 38070.*

3. Though these are excellent exercises for practice, no learner should trust the result of oral work in large numbers without testing its accuracy by the written process.

Multiplying by Units, Tens, Hundreds, etc.

46. 1. Write 17 four times in one column, ten times in another, and fourteen times in a third. Add the columns separately.

Should the sum of the first two footings equal the third? Why?

2. Find 4 times 17 and 10 times 17 by multiplication, setting the work in the following form:

$$\begin{array}{r} 4\times17=\ 68 \\ 10\times17=170 \end{array}$$

How many times 17 is the sum of the two products?

To find 14 times 17, then, let the slate-work stand as follows:

$$\begin{array}{r} 4\times17=\ 68 \\ \underline{10}\times17=\underline{170} \\ 14\times17=238 \end{array}$$

SLATE EXERCISES.

In like manner multiply:

3. 3×23=	**4.** 5×31=	**5.** 4×21=	**6.** 8×63=
10×23=	40×31=	20×21=	30×63=
7. 3×18=	**8.** 6×71=	**9.** 4×26=	**10.** 2×46=
20×18=	50×71=	30×26=	70×46=
300×18=	600×71=	400×26=	200×46=
11. 3×218=	**12.** 4×324=	**13.** 5×381=	**14.** 7×1236=
40×218=	20×324=	60×381=	20×1236=
300×218=	200×324=	200×381=	100×1236=

47. The use of many unnecessary figures can be avoided by writing the multipliers in their proper orders and as one number under the multiplicand, as in the second form below.

15. Multiply 3582 by 4376.

FIRST FORM.

6	times	3582=	21492
70	"	3582=	25074*0*
300	"	3582=	10746*00*
4000	"	3582=	14328*000*
			15674832

SECOND FORM

```
    3582
    4376
   21492
  250740
 1074600
14328000
15674832
```

Though not nearly as many figures and signs are used in the second form as in the first, it must be remembered that both forms mean exactly the same thing. In both we multiply separately by units, tens, hundreds, etc., and then add the products. The products and the way of obtaining them are just the same in one form as in the other.

Definition.

48. The product of the multiplicand by any one term of the multiplier is called a ***Partial product.*** (It is only a *part* of the entire product.)

SLATE EXERCISES.

Arrange the work in both forms.

16. 3 × 2371 =	**17.** 6 × 3582 =	**18.** 3 × 1356 =	**19.** 5 × 4026 =
40 × 2371 =	70 × 3582 =	40 × 1356 =	70 × 4026 =
500 × 2371 =	300 × 3582 =	500 × 1356 =	900 × 4026 =
2000 × 2371 =	1000 × 3582 =	2000 × 1356 =	3000 × 4026 =

49. The positions of the digits in each partial product sufficiently indicate their order; the ciphers at the right hand may therefore be omitted, if great care be taken to set the first figure produced by each multiplication under the figure by which you multiply, as in the example.

THIRD FORM.

```
    3582
    4376
   21492
  25074
 10746
14328
15674832
```

Note.—Frequently read the partial products, not forgetting the 0's.

20. 1776 × 48	**21.** 3028 × 52	**22.** 4958 × 74	**23.** 2888 × 36	**24.** 3675 × 49	**25.** 4788 × 57
26. 6045 × 65	**27.** 5186 × 76	**28.** 2415 × 35	**29.** 3456 × 26	**30.** 2886 × 47	**31.** 3306 × 38
32. 2106 × 79	**33.** 3569 × 41	**34.** 5076 × 58	**35.** 4940 × 65	**36.** 5412 × 438	**37.** 2401 × 492
38. 3536 × 521	**39.** 3436 × 635	**40.** 6882 × 749	**41.** 3884 × 362	**42.** 1102 × 293	**43.** 5605 × 643

44. Multiply 756 by 649; also by 351. How many times 756 in the sum of the products?

45–332. Multiply each number in the following columns by 589; by 376; by 4863; by 6974; by 892; by 3892; by 402; by 2009.

706	483	1858	3128	27228	42471
709	759	6696	5912	59432	60392
245	665	3054	6048	18737	47973
698	534	2968	3582	29735	46795
596	498	2505	2380	61356	12820
737	751	4731	3498	43207	55043

333–386. Multiply the numbers of the first column by 10; by 100; by 1000. Tell how you do it.—Multiply them by 30; by 400; by 6000. Tell how you do it.—Multiply them by 12; by 120; by 1200; and tell how it is done in each case.

---

50. The following is the general rule for multiplication:

Rule **—1. If the multiplier consists of one term, multiply each term of the multiplicand by it, beginning at the right, and carrying as in addition. The result will be the product.**

2. If the multiplier contains more than one term, multiply separately by each term as above, placing the right-hand figure of each partial product in the same order or column as the term by which you are multiplying.

3. Add the partial products together. The sum will be the entire product.

Proof. **—Multiply the multiplier by the multiplicand, and if the result is the same as before, the work is correct.**

Contractions. **—If there are ciphers at the right hand of either factor, omit them in multiplying, and annex as many ciphers to the product as are thus omitted from both factors.**

Examples for Practice and Review.

1. How many letters in a book of 178 pages, if every page contains 2,978 letters?

2. The roof of the main part of a house has 128 rows of shingles, each row containing 129 shingles. How many shingles in that part of the roof?

3. How many lines are there in a work of 15 volumes, if each volume has 348 pages, and each page 46 lines?

4. There are 4 shelves in a drug-store, each containing 18 jars; 6 shelves, each containing 27 jars; and 5 shelves containing 12 jars each. Find how many jars in all.

5. Two school-houses were to be furnished with single desks. One had 18 rooms, each large enough for 52 desks; the other had 14 rooms, each large enough for 64 desks. Find how many desks were required for both houses.

6. A contractor ordered 52 loads of brick of 1,250 each. How many bricks did he order?

7. Multiply any number by 3, by 5 and by 2, and add the products. Why is the sum the same as the product would be if the number were multiplied by 10?

Multiply as indicated below, and find the sum of each column of products. Tell why the sum in each case contains the same digits as the several multiplicands of the example.

8.	9.	10.	11.
$4 \times 327 =$	$5 \times 384 =$	$3 \times 9876 =$	$7 \times 48 =$
$2 \times 327 =$	$3 \times 384 =$	$3 \times 9876 =$	$2 \times 48 =$
$4 \times 327 =$	$2 \times 384 =$	$4 \times 9876 =$	$1 \times 48 =$

Perform the following multiplications, and add the column of products in each example.

12.	13.	14.	15.
$18 \times 356 =$	$15 \times 438 =$	$27 \times 967 =$	23×9807
$19 \times 356 =$	$17 \times 438 =$	$32 \times 967 =$	14×9807
$26 \times 356 =$	$49 \times 438 =$	$12 \times 967 =$	15×9807
$37 \times 356 =$	$19 \times 438 =$	$29 \times 967 =$	48×9807

Compare the sum of products in each case with the several multiplicands of the same example. What do you observe? Why do you think such a result is found?

Suggestion.—Add the column of multipliers in each example.

Perform the following multiplications, add the products as in the preceding examples, and tell what you can about the results.

16.	17.	18.
$103 \times 587 =$	$292 \times 8264 =$	$387 \times 5379 =$
$369 \times 587 =$	$375 \times 8264 =$	$203 \times 5379 =$
$287 \times 587 =$	$109 \times 8264 =$	$141 \times 5379 =$
$241 \times 587 =$	$224 \times 8264 =$	$269 \times 5379 =$

Question.—Is there not a shorter way to find the sums of products as required on this page?

Exercises in Familiar Measures.

51. There are 12 inches (in.) in 1 foot (ft.), 3 feet in 1 yard (yd.).

Note.—If the pupils are not already familiar with these measures, they should become so as soon as possible. They should be required to measure the length and the height of desks, the width of windows, doors; the length and width of the room, of the window-panes, etc., etc. Let the learner become familiar with the weights and measures here presented, but it is not desirable that he should learn any tables at this stage, except by observation and experience.

19. Find how many feet there are in 46 yards.

Analysis.—Since there are 3 ft. in 1 yard, there are 46 times 3 ft. in 46 yards.

$$\begin{array}{r} 46 \\ \underline{3} \\ 138 \end{array}$$

20. How many inches in 16, 79, 63, 479, 200, 571, 325 feet?

21. How many inches in one yard? In 2, 7, 9, 19, 34 yd.?

22. Find first how many feet, then how many inches, there are in 12 yards.—In 24, 36, 112, 324, 420, 500 yd.

---

52. There are 16 ounces (oz.) in a pound (lb.), and 100 pounds in a hundredweight (cwt.). (The *c* stands for *hundred*, and *wt* for *weight*.)

23. How many ounces in 3, 9, 16, 32, 87 pounds?

24. How many ounces in one hundredweight?—In 2, 3 cwt.?

25. How many pounds in 6, 8, 3, 12 cwt.?

26. How many pounds do you weigh? How many ounces? How much would you be worth at $125 a pound?

---

53. There are 2 pints (pt.) in a quart (qt.), and 4 quarts in a gallon (gal.). (Used for measuring liquids.)

27. How many pints are there in 2 quarts?—In 6, 8, 10, 21, 130, 425 qt.?

28. How many quarts in 6 gallons?—In 14, 35, 47, 81, 230, 653 gal.?

29. How many pints in one gallon?—In 2, 6, 13, 27, 35, 87, 237 gal.?

30. How many pints in a barrel containing 30 gallons?

54. There are 8 quarts (qt.) in a peck (pk.), and 4 pecks in a bushel (bu.). (Used for measuring fruit, grain, etc.)

31. How many quarts in 2 pecks ?—In 8, 15, 22, 49, 83, 114 pk. ?

32. How many quarts in 1 bushel ?—In 7, 19, 68, 27, 346 bu. ?

33. How many pecks in 9, 11, 59, 90, 170, 232, 566 bu. ?

34. Tell first how many pecks, then how many quarts, there are in 16, 93, 78, 136, 458 bu. ?

---

55. There are 60 minutes (min.) in an hour (h.), 24 hours in a day (d.), and 7 days in a week (wk.).

35. How many minutes in 3 hours ?—In 14, 21, 17, 23 h. ?

36. How many minutes in 4 h. 56 m. ?—In 16 h. 30 m. ?

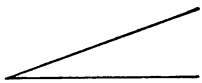
37. How many hours in 2 days ?—In 16, 15, 6, 9, 43 d. ?

38. How many hours in a week ?—In 2 wk. 3 d. ?—In 3 wk. 2 d. ?

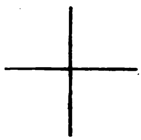
Definitions.

Note.—In the solution of some succeeding problems, the learner will need to know the following definitions:

56. The difference in direction of two lines is an ***angle.***



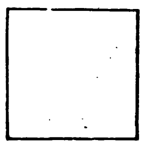
57. If the difference of direction is such that two lines, crossing each other, make four equal angles, the angles are ***right angles.*** (Right angles make square corners.)



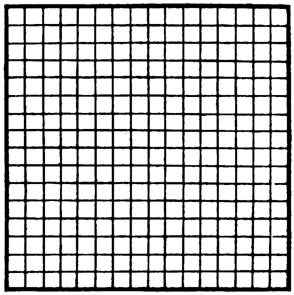
58. A ***rectangle*** is a figure having four straight sides and four right angles.

59. A ***square*** is a figure that has four straight and equal sides and four right angles.

A square inch is a square an inch long and an inch wide. A square foot is a foot long and a foot wide.



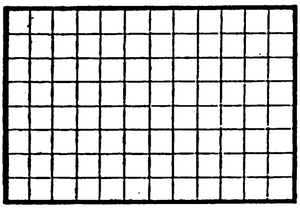
39. How can you find the number of small squares in the larger one without counting them singly? How many are there? How many would there be if it were twice as long and twice as wide?



Note.—For a home exercise let the pupils cut squares of paper measuring a yard, a foot, an inch, on each side, and of other required sizes. There should be a yard-stick in every school-room, and every student of arithmetic should have a foot-rule.

40. How many square pieces, each an inch long and an inch wide, can be cut from a piece of paper that is 4 inches long and 4 wide?

41. How many pieces, each a yard long and a yard wide, in a piece of oil cloth 6 yards long and 6 yards wide?



42. How many small squares in this rectangle? How many square inches would it contain if it were 7 inches long and 8 inches wide?

43. How many square feet if it were 15 feet long and 14 feet wide? How many square yards if it were 7 yards long and 3 wide?

44. How many square inches could you cut from a piece of silk 19 inches long and 12 inches wide?

45. How many square feet are contained in a rectangle 13 feet long and 9 feet wide? 58 feet long and 28 feet wide?

46. How many square inches in a square foot? (A square foot is 12 inches long and 12 inches wide.) How many in a square yard? (A square yard is 36 inches long and 36 inches wide.)

47. How many square feet can be cut out of a newspaper that is 4 feet long and 3 feet wide? How many square inches could be cut out of it? (See preceding Example.)

Miscellaneous Examples.

1. What is the product of 105, 106, 107 and 108?

2. What must be subtracted from 72×763 to leave 34×127?

3. What must be added to 47×436 to make 67×832?

4. Add $24 \times 8{,}277$ and $14 \times 1{,}436$, and from the sum subtract 8,763. What is the remainder?

Perform as many of the following examples as may be directed, and notice the curious results. Notice the sum of the digits in the several multipliers.

5. $45 \times 987{,}654{,}321 =$	6. $45 \times 123{,}456{,}789 =$
7. $54 \times 123{,}456{,}789 =$	8. $54 \times 987{,}654{,}321 =$
9. $27 \times 987{,}654{,}321 =$	10. $72 \times 987{,}654{,}321 =$
11. $36 \times 123{,}456{,}789 =$	12. $36 \times 987{,}654{,}321 =$
13. $63 \times 987{,}654{,}321 =$	14. $63 \times 123{,}456{,}789 =$

Note.—The intention here is not to teach the properties of the numbers, but to afford practice in multiplication. The products may awaken curiosity.

15. From the sum of 8,723, 57, 218, 9,658, 16, and 139, take the difference between 9,165 and 14,320, and multiply the remainder by 167.

16. Mr. Arnold and Mr. Bayard set out from the same place to travel in the same direction. Mr. A. travels 17 m. a day, and Mr. B. 20 m. How far apart will they be in a week (6 days)?

17. If the minuend is 16, and the remainder 7, what is the subtrahend? If the minuend is 90,087, and the remainder 26,089, what is the subtrahend?

18. Three boys together buy a peck of apples for 30¢; A. pays 9¢, B. pays 8¢, what does C. pay? Four men are in partnership with $16,876, of which A's share is $3,421, B's $2,500, and C's $5,693. How much is D's share?

19. An officer has an annual salary of $1,000; he spends $75 a month. How much does he save in one year? In 7, 9, 15, 17 years?

20. A skilled workman earned \$3 a day; he worked 304 days in one year. How much did he earn that year? He spent \$15 a week for himself and family. How much did he lay by?

21. A pipe pours into a reservoir daily 13,410 gallons of water. How many gallons in 30 days?

22. There are a dozen dozen in a gross, how many pencils in 75 gross?

23. A drover buys 12 horses at \$85 apiece, 4 oxen at \$68 apiece, 35 cows at \$56 apiece, 237 sheep at \$4 apiece. How much does he pay for all?

24. Mr. Ambros sells Mr. Dix 516 boxes of soap at \$4 a box, and buys of Mr. Dix 389 bags of flour at \$5 a bag. How much does Mr. Dix owe Mr. Ambros on the transaction?

25. If two boys can dig a row of potatoes in an hour, how long ought it to take one boy to do the same work? A piece of work can be done by 2 men in 7 days. How long would it take one man to do it?

26. A piece of work which can be done in 7 days by 45 men has to be performed by one man. How long will it take him?

27. There are 24 rows of trees in an orchard, and 24 trees in a row, how many trees are there? If there were twice as many rows and twice as many trees in a row, how many trees would there be?

28. I bought an overcoat, a vest, and a hat for \$48. The overcoat and vest cost \$42, the overcoat and hat \$41. How much did each of the three articles cost?

29. Eleven hogsheads of sugar weigh respectively 918, 923, 891, 1022, 976, 889, 1019, 948, 901, 990 and 1080 pounds. How much was the weight of the lot less than it would have been if they had weighed 1086 pounds each?

30. How many toes have 238 camels, 86 ostriches, and 453 canaries. **(Let the pupil take time to find out what he needs to know to perform this example.)**

Original Problems.

To be Composed by the Pupils for the Class.

60. 1. Give the prices of things which you have bought, or which have been bought of you, supposing yourself to be a salesman in a store, at a fair, etc., and ask the cost. (What must you give besides the prices?)

2. Problems almost without number can be made about the number of square feet or yards in the floors, ceilings, blackboard, wall-maps, etc., of the school-room, giving the pupils time to measure for themselves.

Note.—The pupils proposing such questions should first take the necessary measurements. Let them take the nearest *whole* number of feet, inches or yards, according to the length of the line measured.

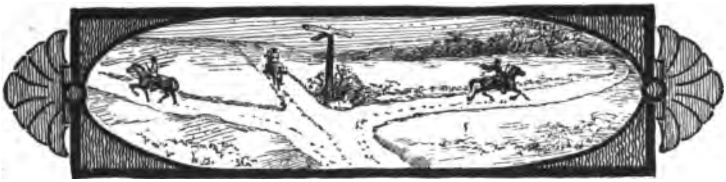
3. Give the number of rows of seats and the number of grown people that can sit in one row, and require the class to find how many can be seated in any church or hall you may name.

4. Borrow a tape-line, measure for yourself, and give the distance from one telegraph-pole to another, and tell the number of poles between any two places you may name; then ask how many feet or yards, from one place to the other.

5. Ask how many trees in Mr. ——'s orchard, after telling the class how many rows there are, and how many trees in a row. How many hills of corn in a field, etc.

6. Require to know *about* how many apples there are in a wagon-load of 50 bushels, say of greenings or any common fruit sold at market. If the class can't tell how it may be done without counting all the apples, tell them.

7. Give such problems as these, to be done in the shortest possible time, changing the numbers from those given here: What is the difference between 8 and 9 times 562? 35 and 45 times 976? Between 132 and 232 times 78? The sum of 36 times and 64 times 84? (Be sure that you see the point yourself.)



CHAPTER V.

DIVISION.

1. Write the letter *a* twenty-four times on slate or paper. How many times 12 a's are there in the 24 a's? How many times 6 a's? How many times 4 a's?

2. At 9¢ apiece, how many lead-pencils can be bought for 72¢? At 12¢ apiece? At 8¢? At 6¢?

Suggestion.—If a boy had 72 one-cent pieces, and knew no more of arithmetic than how to count, he could divide the 72 pieces into lots of 9¢ each, and thus find that for 72¢ he could buy 8 tops at 9¢ apiece, or as many tops as there are times 9¢ in 72¢.

3. With 42 ears of corn, how many horses can be fed if 6 ears are given to each? (Make 42 marks, and divide into groups of 6.)

4. In the same way, show how many dozen there are in 84, in 60, in 96, in 48. (Twelve single things make a dozen.)

5. How many top-strings can be cut from 48 feet of twine, if the strings are made 6 ft. long? If 8 ft. long?

61. A thorough knowledge of the multiplication table supersedes the necessity of marks, or other counters, except for purposes of illustration; for, if we know that twice 12 are 24, we know equally well that in 24 there are two 12's.

Note 1.—For practice at this point let a table, like the one suggested on page 53, be written on the black-board, the first column being omitted. Then a pupil pointing successively to each number in a column, and knowing that the number at the head is one factor, he announces the other; thus under 7 he announces *10, 6, 7,* etc., as rapidly as possible.

Note 2.—Here and elsewhere let counting be absolutely prohibited, except in the way of illustration.

ORAL EXERCISES.

62. Tens and Units.—*Caution.* In the following exercises do not repeat 3 in 18 six times, 3 in 30 ten times, but knowing that you are to tell how many times 3 there are in 18, 30, 27, etc., say at once *6, 10, 9,* etc.

How many times

6. 3 in	**7.** 4 in	**8.** 5 in	**9.** 6 in	**10.** 7 in	**11.** 8 in	**12.** 9 in
18	28	45	54	63	72	81
30	16	30	36	35	40	45
27	32	15	18	49	56	54
21	24	35	42	21	64	63
24	36	40	24	56	32	36
12	48	25	48	28	48	27
15	40	50	60	42	80	72

ILLUSTRATIVE EXERCISES.

Note.—The illustrative exercises introduced here and elsewhere are intended only as examples of what should be done in this direction. Problem after problem should be illustrated till the learner *feels* that in dealing with figures he is dealing with representatives of number, and that the arithmetical processes only indicate what a person ignorant of arithmetic might do to solve similar problems. Let the work be actually performed whenever possible. Labor impresses its lessons more deeply than observation. Mere instruction is not to be compared with it.

63. Hundreds.—How many times 9 in 270?

Write the letter *c* 27 times in a line, and mark them off into groups of 9 each; thus,

c c c c c c c c c, c c c c c c c c c, c c c c c c c c c.

Now, think how many c's there would be in *ten* such lines. How many times 9 c's. If you can not think the answers to these questions without writing the ten lines, write them and count.

64. Thousands.—How many times 4 in 3600?

Write the letter *e* 36 times in a line, and mark them off into groups of 4 e's each; then, think how many e's there would be in 10 such lines; in 100. How many groups of 4 in one line? *In 10 lines?* In 100 lines?

ORAL EXERCISES.

How many times

13. 3 in	**14.** 6 in	**15.** 7 in	**16.** 8 in	**17.** 9 in
210	480	560	480	450
240	360	490	400	630
270	420	630	720	810
150	300	420	880	540
180	540	700	640	720

How many times

18. 6 in	**19.** 7 in	**20.** 8 in	**21.** 9 in
4800	1400	2400	1800
5400	5600	8000	3600
3000	6300	4800	6300
1200	2800	6400	4500

How many times

22. 4 in	**23.** 6 in	**24.** 7 in	**25.** 8 in	**26.** 9 in
32	240	2100	6400	450
160	42	35	5600	63
240	5400	490	64	8100
3600	480	42	720	5400
80	4200	6300	80	720

27. How many times 10 in 80? 110? 70? 120? 60? 90? 20?

28. How many times 11 in 22? 121? 55? 132? 88? 66? 33?

29. How many times 12 in 84? 144? 72? 108? 60? 96? 132?

Applications.—30. How many dozen in 720? 600? 8400? 13200?

31. An army of 96000 men is marching 12 in a rank. How many ranks are there?

32. If pork is worth $7 a cwt., how many cwt. can be bought for $6300? $7700? $5600? 840?

33. If a sheet of paper is folded so as to make 8 pages, how many sheets will be needed for 1600 pages? 5600?

34. If a man walks at the rate of 4 miles an hour, in how many hours will he walk 48 miles? 3600 miles? 400 miles?

ILLUSTRATIVE SLATE EXERCISES.

65. Thousands, and lower orders.—35. How many balls of twine can be bought for $2.73, or 273¢, at 7¢ a ball? **(How many times 7¢ are there in 273¢.)**

Make ten lines of 27 dots each, and beneath them make 3 single dots. Divide each full line of dots into groups of seven, in this way:

(•••••••) (•••••••) (•••••••) ••••••
(•••••••) (•••••••) (•••••••) ••••••
(•••••••) (•••••••) (•••••••) ••••••
(•••••••) (•••••••) (•••••••) ••••••
(•••••••) (•••••••) (•••••••) ••••••
(•••••••) (•••••••) (•••••••) ••••••
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(•••••••) (•••••••) (•••••••) ••••••
•••

When the above work is neatly done, copy the following paragraphs, carefully filling all the blanks:

1. In each full line there are —— dots, which make —— groups (of 7), and — dots over.

2. In the ten lines there are — times as many dots, or —— dots; and — times as many groups, or —— groups; and there are also — times as many over.

3. The ten 6's over and the 3 in the short line beneath make together — dots. Out of 63 dots we can make —— groups of 7 dots each. Hence there are —— times 7 dots in 273, and —— times 7 cents in 273¢. Therefore, at — a ball, we can get — balls of twine for 273¢.

36. How many pine-apples can be bought for $27.78, or 2778¢, at 6¢ apiece?

Make 27 dots, and separate them into groups of 6 each; thus,

(● ● ● ● ● ●) (● ● ● ● ● ●) (● ● ● ● ● ●) (● ● ● ● ● ●) ● ● ●

Note.—With this single line of counters before him, the pupil should now be able to answer the following questions:

1. How many such lines would represent the number of cents expressed by the first two figures of 2778¢?

2. How many dots would there be in 10 such lines? In 100? How many groups of 6 in 10 lines? In 100? How many over in 10 lines? In 100? The 300 dots over and 78 dots would make how many dots, etc.?

(From this point the process is exactly the same as in the preceding Ex. The pupil may complete the work, beginning with the making of 37 dots, if necessary, etc.)

66. The Arithmetical Solution.—6 is not contained in 2778 any *thousands* of times, for even *one* thousand times 6 would be 6000, but it is contained some hundreds of times, for 100 times 6 is only 600. The question is, *how many hundreds* of times?

In a line of 27 dots we made *4* groups of 6 dots each, in ten lines (270 dots) there would be 40 groups, and in 100 lines, 400 groups. Hence, in the arithmetical process we write *4* in hundreds' place. It shows how many *hundred* groups of 6 can be made of 27 *hundred* dots. There are 3 *hundred* remaining. These, with the 78, make 378. How many groups can be made of 378 dots?

```
6)2778
   4..
```

In a line of 37 dots we could make 6 groups, with one dot over, and in ten lines we could make 60 groups, with 10 over. We write the *6* in tens' place. It shows how many times 10 groups of 6 each can be made of 37 *tens*.

```
6)2778
   46.
```

The ten over and the 8 make 18, in which 6 is contained 3 times. Hence, 6 is contained 463 times in 2778.

```
6)2778
   463
```

SLATE EXERCISES.

Find how many times

37. 4 in	38. 6 in	39. 7 in	40. 8 in	41. 9 in
296	258	656	784	399
348	462	483	608	432
264	354	598	527	873
192	408	651	770	778
356	516	619	664	617

How many times

4 in	6 in	7 in	8 in	9 in
42. 3124	50. 3510	58. 7056	66. 1707	74. 5373
43. 4796	51. 2578	59. 4606	67. 9608	75. 2961
44. 5340	52. 1602	60. 6110	68. 7527	76. 4212
45. 9516	53. 5958	61. 2072	69. 5768	77. 8136
46. 6779	54. 1086	62. 5408	70. 3664	78. 7008
47. 8040	55. 2802	63. 4260	71. 6856	79. 4905
48. 1596	56. 4556	64. 1645	72. 2392	80. 2220
49. 7120	57. 3062	65. 1113	73. 2216	81. 2052

Definitions.

67. ***Division*** is a process of finding how many times one number is contained in another, or of finding one factor of a number when the other factor is known.

Division has two uses:

1. To find how many equal parts there are in a number when we know what one part is.

2. To find one of the equal parts of a number when we know how many parts there are.

68. Terms used.—The ***Dividend*** is the number to be divided.

69. The ***Divisor*** is the number by which we divide.

70. The ***Quotient*** is the number of times the divisor is contained in the dividend.

71. A part of the dividend remaining undivided is called the ***Remainder.***

72. Signs.—There are three signs used to indicate division :

1. The divisor is placed on the right of the dividend with the sign ÷ between them ; 84 ÷ 7 is read 84 divided by 7 ; or,

2. The divisor is placed on the left of the dividend, with a curved line between them ; thus, 7)84 is read 84 divided by 7 ; or,

3. The divisor is written beneath the dividend with a horizontal line between them ; thus, $\frac{84}{7}$ is read 84 divided by 7.

Note 1.—Multiplication is taking one number as many times as there are units in another. Division is finding how many times one number is contained in another. Hence division is the reverse of multiplication.

Note 2.—One number can be taken from another as many times as it is contained in it, hence by division we find how many times one number can be subtracted from another.

Division of Numbers into Parts.

73. If a single thing or a number of things is divided into two equal parts or lots, the parts are called halves. Thus, if a boy shares an apple equally with a school-mate, he cuts it into two equal parts : each part is a half of the apple ; or, if he shares a number of chestnuts equally with another, he divides the chestnuts into two equal lots : each lot is a half of the whole number.

If a single thing or number of things is divided into three equal parts, the parts are *thirds ;* if into four equal parts, the parts are *fourths,* etc. Such equal parts are called fractions.

74. *One half* is written in figures thus : $^1/_2$; the figure above the line represents *one,* the figure below represents the word *half.*

One third is expressed by 1 above the line and 3 below it, thus, $^1/_3$; two thirds is written $^2/_3$; the 1 and the 2 represent the number of thirds, the 3 stands for the word *thirds.*

ORAL AND ILLUSTRATIVE EXERCISES.

1. A mother has a basket of pears which she wishes to divide equally among three children : what part of the whole number will she give to each one ? What part would be given to each if

they were equally divided between 2 children? Among 5 children? 4 children?

2. If a father divides 8 one-cent pieces among 4 children, what *part* of the number does each one receive? How *many* is that?

3. If he divides 18¢ among 6 children, how *many* does each one receive? What *part* of the whole number is that?

4. Illustrate with strips of paper, apples, or other objects, what is meant by $^1/_2$, $^1/_3$, $^2/_3$, $^1/_4$, $^2/_4$, $^3/_4$, of anything or number.

5. Show that $^2/_2$, $^3/_3$, $^4/_4$, of anything are equal to the whole of it.

6. Illustrate with objects what is meant by $^3/_4$ of 24; also,

$^1/_8$ of 32,	$^3/_7$ of 28,	$^5/_6$ of 24,
$^1/_3$ of 27,	$^2/_8$ of 56,	$^3/_8$ of 72,
$^1/_6$ of 54,	$^1/_4$ of 36,	$^4/_5$ of 50.

7. *Write in figures.*

four sixths,	one third,	three sevenths,	five eighths,
three sixths,	three thirds,	five sevenths,	eight ninths,
one sixth,	two tenths,	one seventh,	seven eighths,
two sixths,	six sevenths,	seven sevenths,	five ninths,
six sixths,	two sevenths,	six sevenths,	nine tenths,
three tenths,	four tenths,	four eighths,	three ninths,
five sixths,	six fourths,	seven tenths,	seven fifths,

The Two Uses of Division Compared.

75. The two uses of division which are represented in the following problems are often confounded. The figures employed in the arithmetical solutions, and the digits in the answers, are exactly the same for both, yet the answers are really different, and the explanations of the process by which they are obtained should vary accordingly.

1. How many times is 5 contained in 45?

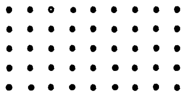
2. One fifth of 45 is how many?

The arithmetical solution to both is the same, thus

$$45 \div 5 = 9.$$

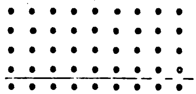
The Solution with Counters.—That the pupil may clearly understand the essential differences between the two problems, and yet why their modes of solution should be the same, let him solve them with counters, thus:

1. To find how many times 5 are contained in 45, he puts down 45 counters, arranging them in groups of 5 as he does so.



The answer to this is *9 groups* of 5, or *9 fives.*

2. To find $^1/_5$ of 45 he does precisely the same as before, *and then takes one counter from each group,* thus getting $^1/_5$ of all the groups, or $^1/_5$ of 45.



The answer to this is *9 units.*

In this way it is shown that *there are as many units in* $^1/_5$ *of 45 as 5 is contained times in 45.*

Show with counters how to find

1. $^1/_3$ of 18	4. $^1/_4$ of 36	7. $^1/_6$ of 18	10. $^1/_9$ of 54
2. $^1/_4$ of 28	5. $^1/_6$ of 42	8. $^1/_7$ of 49	11. $^1/_8$ of 72
3. $^1/_6$ of 30	6. $^1/_5$ of 55	9. $^1/_8$ of 40	12. $^1/_6$ of 54

OBJECTIVE EXERCISES.

Work out the following problems with the aid of counters, without using your knowledge of the multiplication table, and in each case state whether it is your object to find how many there are in a lot; or, how many lots there are.

1. Six gentlemen on a fishing excursion catch 48 fish, and divide them equally among themselves, how many does each one receive? How many poor families would they supply if 6 fish were sent to a family?

2. A dairyman has 96 pounds of butter to be sent to market, how many jars will he need if he puts 8 lbs. in a jar? How many pounds must he pack in a jar if he has but 8 jars?

3. A lady pays $42 for 14 yards of silk, how much does she pay a yard? If instead of the silk she buy velvet at $6 a yard, how many yards would she get for the same money?

4. A teamster hauls 9 barrels of coal oil at a load; how many loads does he make of 126 barrels? He puts them into 3 freight cars; how many in a car?

ORAL AND SLATE EXERCISES.

1. Five ninths of 54 are how many?

Analysis.—$^1/_9$ of 54 is 6; and $^5/_9$ of 54 are 5 times 6 = 30.

How many are

2. $^3/_8$ of 72	3. $^4/_9$ of 54	4. $^3/_4$ of 32	5. $^7/_{10}$ of 90
$^4/_9$ of 36	$^5/_6$ of 42	$^4/_9$ of 27	$^6/_7$ of 28
$^7/_8$ of 48	$^3/_5$ of 45	$^7/_8$ of 32	$^8/_9$ of 81
6. $^5/_6$ of 72	7. $^4/_7$ of 63	8. $^6/_7$ of 35	9. $^3/_7$ of 49
$^5/_9$ of 63	$^5/_8$ of 56	$^3/_8$ of 48	$^4/_9$ of 72
$^5/_8$ of 64	$^3/_4$ of 32	$^4/_5$ of 20	$^5/_{12}$ of 84

How many are

10. $^2/_5$ of 725	15. $^6/_7$ of 8008	20. $^4/_5$ of 18365
11. $^2/_3$ of 891	16. $^5/_9$ of 4392	21. $^7/_8$ of 93208
12. $^5/_6$ of 582	17. $^{11}/_{12}$ of 2196	22. $^5/_7$ of 98098
13. $^4/_7$ of 861	18. $^5/_7$ of 4011	23. $^7/_9$ of 35172
14. $^3/_8$ of 520	19. $^9/_{10}$ of 7680	24. $^2/_7$ of 31738

Remainders, and how to Treat them.

76. 1. To divide seven apples equally between two persons, we would divide 6, the greatest number of them that can be so divided without cutting any, and then we would cut the remaining apple into two equal parts, and give one part to each person.

In like manner the half of 7 is obtained arithmetically by first finding how many there are in the half of 6, and adding thereto one half of the remainder.

$$2)\underline{7}$$
$$3\,^1/_2$$

2. If 11 apples were to be divided equally among three children, we would divide 9, the greatest number of them that can be so divided without cutting, into 3 equal lots; then cutting

each of the two remaining apples into thirds, we would put two of the parts with each lot.

In like manner the $\frac{1}{3}$ of 11 is found arithmetically by finding first the $\frac{1}{3}$ of the greatest number in 11 that can be divided by 3 without a remainder, and then adding thereto $\frac{1}{3}$ of the remainder.

$$3\underline{)11}$$
$$3\tfrac{2}{3}$$

ILLUSTRATIVE EXERCISES.

Show by the division of two strips of paper of the same length and breadth, or by two lines drawn side by side and of the same length, that $\frac{2}{3}$ of 1 are equal to the $\frac{1}{3}$ of 2.

1. Divide 7 sticks of candy equally among 5 children. Illustrate the actual division by lines upon the slate; also perform the example arithmetically.

2. Divide 9 pencils among 4 boys; 7 yards of ribbon among 5 girls. (Illustrate as above.)

3. Divide 11 dollars among 4 persons; 13 dollars among 3 persons; 17 dollars among 5 persons. (Illustrate with counters.)

4. Divide 7 feet into 3, 9 inches into 4, 13 yards into 6, 19 feet into 7 equal parts.

5. Divide 16 min. into 5, 21 h. into 6, 39 d. into 9 equal parts.

SLATE EXERCISES.

These examples may be read thus: find the $\frac{1}{3}$ of 235; $\frac{1}{4}$ of 457, etc., etc., or how many times 3 in 235.

6. 235÷3=	**14.** 493÷3=	**22.** 567÷8=	**30.** 431÷8=
7. 457÷4=	**15.** 743÷6=	**23.** 385÷6=	**31.** 507÷8=
8. 368÷5=	**16.** 621÷7=	**24.** 745÷6=	**32.** 620÷7=
9. 279÷6=	**17.** 349÷3=	**25.** 946÷7=	**33.** 470÷7=
10. 463÷7=	**18.** 573÷4=	**26.** 853÷7=	**34.** 583÷6=
11. 749÷8=	**19.** 831÷4=	**27.** 345÷8=	**35.** 497÷6=
12. 835÷9=	**20.** 756÷5=	**28.** 700÷9=	**36.** 349÷5=
13. 458÷8=	**21.** 492÷5=	**29.** 600÷9=	**37.** 463÷4=

38. 4321÷3=	**48.** 4567÷5=	**58.** 76543÷4=	**68.** 44312÷7=
39. 5678÷4=	**49.** 8765÷4=	**59.** 49021÷5=	**69.** 57368÷8=
40. 4566÷5=	**50.** 9463÷5=	**60.** 74935÷6=	**70.** 49564÷9=
41. 8321÷6=	**51.** 4407÷6=	**61.** 68427÷7=	**71.** 87310÷8=
42. 4720÷7=	**52.** 8371÷7=	**62.** 54379÷8=	**72.** 40302÷7=
43. 5008÷8=	**53.** 9462÷9=	**63.** 48628÷9=	**73.** 50000÷4=
44. 6000÷9=	**54.** 4587÷8=	**64.** 34567÷8=	**74.** 46738÷6=
45. 4725÷8=	**55.** 5349÷9=	**65.** 50021÷7=	**75.** 27493÷3=
46. 9613÷7=	**56.** 4623÷8=	**66.** 38745÷6=	**76.** 56843÷5=
47. 7895÷6=	**57.** 7000÷7=	**67.** 74960÷5=	**77.** 74219÷8=

Applications.—1. How many pounds of beef can be bought for 1854¢, at 9¢ a pound?

2. There are 12 boys on 6 sleds; what part of the boys to each sled? A regiment of 531 men is transported in 9 cars; how many men to each car? What part of the regiment?

3. A man divides $58424 among his 8 children; how much does each one get? What part of the whole is that?

4. A carpenter cuts a strip of molding 192 inches long into 8 equal pieces. How long is each piece? Suppose he cuts it into 4 equal pieces; how long will each be?

5. There are 5280 feet in a mile, how many in 1/4 of a mile?

6. If a boy's hoop measures just 8 feet in circumference (around it), how many times will it revolve (turn round) in a half mile?

7. If I pay 15¢ for 3 lead-pencils, what part of the money do I pay for one? If I pay $15759 for 9 lots, how much do they cost me apiece? What part of the whole sum does each lot cost?

8. If a ship sails 1918 miles in two weeks at a uniform rate of speed, what part of the distance does she sail in one week? How many miles? What part of the whole distance does she sail in one day? How many miles? What part of the whole distance in two days? How many miles?

Long Division.

77. 1. How many times is 37 contained in 9386?

Here we have a divisor which does not occur as a factor in the multiplication table, hence we construct a table specially for it. Having done this, we proceed with the division exactly as we do with divisors less than 10, except, 1st, that we write down the products and remainders because too large to carry in the mind; and 2d, that we place the quotient over the dividend that it may be out of the way of the written work which is to follow.

Table of Multiples of 37.

$1\times37=$	37
$2\times37=$	74
$3\times37=$	111
$4\times37=$	148
$5\times37=$	185
$6\times37=$	222
$7\times37=$	259
$8\times37=$	296
$9\times37=$	333

Process of Division.

$$\begin{array}{r@{}l} & \underline{253\tfrac{25}{37}} \\ 37) & 9386 \\ & \underline{74\mathit{00}} \\ & 198\mathit{6} \\ & \underline{185\mathit{0}} \\ & 136 \\ & \underline{111} \\ & 25 \text{ Rem.} \end{array}$$

We first find in this table that 2 times $37 = 74$; hence we know that 37 is contained two times in 93, and therefore 2 *hundred* times in 93 *hundred.* We place the 2 in the order of hundreds (over hundreds' place in the dividend), and subtract the 74 *hundred* from the 93 *hundred*, and obtain the remainder, 19 *hundred.* Now, instead of carrying this to the 8 mentally, we annex the 8 to the 19, and thus obtain 198 tens for the second partial dividend.

Again, by referring to our table, we find that 37 is contained in 198 (tens) 5 (tens) times, this we write in the order of tens, and, subtracting 185 from 198, we get 13 (tens) for a remainder. To this we bring down the 6 units and proceed as before.

Thus we find that in 9386, 37 is contained 253 times, with a remainder of 25. This can be proved, for 253 times 37 are 9361, and the remainder, 25, being added to 9361, the sum is 9386.

Note.—The terms of the several partial dividends that are at the right of the first figure brought down, and the ciphers annexed to the several products, may be omitted in the work, since they have no effect in the result. In the process above, the figures that may be omitted are printed in italics.

SLATE EXERCISES.

Find how many times 37 in

2. 9860	**5.** 8376	**8.** 2035	**11.** 5817
3. 7935	**6.** 9098	**9.** 8980	**12.** 3542
4. 5047	**7.** 5672	**10.** 6050	**13.** 9273

Note 1.—In the following exercises, 14–40, construct a table of multiples for each divisor. These exercises can be carried to any desirable extent. The divisors remaining the same, the same table of multiples will suffice for thousands of examples. It will be well to practice the pupils in this way till they are thoroughly familiar with the process of long division. They will then find little difficulty in obtaining quotient figures without the aid of tables.

Note 2.—The table of multiples may be formed by adding the divisor to itself for the second multiple, next by adding the divisor to this sum, and so on, till the tenth multiple is reached. If this be the same as the divisor, with a cipher annexed, the result is a good test of the accuracy of the table.

Find how many times

54 in	49 in	64 in	83 in	98 in
14. 872	**19.** 658	**24.** 7856	**29.** 87506	**34.** 296725
15. 953	**20.** 9030	**25.** 4785	**30.** 84378	**35.** 875682
16. 428	**21.** 720	**26.** 9378	**31.** 59643	**36.** 937865
17. 397	**22.** 692	**27.** 2704	**32.** 23232	**37.** 468728
18. 685	**23.** 5377	**28.** 1921	**33.** 12345	**38.** 321485

39–40. How many times 98 in 5764328694761? In 358674930006?

Definitions.

78. When the steps of the solution are all written, as in the preceding examples, the process is called ***Long Division.***

79. Any part of a dividend used to obtain a quotient figure is called a ***Partial Dividend.*** (It is only a part of the entire dividend. See also *partial* product.)

80. The use of the multiple tables is convenient when we have to employ the same divisor many times successively, as in the foregoing exercises, but it would involve a great deal of unnecessary labor to construct one for every divisor we may happen

to have. Hence it is desirable to learn how to obtain a quotient figure without the aid of such a table. In doing this the learner should ask himself, "What must I multiply the divisor by, so as to obtain a product *not greater* than the partial dividend, and *not so small* that the remainder will be equal to or greater than the divisor?"

Note.—If the product is greater than the partial dividend, the term proposed for the quotient is too great, and if the remainder is equal to or greater than the divisor, the proposed term is too small.

SLATE EXERCISES.

1–5.	**6–10.**	**11–15.**	**16–20.**
$234 \div 11 =$	$536 \div 31 =$	$743 \div 62 =$	$3456 \div 51 =$
$543 \div 11 =$	$635 \div 41 =$	$634 \div 72 =$	$2345 \div 51 =$
$754 \div 21 =$	$874 \div 41 =$	$549 \div 82 =$	$3856 \div 51 =$
$638 \div 21 =$	$504 \div 52 =$	$638 \div 53 =$	$7432 \div 61 =$
$497 \div 31 =$	$970 \div 52 =$	$543 \div 95 =$	$1579 \div 61 =$
21–25.	**26–30.**	**31–35.**	**36–40.**
$3842 \div 71 =$	$3461 \div 82 =$	$234 \div 19 =$	$684 \div 69 =$
$6548 \div 71 =$	$7111 \div 73 =$	$765 \div 24 =$	$943 \div 13 =$
$7432 \div 81 =$	$9000 \div 64 =$	$801 \div 35 =$	$976 \div 25 =$
$9465 \div 81 =$	$4050 \div 55 =$	$743 \div 46 =$	$564 \div 36 =$
$4567 \div 91 =$	$6031 \div 46 =$	$257 \div 58 =$	$810 \div 47 =$

41–45.	**46–50.**	**51–55.**	**56–60.**
$240 \div 58 =$	$3654 \div 57 =$	$3579 \div 54 =$	$6492 \div 88 =$
$589 \div 69 =$	$7890 \div 65 =$	$1357 \div 29 =$	$7483 \div 73 =$
$432 \div 88 =$	$2345 \div 78 =$	$4682 \div 37 =$	$6294 \div 97 =$
$345 \div 77 =$	$7937 \div 47 =$	$7038 \div 76 =$	$7385 \div 68 =$
$678 \div 59 =$	$2468 \div 38 =$	$4925 \div 89 =$	$4291 \div 51 =$
61–65.	**66–70.**	**71–75.**	**76–80.**
$406 \div 23 =$	$500 \div 74 =$	$4000 \div 32 =$	$4000 \div 87 =$
$709 \div 34 =$	$400 \div 83 =$	$2000 \div 43 =$	$3000 \div 96 =$
$305 \div 54 =$	$300 \div 92 =$	$6000 \div 54 =$	$4000 \div 65 =$
$407 \div 56 =$	$500 \div 94 =$	$3000 \div 65 =$	$7000 \div 44 =$
$808 \div 65 =$	$800 \div 52 =$	$6000 \div 76 =$	$9000 \div 33 =$

Find how many times

81. 326 in 1630	**86.** 489 in 4375	**91.** 384 in 684
82. 251 in 2362	**87.** 981 in 982	**92.** 721 in 1223
83. 347 in 1829	**88.** 873 in 7756	**93.** 876 in 1676
84. 628 in 2654	**89.** 784 in 7830	**94.** 988 in 9875
85. 592 in 1867	**90.** 892 in 8000	**95.** 876 in 8759

96. 428 in 12415	**99.** 435 in 15781	**102.** 9321 in 993280
97. 326 in 24081	**100.** 1723 in 344680	**103.** 8746 in 785463
98. 234 in 13462	**101.** 2938 in 357264	**104.** 5932 in 593175

General Rule for Division.

81. ***Rule.*—1. Write the divisor at the left of the dividend with a right curve between them.**

2. For the first partial dividend take only as many figures at the left of the given dividend as would, if considered apart from the rest, express a number great enough to contain the divisor.

3. Find the greatest number by which you can multiply the divisor to make a product not greater than this partial dividend, and place it in the quotient.

4. Multiply the divisor by this number, subtract the product from the partial dividend, and to the remainder annex the next figure of the dividend. If the result is equal to or greater than the divisor, it is the second partial dividend, but if less, continue to annex figures from the dividend in their order, placing a cipher in the quotient for each figure brought down, till a partial dividend is formed; or, till all the figures of the dividend have been brought down.

5. Proceed with the second partial dividend as with the first, and so on, till all the terms of the dividend have been used. The result will be the quotient sought.

6. If, after the last division, there be a remainder, place it with the divisor underneath, at the right hand of the quotient.

***Proof.*—Multiply the divisor by the quotient, and to the product add the remainder, if any. The result will be equal to the dividend if the work is correct.**

Note.—The learner should write each term of the quotient over the last figure of the dividend from which it was obtained. It will save him from some mistakes to which he is liable.

Applications.—1. Distribute $13425 equally among 27 sailors. How much will each one receive?

2. If a locomotive can run 513 miles in 19 hours, how far can it run in one hour? In two hours? In 10 hours?

3. The steamer Suevia makes the trip from New York to Hamburg in 12 days. The distance is 3408 miles. How many miles per day does she make? How many in 6 days?

4. How many pounds are there in 352 ounces?

5. How many days in 3567 hours? In that many minutes?

6. How many hours in 4628 minutes? How many days?

7. If 20 horses eat 1940 bushels of oats in a year, how many will one horse eat in the same time?

8. If one boy picks a barrel of apples in an hour, what part of the time ought it to take two boys to do the same work? Three boys? If one man can dig a ditch in 54 hours, how long will it take 9 men to dig it?

9. What number multiplied by 23 will give 36087?

10. How many times can 27 be subtracted from 62397?

11. If the product of two factors is 21015, and one factor is 45, what is the other factor?

12. When potatoes are 75¢ a bushel, how many bushels can I purchase for 675¢?

13. A quire of paper has 24 sheets. How many quires are there in 5631 sheets? In 1436 sheets?

14. What is the price of a barrel of apples, if 36 barrels cost $90?

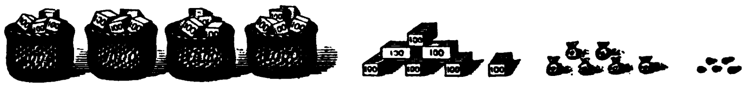
15. At a post-office there are 812 boxes in 14 rows, how many are there in a row?

16. If you weigh 1476 ounces, how many pounds do you weigh? How many pounds does your sister weigh, her weight being 133 ounces less than yours?

17. Bought 897 acres of land for $77142. How much did I pay per acre?

When the Divisor has One or more O's at the Right.

82. A boy employed at a toy-shop had to put a large number of marbles into little canvas bags, which were to be sold with the marbles. He put ten marbles into a bag, and when he had thus filled ten bags, he put them into boxes, and ten of these boxes he put into a basket to be taken to the store-room. When the work was done there were



Express the number of marbles in figures.

ILLUSTRATIVE AND SLATE EXERCISES.

1. *a.* How many *baskets* full in the lot of marbles above represented, and what would be left if they were taken away? How many marbles would remain?

b. How many thousands in 4765, and how many over?

Arithmetical Process.

1|000)4|765
4 — 765 Rem.

c. What do you notice on comparing the figures of the quotient and remainder with the figures in the dividend?

---

2. *a.* How many times 2 baskets full in the lot, and how many would remain if two times 2 baskets full were taken away?

b. How many times 2000 in 4765, and how many over?

Arithmetical Process.

2000)4765(2 Or, 2|000)4|765
4000 2 — 765
765

Note.—The *quotient* figure is the same as if 4 were divided by 2. The figures of the *remainder* are the same as the 3 right-hand figures of the dividend, which stand for the boxes, bags, and single marbles that would be left if 2 times 2 baskets were taken away.

3. *a.* How many times 3 baskets full in the lot, and what would be left if they were taken away?

b. How many times 3000 in 4765, and how many over?

Arithmetical Process.

$$3000)4765(1 \qquad \text{Or,} \qquad 3|000)4|765$$
$$\underline{3000} \qquad\qquad\qquad \overline{1-1765} \text{ Rem.}$$
$$1765 \text{ Rem.}$$

Note.—Here it will be noticed that the result is the same as if 4 were divided by 3 and the remainder prefixed to the figures cut off from the dividend.

4. *a.* How many boxes in the lot, including those in the baskets, and the single boxes represented?

b. How many times 23 boxes in the lot, and what would be left if 2 times 23 boxes were taken away?

c. How many times 2300 in 4765, and how many remaining?

Arithmetical Process.

$$2300)4765(2 \qquad \text{Or,} \qquad 23|00)47|65(2$$
$$\underline{4600} \qquad\qquad\qquad \underline{46}$$
$$165 \text{ Rem.} \qquad\qquad\qquad 165 \text{ Rem.}$$

Note.—The result is the same as if 47 were divided by 23, and the remainder prefixed to the figures cut off from the dividend.

Hence, to shorten the work of division when the lower orders of the divisor are filled with ciphers, we have the following

83. *Rule.*—Cut off the 0's of the divisor, and as many figures at the right of the dividend. Divide as if the parts left were the entire divisor and dividend. The remainder, if any, and the figures cut off from the dividend, form the true remainder.

SLATE EXERCISES.

1. 567÷40=	**7.** 4478÷80=	**13.** 6783÷80=	**19.** 84541÷80=
2. 876÷50=	**8.** 2345÷60=	**14.** 4571÷70=	**20.** 26483÷90=
3. 393÷60=	**9.** 6789÷70=	**15.** 78351÷20=	**21.** 98765÷80=
4. 584÷70=	**10.** 3456÷80=	**16.** 46228÷30=	**22.** 123456÷70=
5. 748÷80=	**11.** 7482÷90=	**17.** 57135÷40=	**23.** 700123÷60=
6. 569÷90=	**12.** 3925÷90=	**18.** 46287÷70=	**24.** 845679÷50=

Shorter Method of Computing in Long Division.

84. Many rapid accountants dispense with written products in long division. They form the remainder by writing down at once what the several terms of the product lack to make up the partial dividend.

Example.—1. How many times 956 in 3681 ?

```
     3
956)3681
    2868
     813
```

Explanation.—The work at the left exhibits the steps of the operation as already learned. How the written product may be dispensed with is shown in the work at the right, for which the following wording is a sufficient explanation.

```
     3
956)3681
     813
```

Wording.—18 and ***3*** are 21 (carry 2); 15, 17, and ***1*** are 18 (carry 1) ; 27, 28, and ***8*** are 36. Don't *say* carry 2, etc., but *do it.*

The numbers in ***heavy italics,*** occurring after the word "and," are written while they are being pronounced.

2. How many times is 217 contained in 7507083 ?

Written Work.

```
    34594 185/217
217)7507083
     997
     1290
      2058
       1053
        185
```

Explanation.—217 being contained 3 times in 750, we multiply, and write down what the product lacks to make up 750.

Wording.—21 and ***9*** are 30, carry 3 ; 3, 6, and ***9*** are 15, carry 1 ; 6, 7.

Annexing the next figure of the dividend, we have 997 for the second partial dividend ; 217 being contained in 997 four times, we set 4 in the quotient, and proceed as before.

Wording.—28 and ***9*** are 37, carry 3 ; 4, 7, and ***2*** are 9 ; 8 and ***1*** are 9.

For Practice in the Shorter Method.

How many times

3. 72 in 856	**6.** 56 in 934	**9.** 333 in 6931
4. 83 in 984	**7.** 111 in 5935	**10.** 235 in 9871
5. 87 in 899	**8.** 222 in 7356	**11.** 354 in 98763

SLATE EXERCISES.

1. 45600÷100=
2. 72400÷300=
3. 45600÷500=
4. 83600÷700=
5. 73500÷900=
6. 47400÷400=
7. 43000÷1000=
8. 234000÷4000=
9. 645000÷6000=
10. 840000÷8000=
11. 375000÷3000=
12. 4687000÷5000=
13. 99120÷ 590=
14. 85800÷ 780=
15. 207900÷ 630=
16. 10800÷ 270=
17. 2090000÷3800=
18. 1617000÷4900=

---

19. 4276÷201=
20. 5318÷102=
21. 3725÷305=
22. 4943÷406=
23. 3756÷507=
24. 35312÷342=
25. 44325÷429=
26. 73812÷368=
27. 44831÷496=
28. 34052÷504=
29. 365212÷7040=
30. 456721÷8050=
31. 843956÷9002=
32. 433421÷6302=
33. 346549÷5900=

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34. 87654÷743=
35. 94615÷685=
36. 34641÷567=
37. 64925÷784=
38. 36287÷1926=
39. 40032÷1835=
40. 50607÷1749=
41. 48325÷1683=
42. 346819÷4297=
43. 543726÷7453=
44. 492570÷6853=
45. 749256÷9469=

---

46. 3654701÷4372=
47. 2043217÷6482=
48. 4700031÷6395=
49. 6127421÷9362=
50. 5432101÷7408=
51. 4382146÷8432=
52. 7040047÷9069=
53. 2468301÷7456=
54. 7654321÷6435=
55. 5043062÷4372=
56. 3489719÷9348=
57. 7154327÷8745=

Self-Testing Problems.

Note.—Divide each dividend by all the divisors. The remainder, if any, will, in each example, be divisible by 9.

1. 53146827 ÷549
2. 61327548 ÷558
3. 128761353÷567
4. 123456789÷576
5. 987654321÷585
6. 963187452÷594
7. 712345689÷711
8. 723918645÷729
9. 791352468÷738
10. 356912478÷747
11. 981762345÷756
12. 765432189÷765
13. 781965423÷774
14. 783993456÷783
15. 792345681÷792
16. 829713456÷828
17. 846123579÷ 846
18. 864123597÷ 864
19. 709005474÷ 882
20. 470049570÷ 918
21. 357114636÷ 936
22. 987654321÷ 954
23. 976548321÷ 972
24. 981234567÷ 981

Applications.—1. If a clock ticks 29,347,095 times in a year of 365 days, how many times does it tick in a day?

2. Divide nine million nine hundred ninety-eight thousand five hundred fifty-seven by seven thousand eight hundred forty-two, and write out the answer in words.

3. The Valley Railroad is 271 miles long, and cost $5,272,305. What was the cost per mile?

4. If a farmer had 138 acres in wheat, from which he harvested 3692 bushels, how many bushels did he raise per acre?

5. A milliner cuts 7 pieces of ribbon, each 10 yards long, into pieces each 27 in. long. How many such pieces has she, and how many and how long are the waste pieces?

6. In a week a boy gathers 192 bushels of apples; how many bushels does he average per day?

7. A farmer raises 1875 bushels of wheat, which he exchanges for flour, at the rate of 5 bushels of wheat for one barrel of flour. How much flour does he receive?

8. Find how many gallons of milk in 8 cans that hold respectively 92, 102, 170, 89, 97, 125, 106, and 56 pints?

9. Suppose that two cans of equal size together hold 376 pints; how many gallons are there in each?

10. How many poor families may be supplied from 37 barrels of flour, allowing 28 pounds to each, a barrel of flour weighing 196 pounds?

11. If you take 86 steps in a minute, how many hours and minutes will it take you to walk 38,270 steps?

12. A train of 28 cars carries 493,920 pounds of flour in barrels, each barrel containing 196 pounds. How many barrels to each car?

13. In 3 weeks, a certain oil-well is said to have produced 35,000 barrels of oil. How much was that per day?

14. How many thousand make one million?

Original Problems.

Note to the Teacher.—Let the yard-stick and foot-rule be as freely used as the circumstances of the school will allow. If the foot or yard measure does not "come out even," let dimensions be given in inches, but let no account be taken of the fractions of an inch at present. No pupil should be allowed to give a problem which he has not solved himself, and the answer to which he does not know.

1. Suppose yourself to be building a pile of cubic blocks, each measuring one inch, foot, or yard on each side, the pile to be as many inches, feet, or yards, on each side, as you please, and ask to know how high you can build it with a given number of blocks.

2. If you see an oil-cloth or carpet with square figures covering a floor, measure one square, count the number on the side and end of the room, give the facts to the class, and ask them to find how long and wide the floor is.

3. Ask questions similar to these : How many states of the size of Delaware might be made out of the state of Georgia? How many cities of the population of Albany (N. Y.) might be made of the city of New York? The members of the class should hunt up the facts for themselves.

4. Give the height of one step of a stair-way, and the distance from the first to the second *floor*, second to third, etc., in some house just building, and ask how many steps will be needed.

5. State the cost of any number of things, as yards of cloth, horses, etc., etc., and the price of one to find the number. State the cost and the number, and ask the price of each.

6. Construct questions about changing hours to weeks, equal parts to wholes, etc.

7. How many days sail from —— to —— for a vessel which runs — miles per day? How many hours run for a railway train from —— to ——, at — miles per hour? (Find distances from your text-book in geography, and rates of sailing from your friends.)

8. A railway train goes from —— to —— in — hours. How many miles an hour?

Note.—The newspapers often suggest interesting problems.

Principles of Division.

85. A convenient number of counters being equally distributed among 6 or 8 members of a class, let the following questions be proposed:

1. If there were twice as many counters equally distributed among the same pupils, how would each one's share be changed? If there were only half as many counters?

2. If the same number of counters had been distributed among twice as many members of the class, how would the share of each be changed? If among half as many?

3. If twice the number of counters had been given to twice as many members of the class, how would the share of each be changed? If one-half as many had been given to half as many persons?

1. How does it affect the value of a quotient to multiply the dividend by 2, by 3, by any number, while the divisor remains unchanged? To divide the dividend by 2, by 3, etc.

2. How does it affect the value of a quotient to multiply the divisor by 2, by 3, by any number, while the dividend remains unchanged? To divide the divisor by 2, by 3, etc.

3. How does it affect the value of a quotient to multiply divisor and dividend by the same number? To divide both divisor and dividend by the same number?

ORAL EXERCISES

1. How many 13's in 78? In 5×78? In $^1/_2$ of 78? In 9×78? In $^1/_3$ of 78?

2. How many times is 18 contained in 360? $^1/_2$ of 18 in 360? $^1/_6$ of 18 in 360? 4×18 in 360? $^1/_9$ of 18 in 360?

3. How many times 24 in 96? 5×24 in 5×96? $^1/_2$ of 24 in $^1/_2$ of 96? 24×24 in 24×96?

4. Divide 224 by 28; 224 by 2×28; 224 by $^1/_4$ of 28; $^1/_2$ of 224 by 28; 3×224 by 3×28; $^1/_7$ of 224 by $^1/_7$ of 28; 13×224 by 13×28.

SLATE EXERCISES.

How many times

1. 3119 in 1197696
2. 24×3119 in 1197696
3. 3119 in $1197696\div24$
4. 4316 in 1031524
5. $4316\div52$ in 1031524
6. 4316 in 1031524×52
7. 4316 in $1031524\div52$
8. 4316×52 in 1031524
9. 4316×718 in 1031524×718
10. $4316\div52$ in $1031524\div52$

Division by Composite Numbers.

86. Let the pupil show, by various examples, that division by a composite number (product of two or more factors) may be performed by dividing successively by its factors. Thus, that

$$\begin{array}{r|l} 7 & 756 \\ 9 & \underline{108} \\ & \;\;12 \end{array} \quad \text{is equivalent to} \quad \begin{array}{l} 63)756(12 \\ \;\;\;\;\underline{63} \\ \;\;\;\;126 \\ \;\;\;\;126 \end{array}$$

Divide in both ways

1. 18576 by 48
2. 37656 by 72
3. 30375 by 81
4. 24678 by 54
5. 391272 by 56
6. 629937 by 63

Why the results of the two methods should be the same, and how to deal with remainders when they occur in the division by factors, is shown in the solution of the following example.

7. Divide 59 by 42.

Solution with Counters.—In 59 counters there are 29 twos and 1 counter remaining; in 29 twos there are 9 sets of 3 twos and 2 twos over; in 9 sets of 3 twos each there are 1 group of 7 sets and 2 sets of 3 twos remaining; all of which is shown as follows.

Arithmetical Process.

$$\begin{array}{r|l} 2 & 59 \\ 3 & \overline{29}\text{—}1 \ldots\ldots\ldots\ 1 \\ 7 & \;\overline{9}\text{—}2\times2=4 \ldots\ 4 \\ & \;\overline{1}\text{—}2\times3\times2=12 \\ & \text{Remainder } \overline{17} \end{array}$$

Explanation.—As may be seen in the illustration, the first divisor is 2 units and the remainder is a *unit;* the second remainder is *2 groups* of 2 each, and the third is *2 larger groups* of 3 twos each. The sum of these remainders is 17, the same as that obtained by dividing 59 at once by 42. Thus we obtain the rule for dividing any number by the factors of composite divisors.

87. ***Rule.***—**1. Divide the dividend by any factor of the divisor, divide the quotient by another factor, and so on, till an entire set of factors has been employed.**

2. If remainders occur, multiply each by all the divisors preceding the one that produced it. The sum of the products, added to the remainder, if any, resulting from the first division, will be the true remainder.

Self-Testing Exercises.

To be Constructed by the Teacher for Dictation.

Addition and Subtraction.—1. Write any set of numbers, each of which shall be greater than the preceding, as, for example,

83, 237, 250, 592, 728, 851, 9872, 18589.

Subtract the first from the second, the second from the third, etc. To the sum of the remainders add the first number. If the work is correct, the sum will equal the last number.

Multiplication.—2. Take any set of numbers the sum of which is 10, 100, 1000, etc., multiply each by any given number, and add together the products. (See examples 7–18, p. 64.)

Division.—3. Take *17* and *19*. Annex *4* ciphers to each. Divide each number thus formed by the sum of *17* and *19*. If the sum of the quotients, including fractions, be expressed by 1, with *4* ciphers annexed, the work is correct.

4. Separate any number expressed by 1 with *4* ciphers annexed into any two parts, each represented by *4* figures. Take any two convenient smaller numbers, as *29* and *38*. Prefix the *29* to either of the former numbers, and the *38* to the other; thus,

*29*3276 and *38*6724 or *38*3276 and *29*6724.

Divide both numbers of either pair by *68*, that is, the sum of *29* and *38* *increased* by 1. The test of accuracy is the same as in 3.

The last method being somewhat complicated, the following additional example is given. We divide by the shorter method for the sake of space.

Explanation.—We take 78314 and 21686 as the two parts of 100000, and prefix 43 to one and 65 to the other. Then we divide both by the sum of 43 + 65 + 1.

$$\begin{array}{r} 40168\frac{2}{109} \\ 109)\overline{43,78314} \\ 183 \\ 741 \\ 874 \\ 2 \end{array} \qquad \begin{array}{r} 59831\frac{107}{109} \\ 109)\overline{65,21686} \\ 1071 \\ 906 \\ 348 \\ 216 \\ 107 \end{array}$$

$$\begin{array}{r} 40168\frac{2}{109} \\ 59831\frac{107}{109} \\ \hline 100000 \end{array}$$ (The teacher adds the quotients.)

Note.—In 3 and 4, other numbers may be substituted for those in italics.

CHAPTER VI.

MISCELLANEOUS EXAMPLES.

Addition, Subtraction, Multiplication, and Division.

1. George Washington was born in 1732, and was 67 years old when he died; what was the year of his death? Abraham Lincoln was born 77 years later than Washington; when was he born? President Lincoln lived 56 years; in what year was he killed?

2. In what number is 244 contained 28 times?

3. How many strokes does the hammer of a clock make from 1 till 12 o'clock, if it strikes only the hours? How many in a day?

4. A man died leaving $5200 to his wife and three children. The widow received $2500, and the children shared the rest equally. How much did each one receive?

5. A dealer proposes to ship 100000 eggs in boxes containing 40 dozen each. How many boxes will he require?

6. One hundred and thirty-eight boxes of equal capacity contain 76176 eggs. How many dozen eggs in each box?

7. If I pay 45¢ for lead-pencils, at 3¢ each, how many pencils do I buy? How many if I pay 5¢ each?

8. A drover has $150. How many cows can he buy at $50 each? $25 each? How many could he buy at $45 each, and how much would he have left?

9. A son is born when his father is 33 years old. When the father is 36 years old, how *much* older is he than the son? How many times as old?

10. Twenty-four sheets of paper make a quire. How many quires in 1824 sheets? In $^1/_2$ as many sheets? In $^1/_4$ as many? In 3 times as many? In 5 times as many?

11. How many hours are there in 9480 minutes? In twice as many minutes? (Always find your answer in the shortest and most convenient way.)

12. How many days are there in 14088 hours? In ten times 14088 hours?

13. Out of 796 logs 3980 planks were sawed. How many planks were cut from each log, supposing them to have been of equal size?

14. Four boys agreed to sweep a school-house two weeks for $24, but at the end of the first week, three of them gave it up, and left the remaining boy to complete the work. How much should the last boy receive? How much each of the others?

15. The managers of an orphan asylum spend $239 per year for each child. The expenses one year were $7170. How many orphans in the asylum that year?

16. The manager of a concert sold 534 tickets at $1 apiece, 936 tickets at $2 apiece, and 257 at $3 apiece. He gave out 34 free tickets. The hall cost him $120 rent, and for gas and fuel he paid $19 extra. How much was left after all expenses were paid, including $2100 for the performers?

17. A congregation intends to build a church, which is to cost $12000. The collections already made are $524, $726, $837, $632, $439. How much is lacking?

18. Mr. Brown earns $28 while Mr. Black earns $15. How much will Mr. Brown earn while Mr. Black earns $105?

19. Express in words the product of the sum and difference of 8765 and 5678.

20. A train of 9 cars has in each car 63 passengers, of whom 4 are children. How many passengers altogether, how many adults, and how many children?

21. January 4th, paid into savings bank, $14; February 1st, paid in $13; February 28th, drew out $11; March 14, paid in $19; March 31st, drew out $25; April 24th, paid in $17; May 3d, paid in $9; May 25th, drew out $15; June 1st, paid in $16. How much had I then in bank?

22. A number of boys in a work-shop earn $7 each per week, and an equal number of younger ones earn $5 each per week. How many boys are there if their wages amount to $132 per week?

Suggestion.—Suppose they work in pairs, an older and a younger boy in a pair, how much would a pair receive? How many pairs?

23. How many times can a 5 gal. pail be filled from a cask containing 150 gal.? How many times from a cistern holding 12 such casks? 24 casks?

24. On Tuesday the Opera was attended by 2486 persons; on Wednesday by 3574 persons. How much more money received on the second day than on the first, at $2 per ticket?

25. Which is the greater, and by how much, one fifteenth of 645, or one sixteenth of 992?

26. If a man takes 92 steps in a minute, how far will he walk in 3 hours if he advances 5 feet in taking 2 steps? At the same rate, how far would he travel in 2 days of 9 hours each?

27. A railroad conductor makes two trips every day (except Sunday) from Philadelphia to New York and back. If these cities are 90 miles apart, what distance does he travel in a week? In a year, if he has two weeks vacation?

28. How many times will a cart-wheel, 16 feet in circumference, revolve in going a mile (5280 feet)?

29. A drover paid $780 for cows and sheep. Of this sum he gave $360 for 9 cows. If a cow cost 8 times as much as a sheep, how many sheep did he buy?

30. How many feet of telegraph wire are needed to connect two stations with a double line, the stations being 37845 yards apart? How many poles would be required if set 45 yards apart?

31. A healthy child's pulse beats 78 times a minute. How often does it beat in an hour? In 4 hours? In 8 hours?

32. Find the smallest number that must be subtracted from 9904, to leave a number that can be divided by 173 without remainder.

33. There are 35 regiments in an army, averaging 693 men in each; how many men in the army?

34. If a man earns $3 a day, how many weeks will it take him, working 6 days a week, to earn $567? $681?

35. A man bought a piece of land for $1564; he built a house on it for $642, a barn for $273, and made other improvements costing $148. He then sold it for $3000. How much did he gain?

36. A railroad 270 miles long has a station every 10 miles; how many stations has it? (There must be a station at both ends of a road.)

37. What is the length of a railroad that has 18 stations, at an average distance of 17 miles apart?

38. A family uses 7¢ worth of milk a day. What was the cost of the milk used the last 4 months of the year? (See 27, p. 49.)

39. If a man pays 7¢ a year for the use of a dollar, how much does he pay for the use of $5 for one year? Of $10, $50, $90, $200, $750? How much for each for a half year?

40. If a man pays 6¢ for the use of a dollar for one year, how much does he pay for the use of $16 for one year? How much for 5, 7, 10 years?

41. If a man pays 8¢ for the use of one dollar for a year, how many dollars can he get the use of, for a year, for 24¢, 32¢, 44¢, 56¢, 68¢, 74¢?

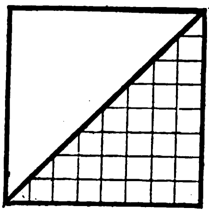
42. Sixteen messenger boys are employed to carry telegraphic dispatches, and receive 2¢ for each dispatch carried. How much does a boy earn in a week who carries 45 dispatches per day? How much do the 16 boys earn in a week if each one averages 38 dispatches per day?

43. The distance from New York city to the Cape of Good Hope is 6670 miles. When a steamer from New York is 957 miles on its way, and a steamer from the Cape is 829 miles on its way, how far apart are they, if both are on the direct line between the ports?

44. A farm has 5 fields, the 1st containing 89 acres, the 2d 101, the 3d 174, the 4th 92, and the 5th 72 acres. If the farm be re-divided into five fields of equal size, how many acres will each contain?

45. If two boys together have 9 apples, and one has 3 more than the other, how many has each? Two candidates together receive 3579 votes. How many has each, if one is elected by a majority of 291?

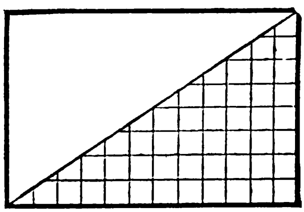
46. If a square sheet of paper be cut, from corner to corner, into two equal parts, we shall have two three-sided pieces, called ***Triangles,*** each of which is one half of the square. From this hint can you reckon how many square inches there are in a triangle such as the one at the left, if the base (the side on which it stands) measures 8 in. and the height 8 in.?



47. How many square inches would such a triangle contain if the base and height were each 87 in.?

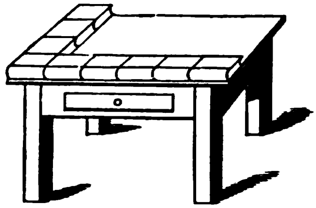
48. If the above figure represents a triangular piece of ground, the base of which is 42 ft., and the height the same, how many square feet does the lot contain?

49. Now, suppose we had such a triangle as the one at the right; how many square inches would it contain if the base were 12 inches and the height 8 inches?



50. How many square yards does a triangle contain, the base being 672 yards, and the height 84 yards?

51. Suppose that you can lay 4 and 6 books along the edges of a table, as in this engraving; how many books can you lay on the table in one layer covering the top? How many times 6 books? How many times 4 books?



52. How many could you place on the table in 2 layers, 5, 7, 10, 12, 16, 24 layers?

53. If you could lay 9 books end to end along the side, and the width of the table were 5 times the width of the book, how many books could you put on the table in 1 layer, 7, 9, 25 layers?

54. How many books can you place on a table that is twice the length and three times the width of a book, if you make the pile 15 books high?

55. How many books can you put in a pile 9 books long, 7 books wide, and 31 books high?

56. How many square blocks, 6 inches long and wide, can be piled on a platform 72 inches long and 48 inches wide, if the pile is made 30 blocks high?

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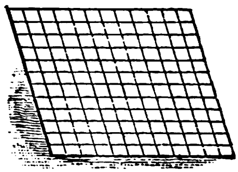
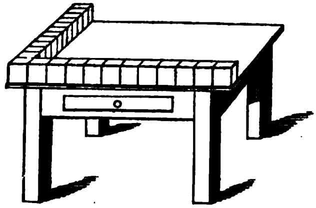
57. This is the picture of a square board divided by lines into small squares, each supposed to be 1 inch long and 1 inch wide. How many inches long is the board? How many inches wide? Count. How many squares on the upper edge? How many rows of squares? How many squares on the whole board?



58. How many square yards in a lot 23 yards long and 10 yards wide? In one 17 yards long and 13 yards wide?

59. How many square yards in a lot 20 yards wide and 30 yards long? In one 180 feet long and 270 feet wide? (How many yards in 180 feet? In 270 feet?)

60. If the blocks represented in the engraving are an inch long, an inch wide, how many inches long and wide is the table? How many such blocks can you place in 1 layer if you cover the top of the table?



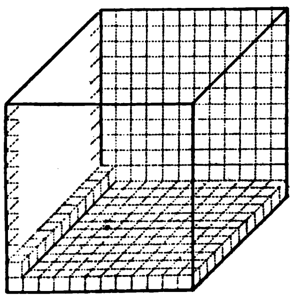
61. How many in 5, 7, 12, 9, 13, 4, 15, 6 layers?

62. Suppose the table to be 3 feet long and 3 feet wide, how many blocks, a foot long and a foot wide, can you place exactly on the front and left edges of the table? How many would exactly cover the top? How many feet high would you have to make the pile of blocks to make it as high as it is long and wide? How many blocks, each a foot high, would there be in such a pile?

63. Could you tell without counting, how many blocks there would be in $\frac{1}{3}$ of the pile? $\frac{2}{3}$?—How many blocks would there be in $\frac{1}{3}$ of one layer? $\frac{2}{3}$ of one layer? $\frac{1}{3}$ of 2 layers? $\frac{2}{3}$ of 2 layers?

64. If with blocks, each 1 foot long, 1 foot wide, and 1 foot high, you make a pile 12 feet long, 10 feet wide, and 6 feet high, how many blocks will there be in one layer? How many in the pile?

65. How many blocks, each an inch long, wide, and high, can be placed in a box measuring on the inside 12 inches in length, width, and depth? If it measures 24 inches each way? 36 inches each way?



66. How many cubic blocks, measuring a foot each way, can be sawed out of a block of stone measuring a yard each way?

Note.—A block measuring a foot long, a foot wide, and a foot high, is called a *cubic foot.* Cut a *cubic inch* out of a piece of wood, or make one of clay. Each side of a cubic inch is a *square inch.* Each edge is an inch long.

67. How many blocks, measuring 1 inch each way, can be cut out of a block of clay measuring 12 inches each way?

68. Find how many cubic feet in a block of marble 5 feet long, 3 feet wide, and 2 feet thick.

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69. A swallow flies on an average 2640 yards per minute. How many miles does it fly in 4 hours?

70. A pipe discharging 9 qt. of water every minute fills a reservoir in 4 hours. How many gallons does the reservoir hold?

71. A milkman brought 12 cans of milk into town. Three of these contained 8 gallons each, 5 contained 18 gallons each, and the others together contained 47 gallons. How many pints of milk did he bring to town?

72. A gentleman once said: "If I had saved only 5¢ a day, since I was 20 years old, my savings would now amount to $730." How many years old was he? (How many cents in $730?)

73. A mill-wheel makes 4 rotations in a minute. How many hours must it revolve to make 1800 rotations?

74. In the center of a room, 12 feet square, lies a rug 6 feet long and 5 feet wide. How many square feet of the floor is uncovered? (Draw a diagram.)

75. A garden, in the form of a rectangle, is 72 yards long and 50 yards wide. In it stands a green-house 22 feet long and 12 feet wide, and a summer-house 10 feet long and 10 feet wide. How much space of the garden is not under cover?

76. Mr. Southworth received 15 boxes of tea, the boxes and the tea together weighing 570 lb. How much did the tea in each box weigh, if the empty boxes weighed each 3 lb.?

77. $2000 is to be distributed among 3 persons. Mr. A. is to receive $^1/_5$ of it, Mr. B. $^1/_4$, and Mr. C. the remainder. How much will each one receive?

78. A certain book contains 35220 lines. How many days will it take you to read it through, at the rate of 587 lines a day?

79. If I give 153 barrels of flour, worth $6 a barrel, in exchange for 54 acres of land, how much do I pay per acre for the land?

80. In a certain year Mississippi produced 997,576 bales of cotton (450 lbs. to the bale). What was its value at 11¢ a pound?

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81. Four boys at a picnic make the following inventory (list) of their property:

William	has	10¢,	20 marbles,	5 arrows,	6 crackers,	4 apples.
Monroe	"	18¢,	16 "	3 "	5 "	2 "
Harry	"	6¢,	50 "	9 "	7 "	2 "
James	"	22¢,	18 "	7 "	2 "	4 "

Find the average number of cents, the average number of marbles, the average number of arrows, etc., which they have; or, in other words, find how much money each would have if the property were equally divided among them. How many marbles, arrows, etc.?

82. The same boys find the weight of each to be as follows: William's, 85 lb.; Monroe's, 78; Harry's, 86; James's, 92. What is their average weight?

83. A butcher buys hogs weighing severally 268, 372, 283, 356, 289, 316, 328 lb. What is their average weight?

84. What is the average value of the hogs at 4¢ per pound? What is the value of the 7 hogs at the rate thus found? Find the value of the hogs separately, and add them together.

85. Four boys wish to find the average age of their fathers. William's father is 52; Monroe's, 48; Harry's, 45; and James, who has made the calculation, says that his father's age makes the average age of the four just 49 years. How old was James's father?

86. The population of Cleveland was 92,829 in 1870; ten years later it was 160,146. What was the average increase per year?

87. Bought 5 yd. of silk, at \$2 per yd.; 13 shirts, at \$2 apiece; 6 pairs of socks, at \$0.50 a pair; and gave the merchant a \$50 bill. What change did I get?

88. One omnibus contains 23 persons, another 32, and a third 26. If two persons leave one of them, and 11 are taken up on the way, how can the party be so divided that the omnibuses shall hold equal numbers?

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89. Add 352, 6324, 497, and 723; subtract 3647 from the sum; multiply the remainder by 84, and divide the product by 114. What is the quotient?

90. Add 2839, 44051, 6273, 78495, 9617; multiply the sum by 27, and from the product subtract the sum of the following numbers: 1827, 3929, 8272, 13764.

91. How much is $^1/_2$ of 96?—$^1/_4$ of 96?—$^1/_8$ of 96? How much is $^3/_4$ of 9856?—$^5/_8$ of 7656?—$^8/_9$ of 3618?

92. From 1000 subtract 367. Multiply 582 by the subtrahend, also by the remainder. Add together the products. Can you find what the sum should be without performing the work in full?

93. Divide the sum of the products of (56×37) and (44×37) by 100, and tell why the digits of the answer should be the same as those of the multiplicand.

94. The great bridge from New York to Brooklyn is suspended from 4 cables, each composed of 6300 wires, each 3578 feet long. How many feet would all these wires extend if laid end to end? How many yards? How many miles? (5280 feet = 1 mile.)

95. If laid end to end, how many times would the wires of the bridge extend from New York to Philadelphia (90 miles)? From New York to Chicago (980 miles)? From New York to San Francisco (3400)? (Work in miles, reject remainders.)

96. In 90 years the total population of the United States increased from 3,929,214 to 50,155,783. What was the average increase per year? Per decade? (A decade is 10 years.)

CHAPTER VII.

UNITED STATES MONEY.

88. The money of the United States consists of coins made at the United States mints, and government or bank notes duly authorized by law.

Coin is the specie or metallic currency, and notes are the paper currency, of the country. (Compare the words current and currency.)

The pupils should be required to write for themselves a list of the coins used, being left to get the information as they can.

89. The following are the denominations of money used in business and in accounts:

10 Mills = 1 Cent, 100 Cents = 1 Dollar.

This table is the correct business form, but the decimal character of our currency is better shown in the following table:

10 Mills = 1 Cent, 10 Cents = 1 Dime, 10 Dimes = 1 Dollar.

The word *mill* is from the Latin *mille, a thousand* (1000 m. = $1). *Cent* is from the Latin *centum, a hundred* (100¢ = $1). *Dime* is from the old French word *disme, ten* (10 dimes = $1).

90. The sign for dollars is $; parts of a dollar are indicated by a point called a separatrix. Both signs are prefixed to the figures to which they belong.

Dollars are read as one number, the first two places to the right of the separatrix as cents, the third as mills.

Thus, $32.875 is read 32 dollars, 87 cents, 5 mills.

Note.—For illustrations the *hundreds* of jackstraws already used will serve to represent dollars; the *tens*, dimes; *single sticks*, cents; and *tenths of a stick*, mills. There is plenty of room for devising other and better illustrations.

ORAL AND SLATE EXERCISES.

1. Read $1.00, $3.05, $5, $1.25, $2.75, $6.50, $4.16, $15.13, $16.12, $71.10, $35.80, $70.65, $40.50, $38.15, $49.35, $586.50, $9828.75, $16782, $0.25, $0.05.

2. Read $1.125, $3.367, $6.875, $4.126, $19.625, $18.163, $3.185, $72.05, $1.05, $1.055, $1.47, $0.876, $2.10, $342.21.

3. Write in figures, and without the aid of words, one cent, two cents, etc., to fifty cents. (Use the sign $, thus $0.01 or $.01. Arrange in columns.)

4. Write 7 dols. 6¢ 3 m., 12 dols. 70¢ 8 m., 88 dols. 60¢ 7 m., 100 dols. 9¢ 9 m., 3426 dols. 1¢ 5 m., 4870 dols. 30¢ 5 m.

Model.—7 dols. 6¢ 3 mills = $7.063.

5. Write in words, 326, $326, $326.01, 26, $0.26, $26.00, $2.60, 845, $8.45, $84.05, $0.845, $16, 160, $0.169, $16.05, $1.605. (Write abstract numbers and sums of money in separate columns.)

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6. How many cents in $1, $2, $3, $4, $5, $9, $6, $8, $7, $10, $11, $15, $17, $25, $81, $67?

7. How many cents in $1, $1.05, $1.10, $2, $2.25, $3.75, $4.85, $6.10, $4.17, $8.25, $10.20, $1.99, $16, $16.87, $35.50?

8. Write the following as cents: $1.75, $6.25, $7.35, $5.45, $7.95, $6.87, $7.21, $18.38, $19.55, $18.76, $27.87, $98.63.

Model.—$21.83 = 2183¢. Read: 21 hundred 83 cents.

9. Read the following as hundreds of cents and cents, and again as dollars and cents: 1675¢ (16 hundred 75 cents, and 16 dollars 75 cents), 500¢, 682¢, 956¢, 1871¢, 1956¢, 4563¢, 627¢, 1856¢, 3983¢, 9987¢, 8672¢, 91374¢.

10. Write the following, separating the dollars from the cents, and use the proper signs: 782¢, 396¢, 950¢, 680¢, 721¢, 354¢, 725¢, 875¢, 1982¢, 1678¢, 2585¢, 4398¢, 1875¢.

Model.—87638¢ = $876.38. (The signs ¢ and m. must be omitted when the dollar mark and the separatrix are used.)

11. How many mills in 1¢, 2¢, 3¢, 4¢, 9¢, 5¢, 8¢, 6¢, 7¢, 10¢ ? Is the mill coined ?

12. How many mills in 20¢, 70¢, 30¢, 60¢, 40¢, 90¢, 50¢, 15¢, 36¢, 58¢, 42¢, 63¢, 51¢ ?

13. How many mills in 100¢, 300¢, 600¢, 400¢, 120¢, 150¢, $1, $2, $3, $16, $9, $1.20, $1.50, 45¢, 450¢, $4.50, $9, $10, $15, $19, $18.25, $25.62, $342.15, $2100.50 ?

14. *Express as mills :*

17¢	163¢	$1.75	$15	$15.12	$185.16	$232.132
20¢	245¢	$2.25	$35	$23.19	$198.21	$830.012
25¢	350¢	$3.50	$78	$62.20	$321.15	$700.019
83¢	425¢	$4.05	$96	$70.00	$568.00	$109.346

Models.—16¢ = 160 m.; $1.35 = 1350 m.; $275.18 = 275180 m.

15. Read the first three columns below as cents, or as cents and mills ; the last three as dollars, cents, and mills :

10 m.	15 m.	152 m.	2645 m.	1325 m.	20175 m.
20 m.	26 m.	235 m.	1875 m.	2515 m.	38625 m.
40 m.	55 m.	428 m.	1625 m.	7628 m.	55825 m.
50 m.	32 m.	632 m.	4935 m.	4932 m.	70362 m.

16. Write the sums expressed above, using the dollar sign and the separatrix.

Model.—10 m. = $0.01. 70362 m. = $70.362.

91. By this time the pupil will perceive that any sum of United States money may be read in different ways. For instance, $1 may be read as 100¢ or as 1000 m.—$16.85 may be read as written, or as 1685¢, or 16850 m.

92. The whole process of changing one denomination in United States money into another consists in annexing 0's, or in placing or removing the separatrix. If any denomination lower than dollars is to be expressed, cents, including dimes, must always have *two places*, even though they be filled with 0's. If the number of cents be less than 10, the figure next to the separatrix must be a 0. It is very important to remember this.

Addition and Subtraction of United States Money.

93. The following rule needs no introduction:

***Rule.*—Write the numbers to be added or subtracted so that the separating points shall be in a vertical line. Then proceed as in addition or subtraction of simple numbers.**

SLATE EXERCISES.

1.	2.	3.	4.	5.
$ 3.75	$ 7.30	$ 3.25	$ 51.32	$661.50
4.52	13.15	114.29	46.39	720.70
13.30	2.99	50.01	54.91	43.35
25.09	48.75	36.36	284.03	438.97

Write in columns, and add:

6. $64.22+$1.01+$6.93+$101+$0.28+$57.05+34.01.

7. $242.32+$4.00+$91.35+$628.21+$5.05+$40+$34.10.

8.	9.	10.	11.	12.
$752.24	$324.40	$ 13.80	$13428.79	$ 12.00
6.23	276.19	14.60	12469.31	10.00
15.62	1328.45	160.32	3872.84	960.05
103.78	9327.71	369.49	92476.75	200.91
923.60	1000.01	765.30	1328.62	1342.78

Find the sum of

13. $3.421+$71.243+$62.103+$5.009+$34.24+$6.328.

14. $20.21+$342.109+$34.497+$603284.679+$9.384.

15.	16.	17.	18.
$ 12.345	$ 10.102	$793.104	$ 1.000
67.890	1.000	38.796	342.060
35.719	100.309	596.647	3.501
864.246	540.701	764.521	917.624
395.713	724.309	990.901	249.387
54.575	99.830	881.725	1800.000
983.600	786.250	9651.062	71.875

19. Add 17 dols. 30¢ 4 m., 123 dols. 45¢ 7 m., 129 dols. 37¢ 9 m., 98 dols. 11¢ 6 m., 59 dols. 24¢ 7 m., 3482 dols. 99¢ 9 m.

20. Find the sum of \$5.097 + \$0.083 + \$5.629 + \$15.912 + \$4.691 + \$59.03 + 16.001 + \$18.875.

21. Find the sum of \$0.187 + \$9.987 + \$4.46 + \$0.365 + \$73.28 + \$83.18 + \$1. + \$5000 + \$3.057 + \$15.01.

22. Add five hundred sixty-nine dols. forty-seven cents, seven thousand two hundred ninety-eight dols. sixty-three cents five mills, ninety-six dols. fifty-three cents seven mills, one hundred dols. forty-five cents seven mills, nine hundred dols. ten cents, one dol. fifty cents.

Applications.—23. A gentleman owes the following: To Mr. Brown, \$16.50; to Mr. Jones, \$79.54; to Mr. Thomas, \$82.35; to Mr. Alfreds, \$46.83. What is the sum of these debts?

24. Mr. Willey gave the following sums to be used for charitable purposes: \$3470.25; \$6945.75; \$1015.00; \$5900.25. How much did he give in all?

25. The property of Mr. Childs was valued as follows: Land, \$3560; house, \$1834; barn, \$690; horse, \$180; carriage, \$135; tools, \$382.95; money in bank, \$987. How much was he worth?

26. Mr. Johnson bought a set of furniture for the parlor, costing \$385.75; one for the sitting-room, costing \$229.55; one for a bed-room, costing \$176.25; one for the dining-room, costing \$194.40. What was the total cost of the four sets?

Subtract:

27.	**28.**	**29.**	**30.**	**31.**
\$642.67	\$648.16	\$3247.67	\$162.38	\$6235.41
83.90	59.23	843.98	37.84	1987.93

32.	**33.**	**34.**	**35.**	**36.**
\$639.10	\$794.13	\$9.00	\$85.00	\$9500.05
234.78	197.25	2.97	3.78	3428.69

37.	**38.**	**39.**	**40.**	**41.**	**42.**
\$953.24	\$123.45	\$523.42	\$6.932	\$173.00	\$11.000
735.38	67.89	94.51	0.478	56.43	9.345

43–50. Deduct $87.93 from each of the following: $100.00, $93.13, $703.24, $103.10, $1834.29, $793.11, $600, $87.94.

51–55. Deduct $934.782 from the footings of examples 8–12.

56–59. Deduct $379.999 from the footings of examples 15–18.

Subtract:

60.	61.	62.	63.
$762.949	$564.120	$24.500	$724.380
438.758	259.437	19.999	149.871

64. Take four thousand six hundred forty-five dollars twenty-eight cents three mills from six thousand two hundred thirty-eight dollars eleven cents.

Applications.—65. If a person's property amounts to $7434.90, and his debts to $1350.78, how much is he worth?

66. Mr. White opened his store with goods worth $6100.50. Six weeks afterward he took an inventory (list) of goods remaining on hand, and found their value to be $4417.46. How much had he disposed of?

67. One season Mr. Hardy, who made a business of dealing in real estate (houses and lands), bought:

a. A city house and lot for $8765, and sold them for $10000;

b. A farm for $16850, and sold it for $17050;

c. A business block for $78500, and sold it for $82000;

d. Two city lots for $30500, and sold one for $13250 and the other for $18525;

e. A city lot for $75000 and sold it for $83500;

f. A theatre for $125250, and sold it for $121750. Find what he gained or lost on each transaction. Did he gain or lose by the business of the season, and how much?

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68. How many cents are there in one half dollar? In one quarter of a dollar? In one fifth of a dollar? In one tenth of a dollar? In three quarters? In four quarters? In two quarters? *In two* fifths? In three tenths?

69. Write the following sums as dollars and cents: $\$1^1/_4$, $\$2\,^1/_2$, $\$3\,^1/_5$, $\$5\,^3/_4$, $\$18\,^1/_2$, $\$22\,^2/_5$, $\$28\,^3/_5$, $\$32\,^3/_4$, $\$42\,^2/_4$, $\$48\,^4/_5$.

Model.—$\$1^1/_4 = \$1.25$.

70. Write the number of mills equivalent to $12\,^1/_2$¢, $18\,^2/_5$¢, $37\,^1/_2$¢, 75¢, $43\,^3/_5$¢.

Model.—$^1/_2$¢ = 5 m., $12^1/_2$¢ = 12¢5 m. = 125 m.

71. Find the sum of $\$1^3/_4$, $\$3\,^1/_2$, $\$5\,^3/_4$, $\$7\,^2/_4$, $\$90\,^1/_2$, $\$10\,^1/_2$. (Write $1.75 for $\$1^3/_4$, etc.)

72. Add $\$7\,^2/_5$, $\$8\,^1/_2$, $\$1^5/_{10}$, $\$3\,^7/_{10}$, $\$6\,^1/_2$, $\$30\,^3/_5$.

Applications.—**73.** William has $\$7\,^1/_2$ in his savings bank, Bertha $\$2\,^3/_4$, Carl $\$1^1/_2$, May $\$3\,^1/_2$, Emma $0.50. How much have they in all?

74. Mrs. Duncan paid the butcher on Monday $0.60; on Tuesday, $0.78; on Wednesday, $^1/_2$ dol.; on Thursday, $^3/_4$ dol.; on Friday, $^2/_5$ dol.; and on Saturday, $1.10. How much in all?

75. My new Reader cost $^4/_5$ dol., my Speller $^2/_5$ dol., my Arithmetic $^1/_2$ dol., and my new slate 15¢. How much did I pay for them all?

76. I bought a pair of shoes for $\$4^1/_2$, a hat for $\$7\,^1/_2$, and an umbrella for $\$1^3/_4$, and gave the merchant a $20 bill. What change did I get?

77. In paying an account of $\$34^4/_5$, I gave the dealer 4 ten-dollar bills. What change was due me? If I had given him a $50 bill, what change should I have received?

78. A boy had bought four articles at the grocery worth $2.25. But not having so much money, he handed back one of the articles costing $^9/_{10}$ of a dollar. How much did he pay for the other three articles?

79. John buys coffee for $\$1^1/_5$, sugar for 70¢, cheese for $^1/_2$ dollar. He sells the grocer potatoes for $1.50. How much does John have to pay, after deducting the price of the potatoes from his bill?

Multiplication of United States Money.

Illustrative Example.—1. If the price of rye is \$1.355 per bu., what will 65 bushels cost?

Suggestive Questions.—If \$1.355 were written in column 65 times, and an addition of all made, where would the separatrix fall? How many places would there be to the right of the point? What denominations would they represent?

Or, How many mills are there in \$1.355? In 65 times \$1.355? Reduce the result to dollars. How many places to the right of the point? What denominations do they represent?

Operation.

\$1.355	price per bu.
65	no. of bushels.
6775	
8130	
\$88.075	cost of 65 bu.

94. *Rule.*—**Multiply as in simple multiplication. The product will be of the same denomination as the lowest order of the multiplicand. If it be in cents or mills reduce to dollars and prefix the dollar mark.**

SLATE EXERCISES.

2–7. Multiply \$87.74 by 27.—\$324.034 by 56.—\$1.95 by 18.—\$27.341 by 35.—\$0.934 by 746.—\$0.34 by 61.

8–22. Multiply each of the following sums of money by 8; by 37; by 368: \$0.398; \$20.03; \$47.731; \$621.70; \$0.604.

23–37. Find first 13, then 49, then 387 times \$34.75; \$967.03; \$309.08; \$7654.60; \$190.10; and subtract \$99.999 from each product.

38. What is the product of \$4.37 by $18 \times 27 \times 15$?

Applications.—39. How much must the government pay for 327 horses, at \$135.50 each?

40, 41. What will 426 sheep cost at \$4.87 1/2 per head? At \$3.62 1/2? (Express fractions of cents in mills.)

42. What is the amount of a contribution if 157 persons contribute each \$5 1/4?

43–45. What will 476 lb. of tea cost at 65¢ a lb.? At 50¢? *At 3/4 dol.?*

46. A merchant bought 3 bales of cloth containing respectively 485, 492, and 497 yards, at \$1.87 $^1/_2$ a yard. Find the cost?

47. The merchant just spoken of sold the two bales first mentioned at \$2.25, and the last one at \$2.75 per yard. What did he receive for the cloth?

48. A grocer buys 38 doz. loaves of bread on Monday, 35 doz. on Tuesday, and 36 doz. on each remaining day of the week, at 60¢ per doz., and sells at 7¢ per loaf. How much does he gain?

49. A grocer bought 9 barrels of cider, each barrel containing 30 gallons, at 12 $^1/_2$¢ per gallon. What did the cider cost?

50. What is the cost of 4 barrels of sugar, weighing 495 lb. each at 7 $^1/_2$¢ a pound?

51. What is the cost of 156 acres of land at \$20.50 per acre?

52. Mr. Jones bought of Messrs. Taylor, Kilpatrick and Co., 16 yd. silk, at \$2.25 a yd.; 6 yd. gingham, at \$0.19 a yd.; 27 yd. linen, at \$0.37 a yd.; 39 yd. muslin, at \$0.13 a yd. What was the amount of the bill?

53. Mr. Taylor bought of Mr. Watts the following implements: a spade for $^3/_4$ dol., a rake for $^3/_5$ dol., a hoe for $^1/_2$ dol., a shovel for 45¢, and a lawn-mower for 14$^3/_4$ dols. Find the amount.

54. Mr. James went to market with \$5. He spent for eggs 45¢, for butter 98¢, for fruit 25¢, and for flour \$1.20. How much had he left of the \$5?

55. A laborer earns \$50 and spends \$39.75 a month. How much will he have saved at the end of six months?

56. What are the profits of a concert, if 3427 tickets are sold at \$1 $^1/_2$ each, and the expenses are \$938.40?

57. Mr. Mills sold 837 shade-trees for \$0.65 each. How much did he receive for the lot?

58. A drover bought 265 head of cattle at \$43.75 a head, and paid \$8.35 a head to get them to market, where he sold them at \$56.80 a head. How much did he gain by the transaction?

Division of United States Money.

ILLUSTRATIVE EXAMPLES.

1. If a builder pays $693.68 for lumber at $2\,^1/_2$¢ per foot, how many feet does he buy?

$$\begin{array}{r} 27747\,^5/_{25} \text{ times.} \\ 25 \text{ m.})\overline{693680} \text{ m.} \\ 50 \\ \hline 193 \\ 175 \\ \hline 186 \\ 175 \\ \hline 118 \\ 100 \\ \hline 180 \\ 175 \\ \hline 5 \end{array}$$

2. If $693.68 be equally divided among 25 men, what will be the exact share of each?

$$\begin{array}{r} \$27.747\,^5/_{25} \\ 25)\overline{\$693.68} \\ 50 \\ \hline 193 \\ 175 \\ \hline 186 \\ 175 \\ \hline 118 \\ 100 \\ \hline 180 \\ 175 \\ \hline 5 \end{array}$$

Note 1.—Every time $2\,^1/_2$¢ can be taken from $693.60, a foot of lumber can be bought. Hence, to find the number of feet, we find how many times $2\,^1/_2$¢ is contained in $693.68. To do this, we change both the $2\,^1/_2$¢ and the $683.68 to mills, and divide the second by the first.

A shorter way would be to change both sums to half cents, and divide, but the first way is generally the better. Try both and see which you like best.

Note 2.—If $693 be divided into 25 equal parts, there will be $27 in each part, and $18 undivided. $18 = 180 dimes, 180 dimes + 6 dimes = 186 dimes. If 186 dimes be divided into 25 equal parts, there will be 7 dimes in each, and 11 dimes will remain undivided. To this we add the 8¢, and then proceed as before till we come to a remainder of 5 mills. For the present, remainders should be disposed of as directed in Art. 76, p. 80.

Note 3.—Thus we find that division of United States money is, first, the process of finding how many times one sum of money is contained in another; and, second, of finding a required part of a given sum. (See also Art. 75, p. 78.)

95. ***Rule I.***—**To find how many times one sum is contained in another, change both to the lowest denomination in either, and divide as in simple division. The quotient will be in integers.**

Rule II.—**To find a required part of a given sum of money, divide the sum by the number of parts, as in simple division. The place of the separatrix in the quotient will be directly over the *separatrix* of the dividend.**

SLATE EXERCISES.

3. Divide $100.50 by 5. Also by 7, by 9, by 65.

4, 5. Four persons are to have equal shares of $4412.88. How much will each one receive? How much would each receive if there were 12 persons?

6–11. Divide $369.009 by 3, by 9.—Also by 8, by 6, by 5, by 15.

12–16. Divide $3759.91 by 19, by 35, by 54, by 67.—Are you here required to find certain parts of $3759.91; or, how many times that sum contains the several divisors?

17. Which is greater, $^1/_6$ or $^1/_2$ of $90.50? How much?

Find

18. $^1/_2$ of $ 97.78.	**22.** $^1/_{16}$ of $8775.36.	**26.** $^{13}/_{16}$ of $10391.52.
19. $^1/_4$ of $ 363.68.	**23.** $^7/_9$ of $ 436.50.	**27.** $^{15}/_{27}$ of $ 8335.71.
20. $^1/_5$ of $ 728.15.	**24.** $^5/_7$ of $ 410.41.	**28.** $^{11}/_{12}$ of $ 5654.76.
21. $^3/_5$ of $8257.25.	**25.** $^3/_8$ of $9873.56.	**29.** $^{18}/_{25}$ of $ 9864.75.

Applications.—30–31. How much sugar can be bought for 99¢ at 11¢ a lb.? At 9¢ a lb.?

32–35. If oranges cost $4^1/_2$¢ apiece, how many can be had for $3.60, for $3.78, for $36.00, for $0.90?

36. A dozen chairs can be bought for $11.40. How much does 1 chair cost?

37. A carpenter has 34 men at work; at the end of the week, 17 of them receive $211.65 wages; the other 17 receive only $184.45. How much does each one of the two classes of workmen receive per week?

38. A street commissioner had 347 men at work. At uniform wages the weekly pay roll amounted to $2602.50. What did each man receive per day?

Six workmen received pay for 26 days' work as follows:

39. The carpenter, $55.25. **40.** Painter, $52. **41.** Bricklayer, $65.
42. Plumber, $68.25. **43.** Laborer, $42.25. **44.** Plasterer, $63.70.

What were the daily wages of each?

Miscellaneous Examples.

1. Mr. Jacobs paid me \$27.34; Mr. Niel, \$79.14; Mr. French, \$34.27; Mr. Myers, \$647.79. My expenses on the tour of collection were \$19.68. When I started out I had \$50.75 in my purse. How much money ought I to have had on my return.

2. In 1873 I paid \$52.23 taxes; in 1874, \$50.79; in 1875, \$46.27; in 1876, \$44.83; in 1877, \$42.21; and in 1878, \$40.90. How much in these 6 years? The repairs on my house in the meantime cost \$238.65. For the first 3 years I received \$650 per year rent, for the last 3 years \$700 per year. How much did I receive in the 6 years clear of expenses?

3. A lady had \$30. She bought a dress for \$9.15, shoes for \$3.40, a bonnet for \$4.50, and 23 yd. muslin at 25¢ a yd. How much did she have left?

4. A farmer owed \$500, and gave in part payment 435 bu. wheat, at \$1.02 a bu. How much money was yet due?

5. If you spend \$0.74 a day, how much will you save in a year if your salary is \$475? (365 days to the year.)

6. If a laborer works 60 days, 10 hours a day, and receives \$135 for his labor, how much does he earn per hour?

7. Mr. Jordan sells Mr. Marsh the following articles: 3 cashmere long shawls, at \$45.75 each; 45 yd. black satin, at \$2.20 a yd.; 12 alpaca umbrellas, at \$1.35 apiece; 60 worsted cord and tassels, at \$0.35 apiece. Find the cost.

Note.—In the following memoranda the price per pair, yard, doz., or single article is given as usually written. The pupil is required to extend the items, and find the footings. The sign @ stands for the word *at* (that is, per pair, per lb., etc.).

8. *Find the cost of*

3 pr. kid gloves,	@	\$1.35
17 yd. Malta lace,	@	.76
4 doz. handkerchiefs,	@	1.80
1 doz. pair linen cuffs,	@	.13
6 yd. silk fringe,	@	1.10
3 ostrich plumes,	@	3.25
3 pr. linen gloves,	@	.65

9. *How much must be paid for*

5 gal. vinegar,	@	\$.27
15 lb. cheese,	@	.09
3 lb. hominy,	@	.45
27 lb. sugar,	@	.07
22 lb. soap,	@	.06
3 gal. molasses,	@	.70
6 lb. prunes,	@	.16

10. If you are sent with a $2 bill to the bakery to get 1 doz. rolls, at 1¢ apiece; 3 loaves of bread, at 8¢ a loaf; 3 doz. cookies, at 10¢ a doz.; and 10¢ worth of caramels, what change will you bring back?

11. Mother makes the following purchase at the crockery store: 1 cream pitcher, 75¢; 1 sugar bowl, 45¢; 1/2 doz. plates, at $1.20 a doz.; 1/2 doz. egg cups, at 90¢ a doz.; 3 cups and saucers, at 60¢ a pair; 2 doz. fruit jars, at 9¢ apiece. What is the bill?

12. If your mother sends you to the grocery with $5 to buy 1/2 lb. of tea, at 90¢ a lb.; 1 lb. of coffee, at 40¢ a lb.; 5 lb. of granulated sugar, at 11¢ a lb.; 3 lb. of lump sugar, at 12¢ a lb.; 1 small bag of salt, at 9¢; 4 loaves of bread, at 6¢ a loaf; 1 peck of apples, at 80¢ a bu., what change will you receive?

13. If you are sent with $2 to buy 3 lb. rice, at 10¢ a lb.; 20 lb. of flour, at 5¢ a lb.; 6 lb. of cheese, at 9¢ a lb.; 5 lb. of prunes, at 16¢ a lb.; 1 gal. coal-oil, at 8¢ a quart, will you have money enough? If not, which article must you omit to keep the sum total within $2?

14. Twelve tons of coal cost $75.00, how much is that per ton? What would 37 tons cost at that rate?

15. A person sells 5 cows at $55 each, and a yoke of oxen at $125. He agrees to take in payment 80 sheep. How much do the sheep cost him per head?

16. What is the cost of 39000 feet of planed pine lumber at $40 per thousand feet? Of 16000 shingles at $2.75 per thousand?

17. *Find the cost of*

60	pr. overshoes,	@	$ .65
15	" boots,	@	4.25
17	" gaiters,	@	3.85
37	" slippers,	@	1.75
230	" mittens,	@	.34
2	doz. pr. slippers,	@	9.00
2	pr. boots,	@	9.50

18. *Required the cost of*

9 lb. lard,	@	$.08
15 lb. butter,	@	.22
18 lb. pork,	@	.06
20 lb. rice,	@	.09
12 lb. raisins,	@	.20
4 cans oysters,	@	.35
10 lb. codfish,	@	.10

19. Mr. White bought 6 tubs of butter, containing 58 lb. each, for \$80.04. How much did he pay per lb.?

20. He sold the butter at a profit of 12¢ a pound. Deducting 7 lb., which he used in his family, how much did he get for it?

21. When coal is \$4¼ per ton, how many tons can be bought for \$238? How much would be saved by buying at \$4 per ton?

22. A farmer buys goods amounting to \$235.75. He pays in cash \$58.25, and agrees to pay the balance in rye, at \$1.25 a bushel. How many bushels will be required?

23. How many pounds of cheese, at 15¢ a lb., must be given in exchange for 14 yd. of gingham, at 30¢ a yard?

24. Subtract \$37.87 from \$237.37; from the remainder subtract \$37.87, and continue subtracting till the remainder is less than the subtrahend. What is the remainder? Is this the shortest way to find the remainder?

25. Multiply \$4.35 by 2; multiply the product by 3; multiply the second product by 4; the next by 5; the next by 6; and the next by 7. What is the last product?

Arrange each line in column, and add:

26. \$13.44, \$300, \$55.25, \$288.39, \$19.50, \$31.67, \$509.07.

27. \$67.31, \$180.61, \$79.03, \$152.70, \$14.23, \$11.12, \$50.22.

28. \$88.75, \$264.16, \$44.56, \$76.82, \$30.50, \$72.39, \$142.33.

29. \$10.13, \$7.56, \$2.18, \$55.44, \$11.19, \$70.25, \$312, \$9.

30. \$13.33, \$72.69, \$15.437, \$34.805, \$125.595, \$77.666.

31. Add together all the sums of money given in examples 26 to 30, inclusive.

32. A store-keeper, who was about to pay some debts, found that he had \$37.45 in change and \$76 in bank-notes in his money-drawer, \$318 in his safe, and \$98.36 in his pocket-book. How much had he left after paying 5 bills of \$56.10, \$38.05, \$48.00, \$213, and \$78.90, respectively?

33. The treasurer of a street railroad took 400 dimes, 800 quarter-dollars, 23 twenty-cent pieces, 600 half-dollars, 1000 five-cent pieces to be exchanged for $5 bills. How many did he get?

34. A grocer exchanged bills for small change. How many 5¢ pieces could he get for $5, $10?—How many dimes for $5, $10? —How many quarters for $25, $45?—How many half-dollars for $37, $54, $96?

Making Change.

96. 1. You buy three pounds of rice at 9¢ a pound, and hand the grocer in payment a dollar bill; how does he count the change due you?

Answer.—Giving you the rice he would count that as 27¢; then placing in your hand successively 3¢, 10¢, 10¢, and 50¢, he would count *30, 40, 50 $1.* This is the most convenient way, and least liable to error. It is similar to the "making up" method in subtraction, which is recommended on page 41.

2. Having only 5¢ and 10¢ pieces, how will the change be counted, taking 15¢ out of $1?

3. Having 1¢, 5¢, 10¢ pieces, and $1 bills, how would you make the change for 35¢ out of $2? For 85¢ out of $5? For 75¢ out of $10? For $1.12 out of $1.50? For $6.03 out of four two-dollar bills? For 67¢ out of $2? For $3.33 out of $5?

4. If the merchant has no change except 25¢, 50¢, and $1 pieces, how can he make change for $2.75 out of $5?

5. A collector presents a bill for $1.90; you have only two $1 bills, one 10¢, and one 5¢ piece = $2.15. The collector has only large bills and quarter dollars. How can the change be made? (If you were to give him your $2.15, could he then make the change?)

6. You owe $2.75, but have only three dollars and a quarter. How can change be made, the collector having none less than a half dollar?

7. If a grocer has only small change, namely, 1¢, 2¢, 3¢, 5¢, 10¢, 20¢, 25¢, 50¢ pieces, how can he make change for $2, the goods you have bought costing 16¢?

8. If a merchant has only \$1, \$2, and \$5 bills, and 1¢ and 5¢ pieces, how will the change for \$3.27 be counted out of a \$20 note?

9. Having 1¢, 5¢, 25¢, and 50¢ pieces, and \$2 bills, how can you count the change for \$1.15 out of \$5? For \$2.23? For \$3.45? For \$1.84? For \$4.66? For \$1.11? For 93¢? For \$4.46?

10. Having only 1¢, 10¢, 25¢, and \$1 pieces, how can you count the change for 27¢ out of \$2? For \$1.34 out of \$5?

Count out the proper change in each of the following transactions:

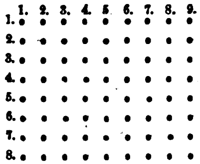
	Goods sold.	Money received.
11.	5 lb. coffee, @ 32¢; 5 lb. sugar, @ 9¢; 3 lb. cheese, @ 14¢.	\$5
12.	2 lb. beef, @ 19¢; 2 lb. butter, @ 42¢; radishes, 10¢.	2
13.	3 lb. soap, @ 10¢; 2 lb. starch, @ 12¢; 1 paper allspice, 12¢.	1
14.	$^1/_2$ lb. tea, @ 90; 4 lb. sugar, @ 11¢; 1 qt. strawberries, 25¢.	2
15.	1 lead-pencil, 10¢; envelopes, 10¢; 1 daily newspaper, 3¢.	5
16.	5 yd. muslin, @ 13¢; 3 spools cotton, @ 6¢; 6 handkerchiefs, @ 28¢.	4
17.	900 lb. pork, @ 3¢; 5 bu. peaches, @ \$2.50; 7 bu. apples, @ 75¢.	50
18.	18 bu. oats, @ 50¢; 12 bu. corn, @ 75¢; 13 cwt. hay, @ \$1.05.	35
19.	3 pk. peaches, @ \$2 per bu.; 5 qt. cherries, @ 18¢; $1^1/_2$ lb. butter, @ 48¢.	5

CHAPTER VIII.

FACTORS AND DIVISORS.

Definition.

97. An ***Integer*** is a whole number. It is so called to distinguish it from a fraction. (This chapter treats of integers only.)



Factors.

Having counted by 8's and 9's to 72, the pupil learned that

8 times 9=72, and
9 " 8=72.

Then he learned—

1. That 8 and 9 are called factors of 72.
2. That 72 is called the product of 8 and 9; and
3. That 72 is called a multiple of 9; also of 8.

98. But since 8 times 9=72 and 9 times 8=72, nine is contained exactly 8 times, and 8 exactly nine times in 72. Thus the factors of a number are exact divisors of that number.

99. Hence, to find the factors of a given number we ascertain by trial what numbers will divide it without a remainder; the divisors and quotients are the factors sought.

Thus we obtain 7 different pairs of factors of 210, as follows:

2)210	3)210	5)210	6)210	7)210	10)210	14)210
105	70	42	35	30	21	15

Definitions.

100. A ***Factor*** of a number is any one of two or more integers which, multiplied together, produce the number.

101. When one number can be divided by another without remainder, the dividend is said to be ***divisible*** by the divisor, and the divisor is called a ***Measure*** or ***Exact Divisor*** of the dividend.

102. A number that is the product of other factors besides *itself* and *one* is called a ***Composite Number.***

Note 1.—A *Composite* number is so called because it is *composed* of other factors.

Note 2.—Since a composite number is divisible by its factors, it may be defined to be a number that is divisible by other numbers besides itself and one.

103. A ***Prime Number*** is one that has no factors, and hence no exact divisor except itself and 1.

104. A ***Prime Factor*** is a factor which is a prime number.

105. An ***Even Number*** is one that is divisible by 2.

106. An ***Odd Number*** is one that is not divisible by 2.

SLATE EXERCISES.

1. Write in columns the numbers in order from 1 to 35; also from 36 to 70, inclusive, and opposite to each write all the pairs of factors that will produce it. Thus

$1=1\times1$	$36=2\times18,\ 3\times12,\ 4\times9,\ 6\times6$
$2=1\times2$	$37=1\times37$
$3=1\times3$	$38=2\times19$
$4=2\times2$	$39=3\times13$
$5=1\times5$	$40=2\times20.\ 4\times10,\ 5\times8$
$6=2\times3$	$41=1\times41$
etc.	etc.

2. In the same manner write the pairs of factors of numbers from 71 to 107, inclusive; also from 108 to 144.

3. Make a table such as the one required in Ex. 5, p. 53, omitting the first line and first column. Give the factors orally.

4. Make a separate list of numbers from 1 to 144 that have

2 for one or more of their factors. *Notice* that the right-hand figure of each is — — — — or —. (?)

5 for one or more of their factors. *Notice* that the right-hand figure of each is — or —. (?)

3 for one or more of their factors. Divide the sum of the digits of each of these numbers by 3, and notice the remainder, if any.

9 for one or more of their factors. Divide the sum of the digits of each by 9, and notice the remainder, if any.

Thus we discover some

AIDS IN FINDING FACTORS.

107. It may be shown to be true of any number that it has

2 for a factor if the right-hand figure is 2, 4, 6, 8, or 0;

5 for a factor if the right-hand figure is 0 or 5;

3 for a factor if the sum of its digits is divisible by 3;

9 for a factor if the sum of its digits is divisible by 9.

Apply the foregoing aids in the following exercises:

1. Tell which of the dividends at the top of p. 82 are divisible by 2; by 5; by 3; by 9.

2. Write 10 numbers of three or more figures each, all of which shall be divisible by 2; by 5; by 3; by 9.

3. Change one figure in each of the following numbers, so as to make the number divisible by 2: (State what change you make, and why.)

4379	6479	5243	7957	4343
5627	8123	2147	8971	5557
8291	4871	9281	3629	4441

4.–6. Change one figure in each, so that the number shall be divisible by 5.—Change the last figure in each, so that the number shall be divisible by 3.—Change the first figure in each, so that the number shall be divisible by 9.

Factoring.

108. Since 7 is a factor of 14, it must be a factor of any number of times 14, as 28, 42, etc. Thus it is true always that

A factor of a factor-of-a-number is a factor of the number itself.

Hence, having obtained one prime factor of a number directly from the number itself, a second one may be obtained from the quotient of the first, and a third, if any, from the quotient of the second, etc. Thus

Solution.

$$\begin{array}{r|l} 2 & 210 \\ \hline 3 & 105 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

Proof.

$2 \times 3 \times 5 \times 7 = 210$

Explanation.—2 being contained 105 times in 210, 2 and 105 are factors of 210. Then dividing the quotient by 3, we find that 3 and 35 are factors of 105, and hence also of 210; and, again, finding that 5 and 7 are factors of 35, we know that they are factors of 105 and also of 210. Thus we derive the

109. ***Rule.*—Divide the given number by any prime factor, and if the quotient is not a prime number, divide it in like manner, and so continue to divide till the quotient is a prime number. The divisors and the last quotient are the factors sought.**

SLATE EXERCISES.

Find the prime factors of

1. 1050	**6.** 5985	**11.** 8140	**16.** 1906	**21.** 2526
2. 2625	**7.** 4620	**12.** 8712	**17.** 1858	**22.** 2978
3. 1820	**8.** 4802	**13.** 1320	**18.** 1478	**23.** 2992
4. 1485	**9.** 5432	**14.** 1768	**19.** 2956	**24.** 3936
5. 1155	**10.** 7000	**15.** 1848	**20.** 2406	**25.** 3430

Note.—The learner will avoid useless labor if he will keep it in mind that the quotient is as much a factor of the dividend as the divisor itself.

Suppose, for instance, that he is working to find the prime factors of 479, as in the last of the preceding examples. He tries successively every prime number from 2 upward, till he comes to 23, when he finds that the quotient has become less than the divisor. Here, if he stops to think, he will say to himself: "It is of no use to try any further. This number can not have an exact divisor greater than 23, for if it had, it would have another less than 23; but I have tried every prime number from 2 to 23, and I know it has none. This number is prime."

Common Factors and Common Divisors.

110. A factor which occurs in each one of two or more numbers is a ***Common Factor*** of those numbers. Thus $15=3\times5$ and $21=3\times7$; the factor 3 is common to 15 and 21.

Note.—A factor is said to be *common* to two or more numbers, just as we may say that the letter *o* is common to the two syllables of the word *common;* or that *i* is common to all the syllables of the word *Mississippi.*

111. *A factor of a number being an exact divisor of the number, a factor that is common to two or more numbers is a common divisor of those numbers.*

Find the prime factors of 252 and 2310, and show that the common factors are common divisors of those numbers.

112. *The products of the prime factors of a number being exact divisors of that number, the products of the prime factors common to two or more numbers are common divisors of those numbers.*

Show that the products of the prime factors that are common to 294 and 315 are common divisors of those numbers.

113. *Since an exact divisor of a number can have no factor which does not occur in that number, the product of all the prime factors that are common to two or more numbers is the* **greatest common divisor** *of those numbers.*

Find, if you can, any other divisors of 396 besides its prime factors, and the products of two or more of them. Can 1820 and 2310 have any other common divisors than their common factors and their products? Try to find one.

114. Hence, to find the greatest common divisor of two or more numbers—

***Rule.*—1. Resolve the given numbers into their prime factors.**

2. Multiply together all the factors that are common to all the numbers. The product will be the greatest common divisor sought.

Example.—Find the g. c. d. of 546 and 910?

Prime Factors Found.

2	546	2	910
3	273	5	455
7	91	7	91
	13		13

Prime Factors Arranged.

$546=2\times3\times7\times13$

$910=2\times5\times7\times13$

Common Prime Factors Multiplied.

$2\times7\times13=$***182*** g. c. d.

115. A shorter Process.—When the prime factors are readily detected, the process is somewhat simplified and considerably shortened by dividing the given numbers only by the factors that are common, and multiplying these together for the greatest common divisor. The principle of this operation is the same as that of the one given at the bottom of the preceding page.

Operation.

2	924	990
3	462	495
11	154	165
	14	15

$2 \times 3 \times 11 = 66$ g. c. d.

Note.—The product of any two of the common prime factors is a common divisor, but it requires the continued product of ALL of them to make the *greatest common divisor.*

ORAL EXERCISES.

Find the greatest common divisor of

1. 14 and 21
2. 26 and 39
3. 40 and 56
4. 72 and 99
5. 80 and 48
6. 57 and 69
7. 15 and 93
8. 50 and 75
9. 56 and 84
10. 21 and 56
11. 32 and 64
12. 34 and 38
13. 58 and 87
14. 18 and 82
15. 81 and 45
16. 46 and 28
17. 36 and 54
18. 49 and 98
19. 54 and 81
20. 63 and 81

SLATE EXERCISES.

Find the greatest common divisor of

21. 323 and 425
22. 228 and 399
23. 615 and 735
24. 819 and 945
25. 949 and 871
26. 7008 and 7968
27. 7568 and 3784
28. 3876 and 1983
29. 7956 and 7668
30. 7378 and 9758
31. 7992 and 9900
32. 8100 and 6300
33. 9864 and 9528
34. 6144 and 6930
35. 4374 and 5508

Find the greatest common divisor of

36. 45, 57, and 81
37. 63, 99, and 126
38. 36, 54, and 56
39. 72, 84, and 90
40. 306, 408, and 510
41. 420, 462, and 84

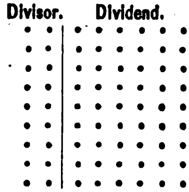
Find the largest number that will exactly divide

42. 546, 462, and 882
43. 900, 936, and 2520
44. 19635, 5355, 8925, and 12495
45. 19782, 16485, and 14287

Cancellation.

116. The principle that dividing divisor and dividend by the same number does not alter the quotient may be demonstrated as follows:

Explanation.—Any divisor and dividend having a common factor may be represented by an equal number of *rows* of dots, as 18 and 54 at the right. Whence it becomes evident that *any part* of the divisor as one or more lines is contained in a *like part* of the dividend, as many times as the *whole* divisor is contained in the *whole* dividend.



117. Hence any factor or number of factors common to divisor and dividend may be rejected without affecting the quotient.

That this principle may be well impressed upon the mind, let many examples, such as the following, be worked out:

Example.—1. (*a*) Divide the product of 7, 3, 13, 5, 7, 19, and 3, by the product of 7, 3, 7, 5, and 3. (*b*) Reject all common factors, find the product of the remaining ones and divide.

Note.—The factor 1 always remains in place of factors omitted. It is not written because it does not affect the result.

Example.—2. How many bushels of oats at 48¢ a bu. can Mr. A. get for 3 crocks of butter containing 8 lb. each at 32¢ a lb.

Solution.

32¢	16
8	48¢)768¢
256¢	48
3	288
$7.68	288

Analysis.—At 32¢ a pound, 8 lb. of butter will cost 8 times 32¢ = 265¢, and 3 crocks containing 8 lb. each will cost 3 times 256¢ = $7.68, and as many bushels can be bought for $7.68 as there are times 48¢ in $7.68 = 16.

But we may indicate this work by writing the factors (makers) of the dividend on the right and the divisor on the left side of a vertical line, and shorten the work by rejecting common factors, as shown at the right.

~~$~~ ~~48~~	~~$2~~ 2
	~~$~~
	8
	16 *Ans.*

118. To ***cancel*** is to erase or cross out, hence the word ***Cancellation*** is applied to erasing or crossing out factors and terms which counterbalance each other in an arithmetical operation.

SLATE EXERCISES.

1. $\frac{15\times57\times36\times35}{12\times19\times25}=$

2. $\frac{17\times95\times8\times23}{85\times38\times5}=$

3. $\frac{42\times18\times65\times11}{35\times45\times13}=$

4. $\frac{91\times36\times94\times54}{78\times14\times18}=$

5. How many cheeses, weighing 49 lb. each, at 12¢ a pound, must be given in exchange for 13 barrels of flour at 4¢ a pound? (196 lb. = 1 barrel of flour.)

6. How many sacks of wheat, containing 3 bu. each, at 96¢ a bushel, must be given for 78 sacks of potatoes, each containing 2 bu. at 64¢ a bu.?

7. A lady bought 9 yards of ribbon at 56¢ per yard, but exchanged it for other ribbon at 32¢ per yard; how many yards did she then get?

8. At $129 for 27 acres of land, what will 180 acres cost?

9. At what price per yard will 5 bales of cloth, containing 12 pieces of 42 yards each, pay for 50 rolls of carpeting, of 75 yards each, at $2.10 per yard?

10. Divide $34\times102\times85$ by $51\times17\times68$.

11. Divide $16773\times13401\times11412$ by $11182\times11912\times8559$.

How many

12. Bu. apples @ 60¢	will pay for	35 lb. tea @ 84¢?
13. Bu. peaches @ $3.50	" " "	25 tons coal @ $12.60?
14. Pieces muslin (39 yd.) @ 12¢	" " "	26 tubs butter (72 lb.) @ 32¢?
15. Tons hay @ $12.18	" " "	3 barrels sugar (232) @ 9¢?
16. Horses @ $91	" " "	7 acres land @ $559?
17. Cows @ $39	" " "	650 sheep @ $3?
18. Mules @ $125	" " "	25 horses @ $160?
19. Tubs butter (54 lb.) @ 28¢	" " "	378 yd. muslin @ 16¢?

At what price will

20. 65 lb. coffee	pay for	26 bu. potatoes @ 45¢?
21. 75 acres land	" "	21 horses @ $125 each?
22. 26 barrels pork	" "	78 bl. flour @ $6?
23. 260 doz. eggs	" "	78 yd. silk @ 90¢?

Factors and Multiples.

119. An integral (or whole) number of times a number is a multiple of that number. (See note, p. 53.)

Note.—A multiple of a number being some *whole times* that number, is, of course, always divisible by it; hence a multiple of a number is sometimes defined to be "a number that is exactly divisible by it."

SLATE EXERCISES.

1. Write in columns the multiples from 1 to 120 of 3, of 4, of 5, of 6, of 7, and of 8. Thus,

3	*4*	*5*	*6*	*7*	*8*
6	8	10	12	14	16
9	12	15	18	21	24
12	16	20	24	28	32
15	20	25	30	35	40
18	24	30	36	42	48
etc.	etc.	etc.	etc.	etc.	etc.

2. Make a list of the numbers from 1 to 120 that are multiples of both 3 and 4; thus—

12, 24, 36, 48, 60, 72, 84, 96, 108, 120.

3. In like manner make a list of the multiples from 1 to 120 that are common to 3 and 5, 3 and 6, etc.

4. Make a list of the multiples from 1 to 120 that are common to 3, 4 and 5; 3, 5 and 6, etc., as far as may be directed.

5. Make a list of the multiples that are common to 3, 4, 5 and 6; to 4, 5, 6 and 7; to 5, 6, 7 and 8, as above.

Note.—The pupil should note the fact that any number may have an unlimited number of multiples, and that any two or more numbers may have an unlimited number of *common* multiples, but only one *least common* multiple.

120. *Any multiple of a number must contain at least all the prime factors of that number.*

Thus 2 and 3 being factors of 6, they must occur as factors in any number of times 6. [3 times 6 is 3 times (2 times 3), and so on, to any number of times 6.] Though the line of marks at the right should be copied millions of times, the *number* of marks could never escape the factors 2 and 3. // // //

121. ***A multiple that is common to two or more numbers must contain at least all the prime factors that enter into each of them.***

Thus, any multiple common to 15 and 21 must contain the factors 3, 5 and 7, for no number that does not contain the factors 3 and 5 can be a multiple of 15, and no number that does not contain the factors 3 and 7 can be a multiple of 21.

Note.—A common multiple of 15 and 21 *may* contain any other factors besides 3, 5, and 7, but these it *must* contain.

122. *The least common multiple of two or more numbers must contain all the prime factors that enter into each of them, and no others.*

Thus, 18 and 24 being resolved (separated) into their prime factors, we have

$$18 = 2 \times 3 \times 3$$
$$24 = 2 \times 2 \times 2 \times 3.$$

Hence, any common multiple of 18 and 24 must contain the factors 2, 2, 2, 3 and 3, and the *least common multiple* must contain no other. If it did contain any other factor it would not be the LEAST common multiple.

ORAL EXERCISES

1. What is the least common multiple of 9, 14, and 21?

Oral Solution.—1. A multiple of 21 must contain the factors 3 and 7. $3 \times 7 = \mathbf{21}$.

2. A multiple of 14 must have the factors 2 and 7, hence the common multiple of 21 and 14 must contain 3, 7, and 2 as factors. $3 \times 7 \times 2 = \mathbf{42}$.

3. A multiple of 9 must have the factors 3 and 3, hence the common multiple of 21, 14, and 9 must have the factors 3, 7, 2, and 3. $3 \times 7 \times 2 \times 3 = \mathbf{126}$.

Hence ***126*** is the *least common multiple* of 9, 14, and 21, because it contains no factor which is not necessary for one or another of the given numbers.

2. Find the least common multiple of 6, 7, 9, 12, 14, 18, 21, 36, and 42.

Oral Solution.—1. Since 36 is a multiple of 6, 9, 12, and 18, any multiple of 36 will be a multiple of these numbers also; and since 42 is a multiple of 7, 14, and 21, any multiple of 42 will be a multiple also of these numbers; hence, the least common multiple of 36 and 42 will be the least common multiple of all the given numbers.

2. The factors of 42 are 2, 3 and 7, but the factors of 36 are 2, 2, 3 and 3, or one 2 and one 3, more than are found in 42; hence we multiply 42 by 2 and by 3, or at once by 6, and obtain the product ***252***, which is the l. c. m. of all the en numbers.

Find the least common multiple

3. Of 2, 4, and 6	**7.** Of 2, 3, 4, and 6	**11.** Of 9, 2, 6, 18, 24
4. " 3, 4, and 6	**8.** " 4, 8, 12, and 16	**12.** " 8, 7, 12, 21, 24
5. " 5, 6, and 15	**9.** " 5, 7, 15, and 21	**13.** " 5, 2, 15, 7, 35
6. " 7, 14, and 21	**10.** " 3, 14, 21, and 28	**14.** " 2, 3, 6, 9, 54

Hence, for finding the least common multiple of two or more numbers, we have the following

123. ***Rule.*—1. Resolve each number into its prime factors.**

2. Take all the prime factors of the greatest given number, and such factors of the others as are not found in it. The continued product of all these factors will be the least common multiple.

Note.—If any of the given numbers are multiples of others, the multiples only need to be considered, for the given number that is a multiple of another contains all its factors.

SLATE EXERCISES.

15. Find the least common multiple of 88, 126, and 330.

Solution.

$88=2\times2\times2\times11$
$126=3\times3\times2\times7$
$330=2\times3\times5\times11$

Least Common Multiple.

$2\times3\times5\times11\times3\times7\times2\times2=27{,}720$

Explanation.—We write out all the factors of the several numbers, so that we may readily see what they are. We then take the factors of 330, and unite with them the factors that occur in the other numbers, and not in 330. The continued product of all is the least common multiple sought.

Note.—Any of the given numbers may be taken at once as the product of its own prime factors, and this being multiplied successively by such of the prime factors of each of the other numbers as are not contained in any preceding number. the product will be the l. c. m. sought. Thus:

$$330\times3\times7\times2\times2=27{,}720$$

In like manner find the l. c. m.

16. Of 27, 24, and 15	**20.** Of 9, 12, 14, and 210	**24.** 19, 27, 36, 63
17. " 63, 27, and 84	**21.** " 60, 15, 24, and 25	**25.** 13, 17, 19, 32
18. " 12, 51, and 68	**22.** " 54, 81, 63, and 14	**26.** 23, 27, 54, 108
19. " 35, 63, and 72	**23.** " 18, 24, 72, and 144	**27.** 14, 17, 105, 110

28. Of 9546, 6364, and 14319 **29.** Of 4862, 2002, and 17017

Problems G. C. D. and L. C. M.

1. One boy's blocks are 2 inches thick, another's 3, and another's 5. The three boys build "towers" of equal heights. How high at least are they? How many blocks does each one use?

2. What is the least sum that can be paid in either 2, 3, 5, 10, 20, or 25¢ pieces?

3. William has 27¢, Mary 36¢, and Harry 51¢, not in one-cent pieces, yet all in coins of one denomination. What is it?

4. In one grammar school there are 504 girls, in another there are 324 boys. It is desired to divide them into classes of equal size. How many pupils will there be in each class, if as large as it can be made?

5. The four sides of a play-ground measure, respectively, 464, 672, 368, and 240 ft. in length. How long must the boards used in fencing it be cut, so that they shall be of equal length, and as long as possible?

6. On the same day a merchant sends out traveling salesmen with instructions to return, respectively, in 1, 2, 3, 4, and 5 weeks. When any one returns he is sent out again at once for the same period as before. In how many weeks will they be together again?

7. In how many weeks would they come in together if sent out for 6, 9, 12, 18, and 36 weeks, respectively?

8. What is the least sum a dealer in live stock must have to be able to invest equal sums in horses at $105, mules at $68, and beeves at $30 per head? How much if he pays $105 per head for horses, $70 for mules, and $30 for beeves?

9. A court-yard 42 ft. 6 in. long, and 31 ft. 8 in. wide, is to be paved with square tiles of equal size, and as large as possible. How long and wide must each tile be?

10. A man having on deposit $695, $417, and $1251, respectively, in three different banks, wishes to draw out the whole in as large equal sums as possible. What is the greatest sum for which he must draw his checks?

CHAPTER IX.

FRACTIONS.

Introductory Exercises.

124. Draw on slate or paper twelve lines of equal length, and about one half inch apart. Divide and subdivide the lines as required by the questions.*

1. If the first line were divided into two equal parts, what would you call each part? How many such parts in a line? How many in 2, 5, 7, 9, 12 lines?

2. How many halves in 2 lines and a half? In 3 lines? In 4 lines and a half? In 11 lines and a half?

3. If each half were divided into two equal parts, how many of the new parts would there be in a whole line? What would you call one part? Two parts? etc. What part of a half is a fourth?

4. If the other lines were divided in the same way, how many fourths in each line? In 3 lines? In 8 lines? etc.

5. How many fourths in 2 lines and 1 fourth? In 5 and 1 fourth? In 6 and 3 fourths? In 7 and 1 fourth? etc.

6. Are 2 halves less or greater than a line? 5 fourths? etc.

7. How many whole lines in 2 halves? 4 halves? etc. How many in 3 fourths? 5 fourths? 9 fourths? etc.

8. If each fourth were divided into two equal parts, how many parts would there be in a line? What would you call them?

(Other questions should here be asked, similar to those on fourths, as above.)

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* Slips of paper of any uniform length, paper squares, etc., etc., are convenient materials for these exercises.

Draw 12 other lines. It would be well if these could be just 12 inches long.

1. If each line were divided into three equal parts, what would the parts be called? Why?

2. How would you express 1 part in figures? 2 parts? 5 parts?

(For reading and writing simple fractions see Art. 73, p. 77.)

3. Are $^4/_3$ greater or less than a whole line? How much? How many thirds in 3, 5, 7, 12 lines?

4. How many thirds in $2^1/_3$, $4^1/_3$, $5^2/_3$, $7^1/_3$, $8^2/_3$, $10^3/_3$?

5. How many whole lines, or how many lines and what parts of a line, are needed to make $^7/_3$, $^9/_3$, $^{14}/_3$, $^{21}/_3$, $^{15}/_3$, $^{30}/_3$, $^{32}/_3$?

6. If each third were divided into two equal parts, how many of the new parts would there be in a whole line? What would you call them? How would you express five of them in figures?

7. How many of these smaller parts are there in a third? In 2 thirds? In 3 thirds? How many sixths in 3 thirds?

8. Which is greater, $^2/_3$ or $^2/_6$ of one of these lines? Why? What part of a third is 1 sixth? Is $^2/_6$ a *part* or the whole of $^1/_3$?

9. How many sixths in $^1/_3$ of a line? In $^1/_2$? In $^2/_3$? In 2 lines? In 6, 8, 10, 11 lines?

10. How many sixths in $1^1/_3$? In $2^5/_6$? In $4^1/_3$? In $7^5/_6$? In $8^4/_6$? In $8^2/_3$? In $7^2/_3$?

11. How many lines, or lines and parts of a line, are needed to make $^{18}/_6$, $^{21}/_6$, $^{24}/_6$, $^{36}/_6$, $^{39}/_6$, $^{42}/_6$, $^{49}/_6$, $^{54}/_6$, $^{58}/_6$?

12. If each sixth were divided into two equal parts, what would the new parts be called? How would you express one or more of them? If the *line* is 12 inches long, what is the length of each *part*?

13. How many of them in $^1/_3$ of a line? In $^2/_3$, $^4/_6$, $^1/_6$, $^1/_3$, $^3/_6$?

14. How many twelfths in 2 lines and $^1/_2$? In $3^3/_{12}$? In $4^5/_{12}$? In $7^1/_{12}$? In $10^7/_{12}$? In $11^{11}/_{12}$? How many twelfths in $1^1/_3$, $2^1/_6$, $4^5/_6$, $7^3/_{12}$, $9^2/_3$ lines?

Questions upon the Rules in the Margin.

125. Note.—The following questions are designed to be only suggestive of exercises that may be given. A foot-rule or a yard-stick will afford many others.

1. Into how many parts is the first of these two measures divided by the horizontal line in the middle? What do you call the parts? Into how many parts is the measure at the right divided by the longest horizontal lines? What do you call these parts? Why?

2. Which is greater, ${}^1/_2$ or ${}^1/_3$? How can you tell without seeing or measuring the parts?

3. What parts of the whole do you get by dividing ${}^1/_3$ into 2 equal parts? Why?

4. How many sixths in ${}^1/_3$, ${}^2/_3$, ${}^3/_3$? What part of ${}^1/_2$ do you get by dividing it into 2 equal parts? What part of the whole is ${}^1/_2$ of ${}^1/_2$?

5. How many parts do you get by dividing each of the fourths into 2 equal parts? What are these new parts called? Why?

6. Which is longer, ${}^1/_6$ or ${}^1/_8$? Suppose you could not see, nor measure, would you know which is the greater, ${}^3/_6$ or ${}^3/_8$? How?

7. The sixths in the second measure are divided each into 2 equal parts. What is their name? Why?

8. Are these twelfths as large as the eighths in the other measure?

9. What are the smallest parts of the second measure? How many are there?

10. Are the smallest parts of the first measure as large as the smallest parts of the second measure? Can you tell by counting them?

ORAL EXERCISES.

1. Explain how it is that $\frac{1}{2}$ is equal to $\frac{2}{4}$.

Note.—Divide any whole thing or number into fourths, and show that one half is equal to 2 fourths.

2. Is $\frac{1}{2}$ equal to $\frac{3}{6}$, to $\frac{4}{8}$, to $\frac{6}{12}$, to $\frac{12}{24}$? State why.

3. Name some other parts equal to $\frac{1}{4}$; also parts equal to $\frac{1}{6}$, to $\frac{1}{8}$, to $\frac{1}{3}$, to $\frac{1}{12}$, to $\frac{2}{3}$, to $\frac{3}{4}$, to $\frac{5}{8}$, to $\frac{5}{6}$, to $\frac{7}{8}$, to $\frac{3}{8}$.

4. If you had a line divided into sixths, how could you change the sixths into twelfths? The twelfths back to sixths?

5. How many sixths in $\frac{1}{3}$, $\frac{2}{3}$? How many eighths in $\frac{3}{4}$, $\frac{1}{2}$?

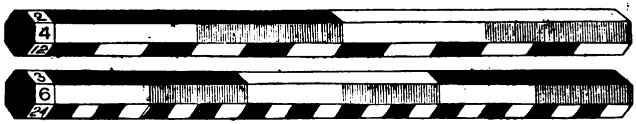
6. How many twelfths in $\frac{1}{6}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{2}$? In $\frac{2}{2}$, $\frac{2}{3}$, $\frac{2}{4}$, $\frac{2}{6}$?

7. How many twenty-fourths in $\frac{3}{8}$? In $\frac{5}{8}$? In $\frac{7}{8}$? How many in $\frac{2}{6}$, $\frac{4}{6}$, $\frac{3}{6}$, $\frac{5}{6}$? How many in $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$?

How much is

8. $\frac{1}{2}$ of $\frac{1}{2}$?	9. $\frac{1}{2}$ of $\frac{1}{8}$?	10. $\frac{1}{4}$ of $\frac{1}{2}$?	11. $\frac{1}{3}$ of $\frac{1}{2}$?
$\frac{1}{2}$ of $\frac{1}{4}$?	$\frac{1}{2}$ of $\frac{1}{6}$?	$\frac{1}{4}$ of $\frac{1}{3}$?	$\frac{1}{8}$ of $\frac{1}{3}$?
$\frac{1}{2}$ of $\frac{1}{3}$?	$\frac{1}{2}$ of $\frac{1}{12}$?	$\frac{1}{3}$ of $\frac{1}{4}$?	$\frac{1}{3}$ of $\frac{1}{3}$?

Note.—The following representation of a fraction rule will suggest other exercises. The figures at the left show how many parts each side is divided into.



12. How many wholes and ninths are in $\frac{15}{9}$, $\frac{29}{9}$, $\frac{21}{9}$, $\frac{30}{9}$, $\frac{47}{9}$?

13. Which makes the larger parts, dividing an apple into 10ths or 12ths? Which is the greater, $\frac{1}{8}$ or $\frac{1}{12}$ of a thing? $\frac{1}{20}$ or $\frac{1}{16}$? $\frac{1}{30}$ or $\frac{1}{31}$? $\frac{1}{4}$ or $\frac{1}{9}$?

14. Which is the greater, $\frac{7}{24}$ or $\frac{17}{24}$? $\frac{4}{7}$ or $\frac{6}{7}$? $\frac{73}{145}$ or $\frac{29}{145}$? $\frac{5}{8}$ or $\frac{3}{8}$? $\frac{7}{18}$ or $\frac{10}{18}$? Why?

15. Draw two lines of equal length. Divide one into thirds, the other into fourths, and find how many more twelfths there are in $\frac{1}{3}$ than in $\frac{1}{4}$.

Definitions.

126. A ***Fraction*** is one or more of the equal parts of a unit or whole.

127. The ***unit of the fraction*** is the unit which is divided. *One* of the equal parts is a ***fractional unit.***

128. Fractions obtained by the division of the unit into tenths, tenths of tenths or hundredths, etc., are called ***Decimal Fractions.*** All others are called ***Common Fractions,*** to distinguish them from decimals.

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129. Common Fractions may be expressed by words, as *two thirds*, or by figures, thus, $^2/_3$, the upper number standing for "two," the *number* of parts, and the lower one for "thirds," the *name* of the parts. (See Art. 73, page 77.)

130. The *number* of parts and the *name* of the parts are called the ***terms*** of the fraction.

131. The term which expresses the number of parts is the ***numerator*** (counter or numberer). The term which indicates the name of the parts is the ***denominator*** (namer).

Note.—Since the denominator indicates the name of the parts by showing *how many* parts there are in a unit, it may be treated as a number as well as a name.

132. A ***simple fraction*** is one whose terms are both integers, as $^9/_9$, $^{17}/_{20}$, etc.

133. A ***proper fraction*** is one whose numerator is less than the denominator, as $^2/_3$, $^3/_4$, etc.

134. An ***improper fraction*** is one whose numerator is equal to or greater than its denominator, as $^6/_6$, $^8/_7$, etc.

135. A ***mixed number*** is one which is composed of an integer and a fraction, as $3\,^1/_2$, $5\,^3/_7$, etc.

136. An integer may be expressed in the form of an improper fraction by writing it as a numerator, with 1 as a denominator. Thus, 5 may be written $^5/_1$, which is read 5 *ones* or 5.

Reductions.

Changes of Form, not of Value.

137. To reduce integers or mixed numbers to improper fractions, and the contrary.

Example.—1. Reduce $67\,{}^3/_5$ to fifths.

Analysis.—In 1 there are 5 fifths, hence in 67 there are 67 times 5 fifths = 335 fifths; 335 fifths + 3 fifths = 338 fifths.

Note.—The result being the same, we multiply 67 by 5, as the shortest way of obtaining 67 times 5.

Slate Work.

$$\begin{array}{r} 67 \\ \underline{5} \\ 335 \\ \underline{3} \\ 338 \end{array}$$

Example.—2. Reduce ${}^{19}/_5$ to an integer or mixed number.

Analysis.—5 fifths = 1. Hence 19 fifths contain as many units as there are times 5 fifths in 19 fifths = $3\,{}^4/_5$.

Suggestion.—For the rules in these cases the pupil may be required to state the processes by which he obtains the results.

Slate Work.

$$\begin{array}{r} 5)\underline{19} \\ 3\,{}^4/_5 \end{array}$$

Note.—In oral exercises the pupil should be required to announce results at once if possible, except when specially directed to give an analysis.

Reduce to improper fractions

3.	4.	5.	6.	7.	8.
$1^3/_5$	$2^2/_5$	$5^7/_8$	$4^1/_3$	$19^2/_3$	$31^1/_4$
$7^1/_8$	$1^7/_8$	$8^5/_7$	$3^3/_5$	$18^3/_4$	$29^1/_2$
$5^1/_2$	$3^1/_7$	$7^5/_8$	$8^2/_7$	$37^1/_2$	$33^1/_3$

Reduce to integers or mixed numbers

9.	10.	11.	12.	13.	14.
${}^5/_3$	${}^{11}/_8$	${}^{16}/_2$	${}^{23}/_7$	${}^{39}/_8$	${}^{33}/_2$
${}^3/_2$	${}^9/_7$	${}^{15}/_3$	${}^{25}/_3$	${}^{55}/_7$	${}^{44}/_2$
${}^5/_4$	${}^8/_3$	${}^{17}/_5$	${}^{32}/_7$	${}^{53}/_9$	${}^{39}/_3$

Reduce mixed numbers to improper fractions, and improper fractions to integers or mixed numbers.

15. $28^7/_{16}$	**18.** ${}^{421}/_7$	**21.** $100^{10}/_{11}$	**24.** ${}^{1828}/_{14}$	**27.** $487^5/_6$
16. $31^6/_{13}$	**19.** ${}^{357}/_6$	**22.** $375^{10}/_{23}$	**25.** ${}^{1327}/_{13}$	**28.** $723^9/_{10}$
17. $24^8/_{17}$	**20.** ${}^{216}/_5$	**23.** $841\,{}^5/_{51}$	**26.** ${}^{4563}/_{15}$	**29.** $891^9/_{12}$

30. How many yards in ${}^{356}/_9$ of a yard?

Reducing to Higher and Lower Terms.

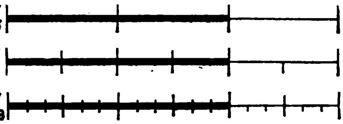
Let it be remembered that *the value of a fraction is not changed by multiplying or dividing both terms by the same number; thus—*

$$\frac{4\times 3}{6\times 3}=\frac{12}{18} \qquad \frac{4\div 2}{6\div 2}=\frac{2}{3}$$

For it is clear that $^2/_3$

is equivalent to $^4/_6$

or to $^{12}/_{18}$



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138. To reduce a fraction to higher terms (greater numerator and denominator).

Example.—1. Reduce $^3/_4$ to twelfths.

Process.

$$\frac{3\times 3}{4\times 3}=\frac{9}{12}$$

Explanation.—If each fourth of a slip of paper be divided into three equal parts, the whole slip will contain 4 times 3 parts, or 12 twelfths, and 3 fourths will contain 3 times 3 parts, or $^9/_{12}$.

Hence the following

***Rule.*—To reduce a fraction to higher terms, divide the required denominator by the denominator of the given fraction, and multiply both of its terms by the quotient.**

ORAL EXERCISES.

Reduce

2. $^1/_2$, $^1/_4$, $^1/_6$, $^1/_3$, to 12ths.

3. 1, $^1/_4$, $^3/_8$, $^1/_2$, " 8ths.

4. $^1/_2$, $^1/_3$, $^1/_6$, $^5/_9$, " 18ths.

5. $^2/_3$, $^3/_4$, $^5/_6$, $^7/_8$, " 24ths.

6. $^3/_4$, $^5/_8$, $^1/_2$, $^5/_{16}$, " 16ths.

Reduce

7. $^2/_3$, $^5/_6$, $^7/_8$, $^7/_{12}$, $^5/_{18}$, to 72ds.

8. $^1/_2$, $^5/_6$, $^4/_9$, $^{14}/_{27}$, $^{14}/_{18}$, " 54ths.

9. $^2/_3$, $^3/_5$, $^7/_9$, $^5/_9$, $^8/_{15}$, " 45ths.

10. $^5/_8$, $^1/_4$, $^9/_{16}$, $^1/_2$, $^{17}/_{24}$, " 48ths.

11. $^7/_9$, $^3/_4$, $^7/_{12}$, $^{11}/_3$, $^{17}/_{18}$, " 36ths.

Let the first three examples be illustrated by division of lines or folding of paper.

12. Change $^1/_2$, $^1/_5$, $^2/_5$, $^3/_5$, $^4/_5$, and $^5/_5$ to tenths.

13. Change to hundredths $^1/_4$, $^2/_5$, $^7/_{10}$, $^3/_{20}$, $^3/_5$, $^7/_{25}$, $^9/_{10}$, $^4/_5$, $^3/_4$, $^{23}/_{25}$.

139. To reduce fractions to lower terms (smaller numerator and denominator).

Note.—In the preceding case (p. 141) we computed the numerical result of dividing any given equal parts of a thing or number into smaller equal parts. In this case, we are to find the result of uniting smaller into larger ones.

Example.—1. Reduce $^9/_{12}$ to lower terms.

Process.

$$3\Big|\frac{9}{12} = {}^3/_4$$

Explanation.—Uniting each 3 twelfths of any object into 1 larger part, we have 4 larger parts (fourths), and in the 9 twelfths there are 3 of them. Hence $^9/_{12} = {}^3/_4$.

Reduce to lower terms

2. $^{12}/_{24}$ $^{64}/_{72}$ $^{5}/_{90}$ $^{66}/_{240}$ $^{59}/_{177}$ $^{10}/_{50}$ $^{48}/_{72}$ $^{15}/_{75}$ $^{64}/_{144}$ $^{41}/_{164}$

3. $^{15}/_{35}$ $^{6}/_{18}$ $^{12}/_{96}$ $^{72}/_{144}$ $^{480}/_{500}$ $^{15}/_{45}$ $^{31}/_{93}$ $^{34}/_{68}$ $^{48}/_{144}$ $^{196}/_{444}$

140. Thus we find that *fractions are reduced to lower terms by dividing both terms by any common factor.*

And, *that they are reduced to their lowest terms by dividing them, successively, by* **all** *the prime factors common to the two;* or, *by the continued product of all, the latter being their greatest common factor, and hence their greatest common divisor.*

141. When the terms of a fraction are large, or not readily resolved into factors, the following method of finding the greatest common divisor will be convenient.

4. Reduce $^{475}/_{589}$ to lowest terms.

Process.

```
475)589(1
    475
    ---
    114)475(4
        456
        ---
         19)114(6
            114
```

$$19\Big|\frac{475}{589} = {}^{25}/_{31}$$

Explanation.—We divide the greater number by the less, and the divisor by the remainder, and so on till there is no remainder. The last divisor (19) is the g. c. d. sought.

Why it is that we can always depend on such a process to find the g. c. d. is not readily understood by the young learner. The demonstration is therefore reserved for the appendix. No formal rule is necessary.

Reduce to lowest terms

5. $^{1645}/_{1833}$ **7.** $^{1363}/_{1739}$ **9.** $^{8903}/_{13201}$ **11.** $^{1261}/_{1649}$ **13.** $^{1989}/_{2873}$

6. $^{1589}/_{2724}$ **8.** $^{8903}/_{10991}$ **10.** $^{1945}/_{3501}$ **12.** $^{2813}/_{3783}$ **14.** $^{1945}/_{3501}$

Addition of Common Fractions.

ORAL EXERCISES.

1. Mary takes $\frac{1}{6}$ of a pie for lunch at school, William takes $\frac{2}{6}$, John $\frac{2}{6}$, and Henry $\frac{2}{6}$. How many sixths do they all take?

Find the sum of

2. $\frac{1}{5}+\frac{3}{5}=$	**3.** $\frac{3}{4}+\frac{1}{4}=$	**4.** $\frac{5}{11}+\frac{8}{11}=$	**5.** $\frac{4}{5}+\frac{3}{5}+\frac{2}{5}=$
$\frac{4}{7}+\frac{2}{7}=$	$\frac{6}{10}+\frac{4}{10}=$	$\frac{6}{12}+\frac{7}{12}=$	$\frac{7}{8}+\frac{5}{8}+\frac{3}{8}=$
$\frac{3}{8}+\frac{4}{8}=$	$\frac{7}{13}+\frac{6}{13}=$	$\frac{9}{17}+\frac{9}{17}=$	$\frac{4}{9}+\frac{5}{9}+\frac{7}{9}=$

6. Sarah has $\frac{5}{8}$ of a yard of ribbon, and Lucy has $\frac{7}{8}$; how many 8ths have they together? How many yards, and what part of a yard?

Find the sum of

7. $7\frac{1}{2}+\frac{1}{2}=$	**8.** $5\frac{6}{7}+\frac{3}{7}=$	**9.** $6\frac{5}{6}+3\frac{1}{6}=$	**10.** $16\frac{7}{16}+\frac{9}{16}=$
$3\frac{2}{5}+\frac{3}{5}=$	$4\frac{3}{8}+\frac{7}{8}=$	$3\frac{9}{11}+5\frac{2}{11}=$	$14\frac{30}{59}+\frac{35}{59}=$
$5\frac{4}{7}+\frac{3}{7}=$	$8\frac{4}{9}+\frac{7}{9}=$	$5\frac{6}{17}+8\frac{11}{17}=$	$12\frac{17}{35}+\frac{19}{35}=$

11. One piece of cloth contains $\frac{3}{4}$ of a yard, and another $\frac{2}{3}$ of a yard. How many yards in the two pieces?

Oral Solution.—$\frac{3}{4}$ is equal to $\frac{9}{12}$, $\frac{2}{3}$ is equal to $\frac{8}{12}$, $\frac{9}{12}$ and $\frac{8}{12}$ together are equal to $\frac{17}{12}$. $\frac{17}{12}=1\frac{5}{12}$; hence, $\frac{3}{4}$ and $\frac{2}{3}$ of a yard of cloth equal $1\frac{5}{12}$ yd.

Illustration.—3 fourths and 2 thirds of a sheet of paper make neither 5 fourths nor 5 thirds, but subdividing both into twelfths we find that they are together equal to $\frac{17}{12}$ or $1\frac{5}{12}$.

142. *Fractions to be added together must have a common denominator.*

Find the sum of

12. $\frac{1}{2}+\frac{1}{3}=$	**13.** $\frac{1}{2}+\frac{3}{4}=$	**14.** $\frac{2}{3}+\frac{2}{6}=$	**15.** $\frac{5}{6}+\frac{7}{12}=$
$\frac{1}{4}+\frac{1}{2}=$	$\frac{1}{4}+\frac{3}{8}=$	$\frac{1}{2}+\frac{4}{5}=$	$\frac{3}{8}+\frac{1}{24}=$
$\frac{1}{3}+\frac{1}{6}=$	$\frac{1}{3}+\frac{5}{6}=$	$\frac{3}{4}+\frac{7}{8}=$	$\frac{5}{7}+\frac{5}{14}=$
$\frac{1}{4}+\frac{1}{8}=$	$\frac{1}{2}+\frac{3}{5}=$	$\frac{2}{3}+\frac{5}{6}=$	$\frac{2}{3}+\frac{7}{15}=$
$\frac{2}{3}+\frac{4}{5}=$	$\frac{4}{5}+\frac{5}{6}=$	$\frac{2}{3}+\frac{7}{8}=$	$\frac{3}{4}+\frac{3}{5}=$

16. $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}=$	**18.** $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{6}=$	**20.** $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}=$
17. $\frac{2}{3}+\frac{5}{6}+\frac{6}{12}=$	**19.** $\frac{1}{2}+\frac{4}{5}+\frac{3}{10}+\frac{7}{15}=$	**21.** $\frac{6}{7}+\frac{2}{3}+\frac{11}{21}=$

WRITTEN WORK.

143. Example.—1. Find the sum of $^7/_{18}$ and $^8/_{15}$.

Solution with Sheets of Paper.—By subdividing 18ths and 15ths each into 2, 3, etc., equal parts,

The 18ths become successively 36ths, 54ths, 72ds, 90ths, 108ths, etc.

The 15ths become successively 30ths, 45ths, 60ths, 75ths, 90ths, etc.

We thus find that 18ths and 15ths can both be changed to 90ths by dividing each 18th into 5, and each 15th into 6 equal parts; and hence that the common denominator of the equivalent fractions must be 90, which is the l. c. m. of 18 and 15.

The arithmetical process of finding the sum of two or more fractions having unlike denominators embraces corresponding steps, viz., *first*, finding the l. c. m. of the denominators; *second*, reducing the given fractions to a common denominator; and, *third*, adding together the numerators. Thus,

The Arithmetical Process.

$18=2\times3\times3$		90
$15=5\times3$	18	$5\times7=35$
	15	$6\times8=48$
Least Com. Mult.		$^{83}/_{90}$
$2\times3\times3\times5=90$		

Explanation.—Having found the l. c. m. we divide it by 18 and 15, and thus find that we must multiply the 7 (eighteenths) by 5, and the 8 (fifteenths) by 6 to change them to 90ths. Having performed these multiplications we add $^{35}/_{90}$ and $^{48}/_{90}$ to obtain the sum $^{83}/_{90}$.

By dispensing with such part of the written work as can be performed without the aid of the pencil, it may be abbreviated as follows:

2. Find the sum of $^3/_4$, $^1/_8$, $^5/_6$, $^1/_{12}$, $^2/_3$.

	24ths
$^3/_4$	18
$^1/_8$	3
$^5/_6$	20
$^1/_{12}$	2
$^2/_3$	16
	$^{59}/_{24}=2\,^{11}/_{24}$

Explanation.—24 being the least common multiple of the denominators, the given fractions may all be reduced to 24ths. In $^1/_4$ there are $^6/_{24}$, and in $^3/_4$ there are 3 times 6 or $^{18}/_{24}$, etc., etc.

144. ***Rule.***—**1. Reduce the fractions, if necessary, to fractions having a common denominator.**

2. Add the numerators, and under the sum write the common denominator.

3. In adding mixed numbers, add first the fractions, then the integers, and unite the results.

Caution.—If any arithmetical process results in a fraction, the work is not complete unless the fraction is expressed in its lowest terms. Improper fractions must be reduced to whole or mixed numbers.

Add by columns, then by lines. Test results.* (See note, p. 32.)

	3.	4.	5.	6.	7.	8.
9.	$\frac{2}{5}+$	$\frac{2}{3}+$	$\frac{1}{2}+$	$\frac{5}{6}+$	$\frac{11}{12}+$	$\frac{7}{24}$
10.	$\frac{3}{4}+$	$\frac{1}{4}+$	$\frac{7}{12}+$	$\frac{1}{2}+$	$\frac{7}{8}+$	$\frac{9}{16}$
11.	$\frac{7}{8}+$	$\frac{2}{3}+$	$\frac{11}{12}+$	$\frac{3}{4}+$	$\frac{17}{24}+$	$\frac{4}{5}$
12.	$\frac{5}{6}+$	$\frac{7}{12}+$	$\frac{5}{14}+$	$\frac{2}{3}+$	$\frac{3}{4}+$	$\frac{1}{2}$

	13.	14.	15.	16.	17.	18.
19.	$\frac{1}{3}+$	$\frac{3}{4}+$	$\frac{2}{3}+$	$\frac{11}{12}+$	$\frac{3}{7}+$	$\frac{5}{6}$
20.	$\frac{5}{8}+$	$\frac{6}{7}+$	$\frac{5}{6}+$	$\frac{3}{5}+$	$\frac{7}{9}+$	$\frac{3}{4}$
21	$\frac{3}{7}+$	$\frac{5}{8}+$	$\frac{6}{11}+$	$\frac{5}{9}+$	$\frac{1}{2}+$	$\frac{1}{6}$
22.	$\frac{5}{6}+$	$\frac{1}{7}+$	$\frac{7}{8}+$	$\frac{9}{11}+$	$\frac{3}{4}+$	$\frac{2}{5}$

23. $24\frac{7}{16}+18\frac{3}{5}+50\frac{3}{10}+18\frac{7}{30}+4\frac{1}{2}+2\frac{5}{6}+59\frac{3}{7}$.

24. $3\frac{4}{9}+5\frac{5}{8}+17\frac{1}{6}+4\frac{5}{72}+37\frac{13}{24}+17\frac{11}{12}+70\frac{5}{9}$.

25. $15\frac{7}{9}+24\frac{3}{4}+38\frac{3}{5}+27\frac{13}{16}+33\frac{14}{25}+19\frac{7}{25}+8\frac{1}{8}$.

	26.	27.	28.	29.	30.	31.
32.	$4\frac{4}{5}+$	$3\frac{1}{5}+$	$4\frac{2}{4}+$	$13\frac{1}{2}+$	$17\frac{1}{60}+$	$14\frac{7}{24}$
33.	$63\frac{3}{4}+$	$21\frac{7}{20}+$	$45\frac{7}{20}+$	$25\frac{3}{4}+$	$14\frac{23}{30}+$	$22\frac{5}{12}$
34.	$227\frac{7}{10}+$	$243\frac{3}{4}+$	$26\frac{9}{10}+$	$36\frac{7}{8}+$	$11\frac{11}{15}+$	$46\frac{3}{8}$
35.	$438\frac{39}{50}+$	$657\frac{43}{50}+$	$38\frac{27}{50}+$	$49\frac{15}{16}+$	$19\frac{11}{12}+$	$39\frac{5}{6}$
36.	$840\frac{1}{2}+$	$760\frac{27}{100}+$	$14\frac{99}{100}+$	$51\frac{5}{8}+$	$15\frac{5}{6}+$	$10\frac{1}{2}$

37. Add $\frac{2}{3}+\frac{5}{8}+\frac{4}{9}+\frac{2}{5}+\frac{3}{4}+\frac{7}{16}+\frac{4}{7}+\frac{3}{16}+\frac{4}{15}$

38. Add $\frac{3}{4}+\frac{4}{5}+\frac{11}{12}+\frac{3}{8}+\frac{4}{15}+\frac{5}{9}+\frac{5}{7}+\frac{4}{15}+\frac{7}{18}$

39. Add $\frac{4}{5}+\frac{5}{6}+\frac{7}{8}+\frac{4}{9}+\frac{5}{12}+\frac{23}{48}+\frac{17}{60}+\frac{7}{48}+\frac{1}{24}$

Applications.—1. Four barrels of cider contain severally $25\frac{3}{4}$ gal., $23\frac{7}{12}$ gal., $29\frac{5}{6}$ gal., $28\frac{5}{6}$ gal. How many gallons in all?

2. What is the total weight of 6 bales, weighing respectively $5\frac{3}{8}$ cwt., $4\frac{7}{10}$ cwt., $6\frac{3}{5}$ cwt., $4\frac{7}{8}$ cwt., $6\frac{19}{20}$ cwt., $6\frac{1}{2}$ cwt.?

3. Mr. Abel has $64\frac{7}{20}$ acres in farm-land, $35\frac{5}{8}$ in meadow-land, $32\frac{19}{50}$ in woodland. How many acres in all?

4. Of 5 brothers the youngest is $11\frac{2}{5}$ years old, the second $3\frac{1}{4}$ years older, the third $2\frac{1}{6}$ years older than the second, the fourth $2\frac{1}{5}$ years older than the third, the fifth $2\frac{7}{9}$ years older than the fourth. How old is each one?

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* Any two or more columns may be assigned for an exercise. Since these examples are self-testing, no answers are given.

Subtraction of Common Fractions.

ORAL EXERCISES.

1. If there are $5/7$ of a pie on a plate, and Harry takes $2/7$, what part of the pie remains?

2.	3.	4.	5.	6.
$7/8 - 5/8 =$	$8/8 - 3/8 =$	$5 - 1/2 =$	$7 - 3/4 =$	$2\,5/8 - 3/8 =$
$17/18 - 11/18 =$	$7/7 - 2/7 =$	$6 - 1/4 =$	$8 - 5/9 =$	$4\,9/10 - 7/10 =$
$21/25 - 12/25 =$	$9/9 - 4/9 =$	$3 - 2/3 =$	$9 - 2/5 =$	$6\,4/5 - 2/5 =$
$81/100 - 25/100 =$	$6/6 - 2/6 =$	$4 - 2/5 =$	$10 - 3/7 =$	$7\,6/7 - 3/7 =$

7.	8.	9.	10.
$2\,3/8 - 5/8 =$	$3\,3/10 - 7/10 =$	$8\,1/4 - 4\,3/4 =$	$7\,3/9 - 3\,7/9 =$
$4\,5/12 - 10/12 =$	$7\,7/20 - 11/20 =$	$59\,2/5 - 4\,4/5 =$	$33\,2/7 - 2\,6/7 =$
$6\,5/9 - 7/9 =$	$6\,4/13 - 5/13 =$	$85\,1/3 - 1\,2/3 =$	$22\,3/16 - 1\,15/16 =$
$5\,5/17 - 8/17 =$	$9\,5/8 - 7/8 =$	$98\,7/11 - 2\,5/11 =$	$43\,4/15 - 1\,14/15 =$
$2\,4/7 - 6/7 =$	$3\,6/27 - 7/27 =$	$14\,3/41 - 3\,5/41 =$	$17\,19/60 - 4\,28/60 =$

11. Subtract the sum of $3/44$, $5/44$, $17/44$, $22/44$, and $13/44$, from $5\,1/44$.

12. Subtract the sum of $6/19$, $3/19$, $5/19$, $17/19$, and $15/19$, from $7\,9/19$.

13. Subtract the sum of $1\,3/100$, $4\,7/100$, $5\,13/100$, $6\,17/100$, and $2\,9/100$, from $49\,7/100$.

14. Sarah has $5/6$ of a yard of velvet. How much will she have left if she uses $3/4$ of a yard for trimming a dress?

Oral Solution.—$5/6$ is equal to $10/12$, $3/4$ is equal to $9/12$; $9/12$ being subtracted from $10/12$ leaves $1/12$. Hence, Sarah would have $1/12$ of a yard remaining.

145. *That one fraction may be subtracted from another, the two must have a common denominator.*

146. *Rule.*—**1. Reduce the fractions, if necessary, to fractions having a common denominator.**

2. Subtract the numerator of the subtrahend from the numerator of the minuend, and write the remainder over the common denominator.

3. In the subtraction of mixed numbers, if the fraction in the subtrahend is greater than that in the minuend, take 1 unit from the whole number in the minuend and add it in fractional form to the fraction of the minuend, and then subtract.

ORAL AND SLATE EXERCISES.

Note.—The oral exercises may be carried as far as the ability of the pupil will permit.

15.	16.	17.	18.	19.
$\frac{1}{2}-\frac{1}{4}=$	$\frac{1}{2}-\frac{3}{8}=$	$\frac{3}{4}-\frac{5}{8}=$	$1\frac{1}{4}-\frac{7}{8}=$	$3\frac{3}{4}-\frac{1}{2}=$
$\frac{1}{3}-\frac{1}{6}=$	$\frac{1}{3}-\frac{1}{12}=$	$\frac{2}{3}-\frac{5}{12}=$	$2\frac{1}{3}-\frac{8}{9}=$	$2\frac{7}{8}-\frac{3}{4}=$
$\frac{1}{4}-\frac{1}{8}=$	$\frac{1}{4}-\frac{5}{24}=$	$\frac{5}{6}-\frac{7}{12}=$	$4\frac{5}{9}-\frac{17}{18}=$	$7\frac{3}{6}-\frac{2}{3}=$
$\frac{1}{5}-\frac{1}{10}=$	$\frac{1}{8}-\frac{1}{16}=$	$\frac{4}{9}-\frac{5}{18}=$	$6\frac{3}{8}-\frac{7}{16}=$	$9\frac{1}{18}-\frac{5}{9}=$
$\frac{1}{2}-\frac{1}{8}=$	$\frac{1}{5}-\frac{2}{25}=$	$\frac{5}{8}-\frac{7}{16}=$	$7\frac{1}{12}-\frac{5}{24}=$	$7\frac{7}{12}-\frac{3}{4}=$
$\frac{1}{3}-\frac{1}{9}=$	$\frac{1}{7}-\frac{2}{21}=$	$\frac{9}{10}-\frac{11}{20}=$	$5\frac{3}{11}-\frac{5}{22}=$	$6\frac{5}{21}-\frac{2}{3}=$
$\frac{1}{8}-\frac{1}{24}=$	$\frac{1}{4}-\frac{3}{16}=$	$\frac{3}{8}-\frac{5}{24}=$	$8\frac{4}{25}-\frac{7}{50}=$	$5\frac{6}{25}-\frac{4}{5}=$

20.	21.	22.	23.
$4\frac{7}{8}-3\frac{5}{16}=$	$7\frac{4}{5}-2\frac{1}{3}=$	$6\frac{2}{3}-1\frac{1}{5}=$	$5\frac{3}{4}-3\frac{2}{3}=$
$3\frac{5}{6}-1\frac{3}{5}=$	$13\frac{5}{8}-1\frac{1}{6}=$	$24\frac{7}{9}-2\frac{3}{8}=$	$7\frac{1}{11}-3\frac{5}{12}=$
$2\frac{1}{6}-1\frac{7}{8}=$	$25\frac{8}{9}-2\frac{7}{8}=$	$37\frac{6}{11}-3\frac{7}{10}=$	$9\frac{1}{2}-5\frac{2}{27}=$

24. Subtract $\$2\frac{3}{4}$ from $\$7\frac{1}{2}$, $\$6\frac{1}{6}$, $\$5\frac{5}{8}$, $\$9\frac{3}{7}$, $\$4\frac{2}{9}$.

25. Subtract $1\frac{5}{8}$ lb. from $6\frac{1}{16}$, $7\frac{3}{8}$, $9\frac{5}{16}$, $11\frac{7}{24}$, $5\frac{5}{12}$ lb.

26. Subtract $3\frac{5}{6}$ qt. from $8\frac{1}{12}$, $9\frac{7}{18}$, $11\frac{5}{8}$, $7\frac{7}{30}$ qt.

27. $2\frac{7}{16}-1\frac{31}{48}=$	**34.** $56\frac{2}{9}-27\frac{5}{6}=$	**41.** $48\frac{3}{8}-7\frac{11}{12}=$
28. $3\frac{3}{20}-1\frac{17}{120}=$	**35.** $165\frac{3}{8}-39\frac{7}{12}=$	**42.** $824\frac{2}{5}-6\frac{7}{12}=$
29. $74\frac{7}{25}-2\frac{19}{50}=$	**36.** $283\frac{5}{6}-46\frac{3}{14}=$	**43.** $936\frac{4}{15}-4\frac{11}{20}=$
30. $68\frac{11}{20}-9\frac{7}{9}=$	**37.** $394\frac{4}{15}-53\frac{7}{10}=$	**44.** $141\frac{7}{20}-3\frac{2}{15}=$
31. $15\frac{13}{18}-9\frac{8}{9}=$	**38.** $443\frac{7}{20}-31\frac{9}{16}=$	**45.** $475\frac{8}{35}-7\frac{9}{28}=$
32. $60\frac{15}{16}-3\frac{1}{7}=$	**39.** $527\frac{6}{17}-13\frac{7}{12}=$	**46.** $718\frac{7}{22}-8\frac{6}{23}=$
33. $25\frac{13}{16}-2\frac{7}{12}=$	**40.** $136\frac{19}{24}-4\frac{3}{4}=$	**47.** $248\frac{18}{25}-5\frac{2}{3}=$

48. From $585\frac{6}{11}+456\frac{15}{32}+354\frac{17}{30}$ take $8\frac{11}{27}+9\frac{13}{15}+1\frac{1}{11}$.

49. Take $7\frac{7}{8}$ inches from $16\frac{5}{6}$ in., from $18\frac{7}{23}$ in., from $21\frac{5}{17}$ in., from $23\frac{7}{24}$ in., from $29\frac{7}{29}$ in.

50. Take $13\frac{7}{11}$ minutes from $23\frac{1}{6}$, $47\frac{1}{19}$, $31\frac{5}{7}$, $56\frac{17}{23}$, $35\frac{6}{23}$ minutes.

51. Subtract each number from the next to the right; add the first number and all the remainders together. (See No. 1, p. 96.)

$\frac{1}{5}$, $2\frac{1}{2}$, $2\frac{5}{8}$, $3\frac{1}{4}$, $3\frac{3}{8}$, $4\frac{3}{4}$, $5\frac{1}{16}$, $6\frac{1}{4}$, 300.

Applications.—1. A train reached Chicago at 10 o'clock; it had made the trip from Milwaukee in $3\frac{5}{6}$ hours. At what time did it start from Milwaukee?

2. A person bought a piece of linen, measuring $60\frac{5}{8}$ yards. After the linen was washed it measured only $59\frac{5}{9}$ yd. How much had it shrunk?

3. A grocer had 2 cheeses, one weighing $38\frac{7}{8}$ and the other $45\frac{17}{32}$ pounds. He sold $7\frac{2}{3}$ lb. of each. What was the difference between the weights before and after the sale?

4. What is the difference in age of two persons, now $73\frac{7}{12}$ and $46\frac{2}{3}$ years old respectively? What will it be 10 years hence?

5. A thermometer showed at noon $73\frac{1}{2}$ degrees above zero. At 6 o'clock P. M. it showed $65\frac{3}{4}$ degrees. How much had it fallen?

6. A grocer received a box of tea weighing $24\frac{7}{32}$ pounds. The weight of the box alone was $3\frac{9}{16}$ lb. How much did the tea weigh?

7. From $\$120\frac{3}{5}$ the following sums were taken: $\$6\frac{1}{2}$, $\$12\frac{2}{5}$, $\$26\frac{17}{20}$, $\$20\frac{11}{50}$. What was the remainder?

8. Last fall we received $17\frac{1}{2}$ tons of hard coal; in the spring $1\frac{5}{8}$ tons were left. How much had we used during the winter?

9. A boy said, "If I had $\$5\frac{2}{3}$ more than I have, I should have $\$21\frac{1}{2}$." How much did he have?

10. A flag-staff $48\frac{3}{4}$ feet high was broken off near the top by a storm, so that it measured only $41\frac{5}{8}$ ft. How long was the piece that had been broken off?

11. A farmer, owning $388\frac{2}{3}$ acres of land, bought in addition $251\frac{5}{8}$, and then sold parcels containing, respectively, $84\frac{3}{4}$, $26\frac{7}{8}$, $38\frac{4}{5}$, $29\frac{11}{12}$, $93\frac{3}{7}$, and $84\frac{9}{10}$ acres. How much had he left?

12. A grocer cut $1\frac{3}{4}$, $2\frac{1}{3}$, $2\frac{7}{8}$, $3\frac{1}{32}$, $1\frac{7}{8}$, $5\frac{15}{16}$, $9\frac{2}{3}$, $1\frac{1}{2}$, $2\frac{3}{4}$, $\frac{1}{16}$, $\frac{13}{32}$, $2\frac{5}{6}$ pounds from a cheese which weighed $43\frac{7}{8}$ lb. How many pounds remained?

13. One man cuts $3\frac{2}{3}$ cords of firewood per day, another $3\frac{3}{8}$ cords per day. The first works 7 days, the second 6 days. How much does one cut more than the other?

Multiplication in Common Fractions.

Example.—1. Three times $^3/_4$ inch are how many inches?

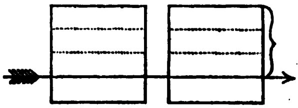
Solution.—$^3/_1 \times {}^3/_4$ in. $= {}^9/_4$ in. $= 2^1/_4$ in.

2. Three times $2^3/_4$ inches are how many inches?

Analysis.—$3 \times 2^3/_4$ in. $= {}^3/_1 \times {}^{11}/_4$ in. $= {}^{33}/_4$ in. $= 8^1/_4$ in.

3. $^3/_4$ of 2 are how many?

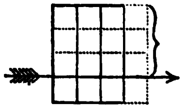
Analysis.—$^3/_4$ of $^2/_1 = {}^6/_4 = 1^1/_2$.



Note.—After a simple fraction the sign $\times$ should be read "of" not "times."

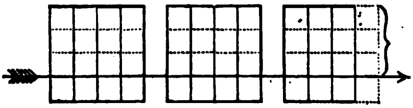
4. $^3/_4$ of $^3/_4$ are how many?

Analysis.—$^1/_4$ of $^1/_4 = {}^1/_{16}$; $^1/_4$ of $^3/_4 = {}^3/_{16}$; $^3/_4$ of $^3/_4 = {}^9/_{16}$.



5. $^3/_4$ of $2^3/_4$ are how many?

Illustration.—The line of the arrow cuts off $^1/_4$ of the $2^3/_4$ squares represented, leaving $^3/_4$ of the $2^3/_4$ squares above it.



Analysis.—$^3/_4$ of $2^3/_4 = {}^3/_4$ of $^{11}/_4$; $^1/_4$ of $^1/_4 = {}^1/_{16}$; $^1/_4$ of $^{11}/_4 = {}^{11}/_{16}$; $^3/_4$ of $^{11}/_4 = {}^{33}/_{16} = 2^1/_{16}$.

6. $3^3/_4 \times 2^3/_4$ are how many?

Illustration.—Copy the last diagram four times, omitting the arrow in each line of squares except the fourth. Thus, above the line of the arrow $3^3/_4$ times $2^3/_4$ squares will be represented, illustrating the following analysis.

Analysis.—$3^3/_4 \times 2^3/_4 = {}^{15}/_4$ of $^{11}/_4$; $^1/_4$ of $^1/_4 = {}^1/_{16}$; $^1/_4$ of $^{11}/_4 = {}^{11}/_{16}$; $^{15}/_4$ of $^{11}/_4 = {}^{165}/_{16} = 10^5/_{16}$.

The parts of the several analyses printed in **heavy type** are the only parts needed in a written solution. Whence the

147. ***Rule.*—Reduce integers and mixed numbers to improper fractions. Multiply the numerators together for the numerator of the product, and the denominators together for the denominator of the product.**

For practice in multiplication of fractions, the pupil may complete the following tables, and construct similar ones. When the multiplier is a fraction, he should substitute the word "of" for "times" in all oral exercises.

1	2	3	4	5	6	7
$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$
$\frac{1}{3}$	$\frac{2}{3}$	1	$1\frac{1}{3}$	$1\frac{2}{3}$	2	
$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1			
$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$				
$\frac{1}{6}$						
$\frac{1}{7}$						

1	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{6}$	$\frac{6}{7}$	$\frac{7}{7}$
2	1	$1\frac{1}{3}$	$1\frac{1}{2}$	$1\frac{3}{5}$			
3	$1\frac{1}{2}$	2	$2\frac{1}{4}$				
4	2	$2\frac{2}{3}$	3				
5	$2\frac{1}{2}$						
6	3						
7							

1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	&c.
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{10}$			
$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{9}$					
$\frac{1}{4}$	$\frac{1}{8}$						
$\frac{1}{5}$							
&c.							

1	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{3}{7}$	$\frac{4}{7}$	$\frac{5}{7}$	$\frac{6}{7}$	$\frac{7}{7}$
$\frac{1}{5}$	$\frac{1}{35}$	$\frac{2}{35}$	$\frac{3}{35}$	$\frac{4}{35}$			
$\frac{2}{5}$	$\frac{2}{35}$	$\frac{4}{35}$					
$\frac{3}{5}$	$\frac{3}{35}$				$\frac{3}{7}$		
$\frac{4}{5}$	$\frac{4}{25}$						$\frac{4}{5}$
$\frac{5}{5}$	$\frac{1}{7}$	$\frac{2}{7}$					1

ORAL EXERCISES.

Note.—Let the oral exercises be carried as far as possible.

1.	2.	3.	4.
$7\times\frac{1}{2}=$	$\frac{1}{24}\times26=$	$6\times\frac{2}{3}=$	$\frac{7}{10}\times23=$
$5\times\frac{1}{3}=$	$\frac{1}{61}\times72=$	$7\times\frac{4}{5}=$	$\frac{11}{15}\times60=$
$4\times\frac{1}{5}=$	$\frac{1}{25}\times36=$	$9\times\frac{2}{7}=$	$\frac{7}{12}\times29=$
$6\times\frac{1}{4}=$	$\frac{1}{38}\times47=$	$3\times\frac{3}{4}=$	$\frac{5}{8}\times13=$
$8\times\frac{1}{6}=$	$\frac{1}{27}\times54=$	$5\times\frac{6}{7}=$	$\frac{9}{17}\times15=$
$9\times\frac{1}{7}=$	$\frac{1}{56}\times63=$	$4\times\frac{5}{6}=$	$\frac{4}{7}\times25=$
$10\times\frac{1}{9}=$	$\frac{1}{21}\times33=$	$8\times\frac{7}{8}=$	$\frac{6}{25}\times39=$

Note.—In written work, always cancel factors that are common to both numerator and denominator.

5. How much is $6 \times \frac{1}{24}$ day? $\frac{1}{6}$ d.? $\frac{1}{4}$ d.?
6. How much is $9 \times \frac{3}{8}$ lb.? $\frac{3}{4}$ lb.? $\frac{17}{16}$ lb.?
7. How much is $17 \times \$\frac{1}{4}$? $\$\frac{7}{10}$? $\$\frac{3}{5}$?
8. What is $\frac{3}{4}$ of 1 hour? 6 h.? 9 h.?
9. What is $\frac{7}{12}$ of 3, 5, 7 feet?

10.	11.	12.	13.
$\frac{1}{2}$ of $\frac{1}{2}=$	$\frac{2}{3} \times \frac{1}{4}=$	$\frac{3}{8}$ of $\frac{2}{5}=$	$\frac{4}{21} \times 17=$
$\frac{1}{2}$ of $\frac{1}{3}=$	$\frac{3}{12} \times \frac{1}{2}=$	$\frac{2}{5}$ " $\frac{3}{4}=$	$19 \times \frac{13}{25}=$
$\frac{1}{3}$ of $\frac{1}{3}=$	$\frac{3}{4} \times \frac{1}{5}=$	$\frac{4}{9}$ " $\frac{3}{8}=$	$\frac{3}{11} \times 28=$
$\frac{1}{4}$ of $\frac{1}{4}=$	$\frac{5}{12} \times \frac{1}{5}=$	$\frac{2}{3}$ " $\frac{5}{6}=$	$51 \times \frac{14}{29}=$
$\frac{1}{5}$ of $\frac{1}{3}=$	$\frac{3}{4} \times \frac{1}{3}=$	$\frac{5}{6}$ " $\frac{4}{5}=$	$\frac{27}{63} \times 35=$
$\frac{1}{3}$ of $\frac{1}{4}=$	$\frac{5}{6} \times \frac{1}{3}=$	$\frac{4}{7}$ " $\frac{3}{5}=$	$46 \times \frac{33}{34}=$
$\frac{1}{6}$ of $\frac{1}{4}=$	$\frac{5}{8} \times \frac{1}{6}=$	$\frac{4}{5}$ " $\frac{5}{8}=$	$\frac{3}{8} \times 29=$

14.	15.	16.	17.
$3\frac{1}{8} \times 5=$	$6\frac{5}{12} \times 8=$	$\frac{13}{3} \times \frac{17}{6}=$	$\frac{15}{2} \times \frac{19}{8}=$
$4\frac{1}{5} \times 6=$	$7\frac{7}{8} \times 9=$	$\frac{19}{5} \times \frac{19}{8}=$	$\frac{29}{7} \times \frac{13}{9}=$
$5\frac{4}{9} \times 7=$	$9\frac{3}{4} \times 8=$	$\frac{21}{6} \times \frac{11}{7}=$	$\frac{38}{8} \times \frac{31}{2}=$

18. What is $\frac{1}{5}$ of $\$\frac{1}{4}$? $\frac{1}{5}$ of $\$\frac{1}{10}$? $\frac{1}{5}$ of $\$\frac{1}{2}$? $\frac{1}{3}$ of $\$\frac{1}{8}$?
19. What is $\frac{1}{6}$ of $\frac{1}{2}$ hour? $\frac{1}{6}$ of $\frac{1}{5}$ h.? $\frac{1}{5}$ of $\frac{1}{6}$ h.?
20. What is $\frac{1}{16}$ of $\frac{1}{2}$ lb.? $\frac{1}{16}$ of $\frac{1}{4}$ lb.? $\frac{1}{16}$ of $\frac{1}{8}$ lb.?
21. What is $\frac{2}{3}$ of $\frac{1}{4}$ ft.? $\frac{3}{4}$ of $\frac{1}{2}$ ft.? $\frac{5}{8}$ of $\frac{1}{3}$ ft.?
22. What is $\frac{4}{7}$ of $\frac{1}{2}$ week? $\frac{1}{4}$ of $\frac{3}{7}$ wk.? $\frac{3}{5}$ of $\frac{5}{6}$ wk.?
23. What is $\frac{3}{5}$ of $\frac{1}{8}$ qt.? $\frac{3}{4}$ of $\frac{6}{7}$ qt.? $\frac{7}{8}$ of $\frac{9}{10}$ qt.?
24. Multiply $\frac{2}{7}$ by $1\frac{1}{2}$; $\frac{3}{5}$ by $2\frac{3}{4}$; $\frac{6}{7}$ by $3\frac{3}{5}$; $\frac{6}{11}$ by $4\frac{4}{7}$.

25.	26.	27.	28.
$3\frac{1}{2} \times \frac{1}{4}=$	$\frac{1}{7} \times 7\frac{3}{4}=$	$8\frac{2}{3} \times \frac{5}{6}=$	$\frac{3}{8} \times 6\frac{7}{9}=$
$2\frac{2}{3} \times \frac{1}{5}=$	$\frac{1}{9} \times 3\frac{2}{5}=$	$7\frac{1}{4} \times \frac{2}{5}=$	$\frac{2}{5} \times 2\frac{5}{7}=$
$4\frac{2}{5} \times \frac{1}{6}=$	$\frac{1}{3} \times 5\frac{1}{2}=$	$54\frac{2}{7} \times \frac{3}{4}=$	$\frac{4}{9} \times 8\frac{3}{5}=$
$3\frac{3}{4} \times \frac{1}{8}=$	$\frac{1}{5} \times 4\frac{2}{5}=$	$25\frac{3}{8} \times \frac{5}{6}=$	$\frac{7}{11} \times 7\frac{4}{7}=$

29. Multiply $3\frac{3}{8}$, $7\frac{4}{5}$, $9\frac{3}{7}$, $6\frac{2}{3}$, $8\frac{3}{4}$, $5\frac{7}{8}$, $4\frac{7}{9}$, each by $\frac{11}{23}$.
30. What is $\frac{3}{4}$ of $3\frac{11}{12}$? of $5\frac{9}{11}$? of $6\frac{2}{7}$? of $10\frac{2}{3}$? of $9\frac{3}{5}$?

148. **Since mixed numbers may be reduced to improper fractions, and integers may be expressed in fractional form, the general rule may be applied to all cases of multiplication in which fractions are involved, but when the numbers are large, the method is awkward, and requires more figures than the following processes. In business calculations, the rule is seldom followed.**

Example.—1. Multiply 85 by $17^2/_3$.

$$\begin{array}{rrl} & 85 & \\ & \underline{17^2/_3} & \text{Analysis.} \\ 3) & \underline{170} & =2\times 85 \\ & 56^2/_3 & =^2/_3\times 85 \\ & 595 & \rbrace =17\times 85 \\ & \underline{85} & \\ & 1501^2/_3 & =17^2/_3\times 85 \end{array}$$

2. Multiply $58^3/_4$ by 29.

$$\begin{array}{rrl} & 58^3/_4 & \\ & \underline{29} & \text{Analysis.} \\ 4) & \underline{87} & =29\times 3 \\ & 21^3/_4 & =29\times ^3/_4 \\ & 522 & \rbrace =29\times 58 \\ & \underline{116} & \\ & 1703^3/_4 & =29\times 58^3/_4 \end{array}$$

3. Multiply $645^2/_3$ by $328^3/_4$.

$$\begin{array}{lrl} & 645^2/_3 & \\ & \underline{328^3/_4} & \text{Analysis.} \\ & ^1/_2 & =^3/_4\times ^2/_3 \\ 4)1935= & 483^3/_4 & =^3/_4\times 645 \\ 3)\ 656= & 218^2/_3 & =328\times ^2/_3 \\ & 5160 & \\ & 1290 & \rbrace =328\times 645 \\ & \underline{1935} & \\ & 212262^{11}/_{12} & =328^3/_4\times 645^2/_3 \end{array}$$

Explanation.—Beginning at the right, as in multiplication of integers, we multiply separately the fraction and integer of the multiplicand, *first* by the fraction and *second* by the integer of the multiplier.

The work at the left indicates the steps by which we obtain the product of the integers and fractions.

ORAL EXERCISES.

Multiply

1. $8^1/_3$ by 12	2. $13^1/_3$ by 9	3. 6 by $2^1/_3$	4. 12 by $7^2/_3$
$15^1/_4$ by 16	$17^2/_3$ by 6	12 by $3^2/_3$	18 by $4^5/_6$
$7^2/_3$ by 9	$18^3/_4$ by 8	14 by $6^5/_7$	21 by $6^5/_7$
$6^1/_2$ by 14	$15^3/_4$ by 12	16 by $4^5/_8$	15 by $7^3/_5$
5. $2^1/_2$ by 3	6. $4^1/_6$ by 5	7. 9 by $3^1/_2$	8. 37 by $2^1/_4$
$4^2/_3$ by 5	$6^5/_7$ by 8	13 by $6^2/_5$	16 by $6^3/_7$
$7^1/_5$ by 6	$8^5/_{12}$ by 7	18 by $3^1/_8$	27 by $2^3/_5$

SLATE EXERCISES.

1.	2.	3.	4.
$3\frac{1}{5}\times 6\frac{2}{3}=$	$6\frac{1}{5}\times 4\frac{2}{5}=$	$14\frac{2}{3}\times 11\frac{3}{10}=$	$1\frac{17}{25}\times 3\frac{10}{23}=$
$2\frac{3}{4}\times 3\frac{1}{2}=$	$44\frac{2}{5}\times 5\frac{1}{6}=$	$13\frac{1}{8}\times 25\frac{7}{9}=$	$7\frac{13}{16}\times 6\frac{19}{27}=$
$5\frac{2}{3}\times 2\frac{2}{5}=$	$82\frac{3}{8}\times 7\frac{1}{2}=$	$16\frac{2}{5}\times 13\frac{3}{7}=$	$6\frac{23}{27}\times 9\frac{15}{16}=$
$7\frac{1}{2}\times 3\frac{3}{4}=$	$27\frac{6}{7}\times 3\frac{3}{8}=$	$24\frac{3}{8}\times 28\frac{5}{9}=$	$9\frac{18}{19}\times 7\frac{28}{29}=$

5. Find the product of $2\frac{2}{5}\times\$30\frac{7}{10}$; $1\frac{4}{5}\times\$25\frac{5}{9}$; $4\frac{7}{8}\times\$19\frac{2}{3}$.

6. Multiply $\$18\frac{2}{3}$ by $15\frac{3}{7}$; $\$27\frac{13}{21}$ by $38\frac{17}{20}$; $41\frac{1}{2}$ by $20\frac{1}{2}$.

7. $2\frac{1}{25}\times 5\frac{3}{4}\times 16\frac{2}{3}=?$ $\frac{3}{4}$ of $\frac{2}{5}$ of $\frac{5}{6}$ of $\frac{8}{21}$ of $\frac{9}{5}=?$

8. What number is $3\frac{7}{9}$ times $35\frac{1}{2}$? $5\frac{4}{7}$ times $6\frac{3}{8}$?

9. There are $4\frac{1}{2}$ lb. in a package. How many pounds in $8\frac{1}{4}$ such packages?

10. Reduce $\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times\frac{5}{9}\times\frac{9}{32}\times\frac{16}{17}$ to a simple expression.

Applications.—Note.—In business it is customary to drop a fraction in the result, if less than $\frac{1}{2}$¢, and to add 1¢ to the integer, if the fraction is equal to or greater than $\frac{1}{2}$¢. The pupil should here be required to obtain the exact answer, and to write the result in business form under it.

11–38. *What is the cost of*

$37\frac{1}{2}$ bushels of potatoes at $75\frac{1}{2}$¢?
345 yards of cloth at $90\frac{3}{4}$¢?
$17\frac{1}{2}$ feet of oilcloth at $3\frac{1}{5}$¢?
$43\frac{1}{2}$ quarts of cider at $5\frac{1}{6}$¢?
387 bushels of oats at $43\frac{3}{4}$¢?
$40\frac{1}{16}$ pounds of starch at $18\frac{1}{2}$¢?
345 barrels of apples at $\$3\frac{2}{5}$?
$47\frac{1}{2}$ sacks of flour at $\$2\frac{3}{5}$?
$53\frac{1}{2}$ pounds of cheese at $9\frac{3}{4}$¢?
17 sacks of rice at $\$14\frac{3}{5}$?
64 pecks of beans at $17\frac{3}{5}$¢?
$27\frac{4}{5}$ pecks of potatoes at $23\frac{1}{2}$¢?
$328\frac{3}{4}$ pounds of butter at $43\frac{3}{4}$¢?
$17\frac{9}{16}$ pounds of bacon at $12\frac{1}{2}$¢?
$56\frac{1}{2}$ pounds of tea at $\$\frac{3}{4}$?
$6\frac{3}{4}$ tons of coal at $\$3\frac{3}{4}$?
$15\frac{1}{2}$ yards of ribbon at $37\frac{1}{2}$¢?
$24\frac{1}{4}$ gallons of oil at 85¢?
3 quarts of nuts at $12\frac{1}{2}$¢
300 bushels of rye at $94\frac{1}{2}$¢?
$12\frac{1}{2}$ yards of lace at $\$5\frac{1}{4}$?
$17\frac{3}{4}$ pounds of honey at $18\frac{3}{4}$¢?
$4\frac{1}{2}$ barrels of herring at $\$3\frac{1}{4}$?
325 pounds of beef at $11\frac{1}{2}$¢?
63 bl. of cranberries at $\$12\frac{1}{2}$?
$17\frac{3}{8}$ bu. of strawberries at $\$4\frac{5}{8}$?
$37\frac{5}{8}$ yards of velvet at $\$4\frac{3}{8}$?
$48\frac{3}{4}$ yards of carpet at $\$1\frac{3}{4}$?

39. How many square feet in a square, each side of which measures $3\frac{7}{8}$ feet? $3\frac{7}{8}$ inches? $3\frac{7}{8}$ yards?

Draw a square, each side measuring $2\frac{3}{4}$ inches. Divide each side into fourths of an inch, and draw lines, cutting the square into small ones, each a fourth of an inch square. How many of these small squares? How many square inches?

40. Find the number of square inches in a rectangle $5\frac{7}{8}$ ft. long and $7\frac{3}{4}$ inches wide.

41. What is the cost of 9 tons of coal at $\$3\frac{3}{5}$, with cartage at $\$\frac{1}{4}$ per ton?

42. If the salary of an officer is $1700, how much does he receive in $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{10}$, $\frac{1}{12}$ year? How much in $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{7}{12}$ year?

43. A laborer earns $\$20\frac{3}{5}$ a week. How much in $\frac{3}{5}$ year? In $1\frac{4}{5}$ yr.? (Count 52 weeks.)

44. My neighbor pays $700 rent per year. How much is that per month?

45. I buy some lots containing respectively $\frac{3}{4}$, $\frac{4}{5}$, $\frac{13}{25}$, $\frac{47}{50}$, $6\frac{3}{10}$, $19\frac{13}{20}$ acres. What is the cost of the whole, at $48 per acre?

46. How many square feet in the surface of a stone slab $2\frac{7}{8}$ feet wide and $4\frac{5}{6}$ feet long?

47. Four boxes of hardware, weighing respectively $3\frac{1}{4}$ cwt., $4\frac{3}{5}$ cwt., $4\frac{1}{2}$ cwt., and $4\frac{3}{10}$ cwt., cost 16¢ freight per cwt. What is the freight on each box, and on the 4 boxes?

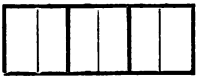
Find the sum to be paid for

48.
$5\frac{1}{2}$ lb. sugar, at $9\frac{1}{2}$¢
$6\frac{3}{4}$ lb. coffee, at $32\frac{1}{2}$¢
$2\frac{4}{5}$ lb. rice, at $8\frac{4}{7}$¢
$14\frac{3}{8}$ lb. flour, at $4\frac{1}{5}$¢
$3\frac{3}{4}$ lb. butter, at $23\frac{1}{2}$¢
$2\frac{1}{2}$ lb. cheese, at $11\frac{3}{4}$¢
$3\frac{5}{6}$ doz. eggs, at 20¢
$2\frac{4}{5}$ lb. starch, at $12\frac{1}{2}$¢

49.
18 yd. calico, at 9¢
$20\frac{1}{2}$ yd. alpaca, at $42\frac{1}{2}$¢
$19\frac{1}{4}$ yd. shirting, at 17¢
25 yd. ribbon, at $33\frac{1}{2}$¢
10 doz. buttons, at $\$\frac{1}{4}$
3 cloaks, at $\$18\frac{2}{5}$
10 yd. velvet, at $\$3\frac{3}{4}$
$22\frac{1}{2}$ yd. velveteen, at $\$1\frac{4}{5}$

Division in Fractions.

Note.—In the first two exercises the square is the unit. In the third exercise the linear inch is the unit.



1. *a.* How many times 1 in 3, $2\frac{1}{2}$, 2, $1\frac{1}{2}$? What part of 1 is contained in $\frac{1}{2}$? What part of 2 in $\frac{1}{2}$? In $1\frac{1}{2}$? etc.

b. How many times $\frac{1}{2}$ in 1? In 2? In 3? How many times $1\frac{1}{2}$ in 3? $2\frac{1}{2}$ in 3? ($\frac{5}{2}$ in $\frac{6}{2}$?)

c. How many times $\frac{1}{2}$ in $1\frac{1}{2}$? In $2\frac{1}{2}$? $1\frac{1}{2}$ in $2\frac{1}{2}$?

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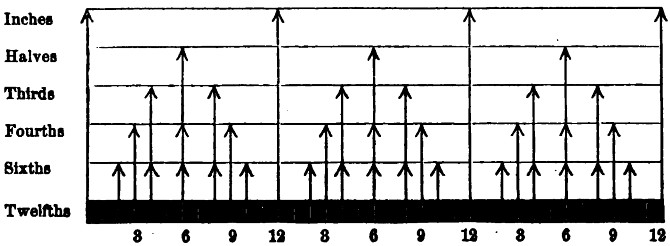
2. *a.* How many times 1 in $1\frac{2}{3}$? In $2\frac{1}{3}$? What part of 1 is in $\frac{2}{3}$? What part of 2? What part of 2 is in $\frac{1}{3}$? In $1\frac{1}{3}$?

b. How many times $\frac{1}{3}$ in 1? In 2? How many times $\frac{2}{3}$ in 2? In 3? How many times $1\frac{2}{3}$ in 3? $2\frac{1}{3}$ in 3?

c. How many times $\frac{1}{3}$ in $\frac{5}{3}$? In $2\frac{2}{3}$? How many times $1\frac{1}{3}$ in $2\frac{2}{3}$? $\frac{2}{3}$ in $2\frac{2}{3}$? $1\frac{2}{3}$ in 3? $1\frac{1}{3}$ in $2\frac{1}{3}$?

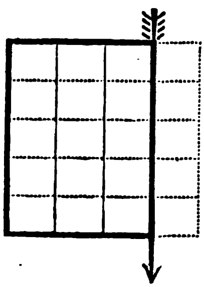


Note.—The points of arrows divide the inches into halves, thirds, etc. By following the shafts downward equivalents in smaller parts are found.

3. *a.* How many times 2 in $2\frac{3}{4}$? What part of 3 in $\frac{1}{2}$, $1\frac{2}{3}$?

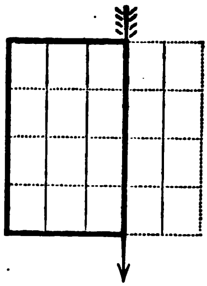
b. How many times $\frac{1}{6}$ in 2? In $2\frac{1}{2}$? In 3? How many times $\frac{3}{4}$ in 1? In $\frac{7}{12}$? In 2? How many times $\frac{5}{6}$ in 3? What part of 2 is $\frac{1}{4}$, $\frac{2}{3}$? What part of 3 is $1\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{3}$?

c. How many times $\frac{1}{4}$ in $\frac{1}{2}$? $\frac{5}{12}$ in $\frac{2}{3}$? $\frac{1}{3}$ in $\frac{3}{4}$? $\frac{1}{4}$ in $\frac{2}{3}$? What part of $\frac{1}{2}$ is in $\frac{1}{4}$, $\frac{1}{3}$? What part of $\frac{2}{3}$ in $\frac{3}{4}$? etc.



4. Divide $^3/_4$ by $^3/_5$.

Illustration.—To find how many times $^3/_5$ is contained in $^3/_4$, the 5ths and 4ths are subdivided into parts of the same size by dividing each 5th into four and each 4th into five equal parts, thus making $^{12}/_{20}$ and $^{15}/_{20}$, whence we see that $^3/_5$ is contained in $^3/_4$ as many times as 12 is contained in $15 = {}^{15}/_{12} = 1\,^1/_4$.



149. The arithmetical solution is performed on the same principle as the solution by diagram, the folding of paper, etc., thus:

Indicating the division by writing the divisor under the dividend (see Art. 72, § 3), we reduce both fractions to 20ths, 20 being the least common multiple of 4 and 5, and divide the numerator of the dividend by the numerator of the divisor.

$$\frac{\dfrac{3\times5}{\cancel{4\times5}}=\dfrac{15}{\cancel{20}}}{\dfrac{3\times4}{\cancel{5\times4}}=\dfrac{12}{\cancel{20}}}=\frac{15}{12}=1\tfrac{1}{4}$$

But, since the common denominator does not affect the quotient ($^{15}/_{12}$), the work, printed in italics, by which it is obtained may be omitted. The written process would then appear as at the right.

$$\frac{3}{4}\times\frac{5}{3}=\frac{15}{12}=1\tfrac{1}{4}$$

150. ***Rule.*—Invert the divisor and multiply the numerators together for the numerator of the quotient, and the denominators for the denominator of the quotient.**

Note.—Since integers and mixed numbers may be expressed in the form of improper fractions, this rule applies to all cases of Division in Common Fractions.

151. The following analysis leads to an equivalent rule:

$^3/_5$ is contained in 1, or $^5/_5$, five thirds times, and in $^3/_4$ it is contained $^3/_4$ of $^5/_3$ times; hence we have

$$\frac{3}{4}\div\frac{3}{5}=\frac{3}{4}\times\frac{5}{3}=\frac{15}{12}=1\tfrac{1}{4}.$$

152. An inverted fraction shows how many times the fraction is contained in one, and is called the ***reciprocal*** of the fraction. Hence, for dividing one fraction by another, we have the

***Rule.*—Multiply the reciprocal of the divisor by the dividend.**

153. In dividing a fraction by an integer, a part of the written work required by the rule may be omitted, as follows:

First, when the numerator of the fraction is divisible by the integer, as in

Example.—1. Divide $^6/_7$ by 3. (What part of 3 in $^6/_7$?)

The process of division, according to the rule, would be

$$^6/_7 \div 3 = {}^1/_{\cancel{3}} \times {}^{\overset{2}{\cancel{6}}}/_7 = {}^2/_7,$$

but this is equivalent to dividing the numerator directly by the integer, thus,

$$^6/_7 \div 3 = {}^2/_7.$$

Second, when the numerator of the fraction is not divisible by the integer, as in

Example.—2. Divide $^5/_7$ by 3. (What part of 3 in $^5/_7$?)

The process of division, under the rule, would be

$$^5/_7 \div 3 = {}^1/_3 \times {}^5/_7 = {}^5/_{21},$$

but we can multiply the denominator directly by 3 without writing out the second step; hence, we need to put down only

$$^5/_7 \div 3 = {}^5/_{21}.$$

154. *Hence, dividing the numerator or multiplying the denominator of a fraction divides the value of the fraction.*

155. In dividing a mixed number by an integer, by a fraction, or by a mixed number, the written work may be as follows:

Example.—3. Divide $379^3/_4$ by 6.

$$\begin{array}{r} 6\underline{)379^3/_4} \\ 63^7/_{24} \end{array}$$

Explanation.—Six is contained in $379^3/_4$ 63 times, with a remainder of $1^3/_4$.

$1^3/_4 = {}^7/_4$; $^7/_4 \div 6 = {}^7/_{24}$, which, being annexed to 63, gives the entire quotient, $63^7/_{24}$.

Note.—In the two following examples we multiply both divisor and dividend by the denominator of the divisor in order to get rid of the fraction in the divisor. This makes the division more convenient and does not alter the value of the quotient. The process then becomes the same as in the preceding solution.

4. Divide $379^1/_4$ by $^2/_3$.

$$\begin{array}{ll} ^2/_3) & 379^1/_4 \\ \underline{3} & \underline{3} \\ 6) & \underline{1137^3/_4} \\ & 568^7/_8 \end{array}$$

5. Divide $349^2/_3$ by $2^1/_4$.

$$\begin{array}{ll} 2^1/_4) & 349^2/_3 \\ \underline{4} & \underline{4} \\ 9) & \underline{1398^2/_3} \\ & 155^{11}/_{27} \end{array}$$

ORAL EXERCISES.

Divide

1.	2.	3.	4.
$\frac{1}{4}$ by $\frac{1}{36}$	$\frac{1}{6}$ by $\frac{1}{12}$	$\frac{2}{3}$ by $\frac{1}{18}$	$\frac{1}{2}$ by $\frac{1}{36}$
$\frac{1}{4}$ by $\frac{1}{28}$	$\frac{1}{6}$ by $\frac{1}{18}$	$\frac{2}{3}$ by $\frac{1}{15}$	$\frac{1}{2}$ by $\frac{1}{34}$
$\frac{1}{4}$ by $\frac{1}{24}$	$\frac{1}{6}$ by $\frac{1}{24}$	$\frac{2}{3}$ by $\frac{1}{12}$	$\frac{1}{2}$ by $\frac{1}{32}$
$\frac{1}{4}$ by $\frac{1}{16}$	$\frac{1}{6}$ by $\frac{1}{36}$	$\frac{2}{3}$ by $\frac{1}{9}$	$\frac{1}{2}$ by $\frac{1}{30}$

Question.—In which of the above columns do the quotients increase as you descend? Why do they increase? In which do they decrease? Why?

5. Divide $\frac{3}{4}$ by $\frac{1}{6}$; $\frac{2}{3}$ by $\frac{1}{5}$; $\frac{3}{5}$ by $\frac{1}{4}$; $\frac{5}{6}$ by $\frac{1}{4}$; $\frac{6}{11}$ by $\frac{5}{11}$.

6. " $\frac{3}{4}$ by $\frac{1}{7}$; $\frac{5}{8}$ by $\frac{5}{9}$; $\frac{3}{5}$ by $\frac{8}{11}$; $\frac{2}{47}$ by $\frac{5}{47}$; $\frac{7}{8}$ by $\frac{1}{9}$.

7. " $\frac{2}{3}$ by $\frac{9}{32}$; $\frac{5}{8}$ by $\frac{7}{12}$; $\frac{9}{20}$ by $\frac{1}{4}$; $\frac{7}{23}$ by $\frac{2}{6}$; $\frac{1}{4}$ by $\frac{1}{27}$.

8. How many times may $\frac{1}{3}$ of a pound be taken from $\frac{6}{7}$ lb., $\frac{3}{5}$ lb., $\frac{7}{8}$ lb., $\frac{9}{10}$ lb.?

9.	10.	11.	12.
$\frac{3}{4} \div 4 =$	$\frac{4}{19} \div 2 =$	$\frac{7}{8} \div 4 =$	$\frac{11}{15} \div 7 =$
$\frac{1}{2} \div 3 =$	$\frac{8}{13} \div 9 =$	$\frac{65}{66} \div 13 =$	$\frac{19}{27} \div 3 =$
$\frac{5}{6} \div 7 =$	$\frac{5}{9} \div 2 =$	$\frac{63}{71} \div 7 =$	$\frac{1}{2} \div 4 =$
$\frac{9}{11} \div 3 =$	$\frac{3}{4} \div 3 =$	$\frac{64}{75} \div 16 =$	$\frac{3}{4} \div 9 =$
$\frac{5}{9} \div 6 =$	$\frac{12}{13} \div 4 =$	$\frac{6}{7} \div 8 =$	$\frac{4}{5} \div 8 =$

13. Divide $6\frac{2}{3}$ by $4\frac{1}{8}$; $5\frac{1}{5}$ by $2\frac{7}{9}$; $3\frac{8}{11}$ by $1\frac{1}{13}$; $2\frac{1}{2}$ by $\frac{1}{4}$.

14. Divide $9\frac{9}{10}$ by $4\frac{1}{7}$; $2\frac{3}{5}$ by $9\frac{9}{10}$; $7\frac{7}{8}$ by $2\frac{4}{5}$; $6\frac{7}{9}$ by $3\frac{3}{4}$.

15.	16.	17.	18.
$2\frac{3}{4} \div 7 =$	$5\frac{6}{7} \div 9 =$	$9\frac{4}{5} \div 8 =$	$8\frac{1}{3} \div 5 =$
$6\frac{1}{5} \div 8 =$	$3\frac{1}{3} \div 7 =$	$8\frac{1}{7} \div 8 =$	$3\frac{4}{7} \div 5 =$
$9\frac{2}{3} \div 5 =$	$7\frac{2}{5} \div 6 =$	$7\frac{5}{8} \div 6 =$	$4\frac{5}{6} \div 8 =$

19. Divide 5 by $\frac{3}{4}$; 7 by $\frac{9}{11}$; 6 by $\frac{5}{6}$; 12 by $\frac{3}{4}$.

20.	21.	22.	23.
$18 \div \frac{7}{8} =$	$14 \div \frac{7}{8} =$	$39 \div \frac{6}{11} =$	$27 \div \frac{7}{13} =$
$36 \div \frac{7}{9} =$	$41 \div \frac{7}{19} =$	$23 \div \frac{5}{13} =$	$20 \div \frac{7}{10} =$
$84 \div \frac{2}{3} =$	$35 \div \frac{7}{9} =$	$49 \div \frac{4}{7} =$	$19 \div \frac{7}{15} =$
$17 \div \frac{5}{6} =$	$71 \div \frac{3}{5} =$	$21 \div \frac{5}{8} =$	$23 \div \frac{8}{17} =$

24. Divide 23 by $4\frac{1}{2}$; 17 by $5\frac{2}{3}$; 48 by $3\frac{7}{8}$; 59 by $7\frac{1}{5}$.

25. Divide 25 by $3\frac{1}{3}$; 24 by $7\frac{1}{6}$; 39 by $8\frac{2}{3}$; 72 by $2\frac{5}{6}$.

ORAL AND SLATE EXERCISES.

Note.—Let the oral exercises be carried as far as time and the ability of the pupils will permit.

1. $1\div 6=$	**2.** $4\div 3=$	**3.** $4\div\frac{1}{2}=$	**4.** $5\div\frac{2}{3}=$	**5.** $3\div 1\frac{1}{2}=$
$1\div 8=$	$7\div 4=$	$3\div\frac{1}{4}=$	$6\div\frac{3}{7}=$	$5\div 2\frac{2}{3}=$
$2\div 3=$	$6\div 5=$	$2\div\frac{1}{6}=$	$8\div\frac{5}{8}=$	$27\div 7\frac{3}{4}=$
$2\div 4=$	$14\div 8=$	$5\div\frac{1}{9}=$	$4\div\frac{4}{5}=$	$15\div 3\frac{2}{5}=$
$5\div 7=$	$8\div 5=$	$7\div\frac{1}{8}=$	$10\div\frac{2}{9}=$	$39\div 9\frac{3}{5}=$

6. $\frac{2}{3}\div 4=$	**7.** $\frac{7}{8}\div\frac{1}{8}=$	**8.** $2\frac{2}{5}\div\frac{3}{5}=$	**9.** $15\frac{1}{2}\div 4\frac{1}{2}=$
$\frac{3}{4}\div 5=$	$\frac{8}{9}\div\frac{2}{9}=$	$3\frac{3}{4}\div\frac{3}{4}=$	$17\frac{4}{5}\div 3\frac{2}{5}=$
$\frac{4}{6}\div 3=$	$\frac{6}{7}\div\frac{3}{7}=$	$3\frac{9}{11}\div\frac{7}{11}=$	$16\frac{2}{3}\div 5\frac{2}{3}=$
$\frac{3}{7}\div 6=$	$\frac{14}{15}\div\frac{7}{15}=$	$5\frac{5}{9}\div\frac{5}{9}=$	$29\frac{4}{7}\div 9\frac{3}{7}=$

10. How many times \$$4\frac{3}{4}$ in \$$26\frac{1}{4}$? In \$$37\frac{3}{4}$? In \$$45\frac{2}{4}$?

11. $3\frac{3}{7}$ hours is what part of $19\frac{1}{7}$, $27\frac{2}{7}$, $38\frac{3}{7}$ hours?

12. What part of $18\frac{9}{16}$, $24\frac{12}{16}$, $30\frac{15}{16}$, $49\frac{8}{16}$ lb. is $6\frac{3}{16}$ lb.?

13. $\frac{1}{2}\div\frac{1}{4}=$	**14.** $\frac{10}{21}\div\frac{2}{3}=$	**15.** $\frac{5}{6}\div\frac{1}{18}=$	**16.** $\frac{4}{5}\div\frac{2}{25}=$
$\frac{1}{4}\div\frac{1}{8}=$	$\frac{12}{24}\div\frac{4}{6}=$	$\frac{4}{5}\div\frac{1}{15}=$	$\frac{8}{21}\div\frac{4}{63}=$
$\frac{1}{9}\div\frac{1}{36}=$	$\frac{20}{35}\div\frac{5}{7}=$	$\frac{5}{6}\div\frac{1}{9}=$	$\frac{25}{36}\div\frac{5}{72}=$
$\frac{1}{4}\div\frac{1}{20}=$	$\frac{35}{42}\div\frac{5}{6}=$	$\frac{4}{5}\div\frac{1}{25}=$	$\frac{4}{5}\div\frac{5}{25}=$
$\frac{1}{3}\div\frac{1}{9}=$	$\frac{18}{33}\div\frac{6}{11}=$	$\frac{5}{7}\div\frac{1}{28}=$	$\frac{7}{18}\div\frac{7}{54}=$

17. $\frac{1}{6}\div\frac{1}{12}$; $\frac{1}{72}\div\frac{1}{18}$; $\frac{1}{75}\div\frac{1}{15}$; $\frac{1}{64}\div\frac{1}{16}$; $\frac{5}{36}\div\frac{5}{9}$.

18. $\frac{1}{3}\div\frac{1}{5}$; $\frac{1}{2}\div\frac{1}{6}$; $\frac{1}{4}\div\frac{1}{15}$; $\frac{1}{18}\div\frac{1}{9}$; $\frac{1}{3}\div\frac{3}{4}$.

19. $\frac{1}{9}\div\frac{2}{7}$; $\frac{3}{8}\div\frac{5}{6}$; $\frac{3}{5}\div\frac{2}{5}$; $\frac{2}{7}\div\frac{4}{9}$; $\frac{3}{7}\div\frac{5}{12}$.

20. $1\frac{1}{3}\div\frac{2}{9}=$	**21.** $11\frac{1}{3}\div 1\frac{2}{15}=$	**22.** $3\frac{2}{3}\div\frac{5}{7}=$	**23.** $5\frac{1}{12}\div 4\frac{1}{5}=$
$3\frac{3}{4}\div\frac{5}{12}=$	$23\frac{1}{2}\div 2\frac{1}{8}=$	$2\frac{1}{14}\div\frac{4}{9}=$	$3\frac{1}{4}\div 2\frac{1}{8}=$
$2\frac{1}{2}\div\frac{5}{16}=$	$45\frac{2}{5}\div 3\frac{2}{25}=$	$1\frac{1}{6}\div\frac{3}{5}=$	$5\frac{1}{10}\div 1\frac{4}{5}=$
$4\frac{4}{5}\div\frac{4}{15}=$	$25\frac{1}{2}\div 4\frac{1}{4}=$	$1\frac{4}{5}\div\frac{7}{13}=$	$6\frac{2}{3}\div 3\frac{2}{15}=$
$1\frac{3}{4}\div\frac{7}{20}=$	$27\frac{1}{3}\div 2\frac{1}{6}=$	$2\frac{7}{15}\div\frac{4}{51}=$	$7\frac{1}{3}\div 4\frac{1}{16}=$

24. Divide $\frac{4}{7}$ by $2\frac{2}{3}$; $\frac{1}{8}$ by $3\frac{3}{4}$; $\frac{2}{3}$ by $1\frac{4}{9}$; $\frac{1}{9}$ by $2\frac{2}{5}$; $\frac{1}{5}$ by $3\frac{3}{4}$.

25. Divide $\frac{3}{4}$ by $7\frac{4}{5}$; $\frac{4}{9}$ by $3\frac{3}{4}$; $\frac{5}{7}$ by $4\frac{4}{5}$; $\frac{5}{8}$ by $5\frac{5}{6}$; $\frac{3}{8}$ by $3\frac{2}{5}$.

Applications.—1. If 4 yards of ribbon cost $\$^5/_7$, what will 1 yard cost?

Analysis.—If 4 yards cost $\$^5/_7$, one yard will cost $^1/_4$ of $\$^5/_7 = \$^5/_{28}$.

2. A farmer sold 5 dozen eggs for $\$^{11}/_{20}$. How much was that per dozen?

3. In six days a man plows $^5/_{14}$ of a field. At this rate, how much does he plow in 1 day?

4. If a weaver earns $\$9^3/_{20}$ per week (6 days), what does she earn per day?

5. If $^4/_5$ lb. of coffee cost $\$^3/_8$, what will 1 lb. cost?

Analysis.—If $^4/_5$ lb. cost $\$^3/_8$, $^1/_5$ will cost $^1/_4 \times \$^3/_8 = \$^3/_{32}$, and $^5/_5$ lb. will cost $5 \times {}^3/_{32} = \$^{15}/_{32}$.

6. If a traveler can make $^5/_6$ of his journey in $^3/_7$ of a month, what time will the entire journey require?

7. If $^2/_9$ of a bar of gold weighs $^5/_{12}$ lb., what is the weight of the bar?

8. How many are $^2/_3$ of $2^1/_2$ dozen? $^2/_3$ of $2^1/_2$ gross?

9. A garden, containing $148^4/_5$ □ yd., is to be divided up into beds of $12^2/_5$ □ yd. each. How many such beds will there be?

10. A quantity of grain, weighing $110^1/_4$ cwt., is to be put into bags, each holding $1^3/_4$ cwt. How many bags are required?

11. How many yards of cloth at $\$^3/_8$ a yard can be bought for \$2, \$5, \$7, \$9, \$4, \$23?

12. How many bushels of potatoes at $\$^{24}/_{25}$ per bu. can be bought for \$6, \$8, \$11, \$13?

13. How many times may $1^8/_9$ quarts be drawn from a can holding 17 quarts? From one holding $22^2/_3$ quarts?

14. If a laborer can mow a field in $7^{14}/_{19}$ days, how much of it can he mow in 1 day?

15. Divide \$22,500 among the 7 members of a family so that each of the 4 older ones may receive a third more than one of the younger.

Complex Fractions.

If a slip of paper, a melon, or other object, is cut into 3 equal parts, one and a half of these parts are $1\frac{1}{2}$ thirds, $2\frac{1}{2}$ parts are $2\frac{1}{2}$ thirds, etc.

Show by the use of objects what is meant by $\frac{2\frac{1}{3}}{5}$, $\frac{1\frac{3}{4}}{3}$, etc.

156. ***A complex fraction*** is a fraction having a fraction or mixed number for its numerator and an integer for its denominator.

Reducing Complex to Simple Fractions. Example.—1. Reduce $\frac{2\frac{1}{3}}{4}$ to a simple fraction.

Analysis.—If each fourth of an object be divided into 3 equal parts, 4 fourths, or the whole, will contain 4 times 3, or 12, and $2\frac{1}{3}$ will contain $2\frac{1}{3}$ times 3, or 7, of them, hence $2\frac{1}{3}$ fourths are equal to $\frac{7}{12}$.

Written Work.

$$\frac{2\frac{1}{3}}{4}=\frac{2\frac{1}{3}\times3}{4\times3}=\frac{7}{12}$$

Illustration.—The mode of demonstrating the foregoing analysis by means of objects is sufficiently indicated by the analysis.

Reduce to simple fractions:

1. $\frac{2\frac{1}{3}}{6}$ **2.** $\frac{3\frac{1}{2}}{8}$ **3.** $\frac{1\frac{2}{3}}{7}$ **4.** $\frac{5\frac{1}{2}}{9}$ **5.** $\frac{2\frac{3}{4}}{3}$ **6.** $\frac{3\frac{3}{4}}{2}$

7. $\frac{\frac{2}{5}}{7}$ **8.** $\frac{4\frac{1}{7}}{8}$ **9.** $\frac{6\frac{1}{3}}{11}$ **10.** $\frac{\frac{3}{9}}{15}$ **11.** $\frac{\frac{1}{8}}{12}$ **12.** $\frac{7\frac{3}{8}}{10}$

13. $\frac{2356\frac{7}{9}}{3872}$ **14.** $\frac{483\frac{1}{13}}{286}$ **15.** $\frac{\frac{3}{5}}{989}$

157. Expressions in which fractions or mixed numbers occur as denominators, as $\frac{4}{5\frac{1}{2}}$, are not properly fractions, though they are commonly classified as such. They only indicate division (see Art. 72), and are reduced by performing the division indicated, or by multiplying divisor and dividend of the complex expression by the least common multiple of the denominators of their fractional parts (see Art. 85).

SLATE EXERCISES.

Reduce to simple fractions:

1. $\dfrac{2\frac{1}{3}}{3\frac{1}{4}}$	2. $\dfrac{5\frac{1}{2}}{7\frac{3}{5}}$	3. $\dfrac{6\frac{2}{3}}{3\frac{1}{3}}$	4. $\dfrac{33\frac{1}{8}}{51\frac{2}{11}}$	5. $\dfrac{68\frac{1}{5}}{97\frac{2}{3}}$
6. $\dfrac{17\frac{3}{7}}{18\frac{1}{7}}$	7. $\dfrac{32\frac{3}{4}}{33\frac{3}{8}}$	8. $\dfrac{6\frac{5}{12}}{9\frac{3}{4}}$	9. $\dfrac{13\frac{2}{3}}{68\frac{1}{3}}$	10. $\dfrac{17\frac{1}{7}}{23\frac{2}{3}}$

The reduction of the following complicated expressions will afford exercises in addition, subtraction, multiplication, and division of fractions.

1. $\frac{3}{5}+(2\frac{1}{3} \text{ of } 2\frac{1}{5})+(\frac{4}{5}\times 2\frac{1}{3})+(6\frac{3}{13}\div\frac{45}{91})$

2. $4\frac{2}{9}\times 6\frac{3}{7}\div\dfrac{2\frac{1}{2}}{7}$

3. $\dfrac{\frac{5}{7}+\frac{1}{4}}{2\frac{1}{2}-1\frac{6}{7}}$

4. $\dfrac{\frac{14}{27}\times\frac{18}{35}}{1\frac{23}{25} \text{ of } \frac{9}{36}}$

5. $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\frac{1}{2\frac{1}{2}}+\frac{1}{3\frac{1}{2}}+\frac{1}{4\frac{1}{2}}}$

6. $6\frac{2}{3}\div\dfrac{3-\frac{16}{31}}{\frac{7}{8}-\frac{3}{5}}$

7. $\dfrac{1}{6+1\frac{7}{8}}$

8. $\dfrac{1\frac{4}{17}\times 6\frac{4}{5}}{3\frac{3}{5}-1\frac{2}{25}}+(\frac{2}{11} \text{ of } 2\frac{2}{5})-\frac{3}{4}-\dfrac{1\frac{3}{5}}{12}$

9. $3\frac{1}{3}\div\dfrac{1+\frac{15}{16}}{\frac{1}{5}+\frac{1}{8}}$

10. $\dfrac{\frac{1}{3}+\frac{4}{9}+\frac{11}{27}}{\frac{1}{4}+\frac{5}{6}+\frac{11}{12}} \text{ of } \dfrac{\frac{5}{8} \text{ of } 2\frac{1}{3}\div 1\frac{11}{24}}{\frac{16}{27} \text{ of } 4\frac{5}{8}\div 9\frac{1}{4}}$

11. $\dfrac{15\frac{3}{4}-(4\frac{2}{3}\times 1\frac{5}{6})}{(\frac{1}{5}\div\frac{3}{70})+2\frac{19}{36}}$

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158. Applications.—To find what part one given number is of another.

Example.—1. What part of 8 is 3?

Analysis.—3 is $\frac{3}{8}$ of 8 because it is 3 of the 8 equal parts into which 8 can be divided. Illustrate by the use of counters.

What part of

2. 7 is 5? 16 is 12? 18 is 15? 21 is 14? 30 is 20?

3. 39 is 26? 42 is 28? 48 is 36? 72 is 48? 32 is 24?

Example.—4. What part of 5 is $\frac{2}{3}$?

Analysis.—$5 = \frac{15}{3}$, and $\frac{2}{3}$ is $\frac{2}{15}$ of $\frac{15}{3}$ because it is 2 of the 15 equal parts into which 15 thirds can be divided.

Note.—Illustrate with objects. By cutting 5 wholes into thirds, and taking 2 of the fifteen, we see how nearly this problem is like the preceding.

What part of

5. 4 is $^3/_4$? 12 is $^2/_3$? 15 is $^5/_9$? 18 is $^2/_3$? 25 is $^5/_8$?

6. 39 is $^3/_5$? 16 is $^4/_5$? 21 is $^3/_7$? 28 is $^4/_9$? 36 is $^{12}/_{13}$?

Example.—7. What part of $^4/_5$ is $^2/_3$?

Analysis.—$^4/_5 = {}^{12}/_{15}$ and $^2/_3 = {}^{10}/_{15}$, $^{10}/_{15}$ is $^{10}/_{12}$ of $^{12}/_{15}$ because it is 10 of the 12 equal parts into which 12 fifteenths can be divided. Illustrate with objects.

What part of

8. $^5/_6$ is $^3/_8$? $^6/_7$ is $^2/_3$? $^3/_5$ is $^5/_9$? $^3/_4$ is $^4/_7$? $^5/_{11}$ is $^1/_5$?

9. $3^2/_3$ is $1^1/_2$? $2^1/_3$ is $1^1/_4$? $4^1/_2$ is $2^1/_3$? $4^7/_8$ is $3^1/_4$? $5^2/_3$ is $4^1/_3$?

10. $6^1/_8$ is $5^3/_4$? $5^1/_8$ is $4^3/_7$? 11 is $5^1/_2$? $5^9/_{10}$ is $2^3/_5$? $7^1/_2$ is $6^1/_3$?

159. In problems such as the preceding, the results may be reached by taking the number representing the *part* for the *numerator* and the number representing the *whole* for the *denominator* of a fraction, and reducing as suggested in Art. 157, or, by dividing the former by the latter. Thus:

1. What part of 4 is $1^2/_3$? **Solution:** $\dfrac{1^2/_3 \times 3}{4 \times 3} = \dfrac{5}{12}$

2. What part is $5^3/_4$ of $7^5/_6$? **Solution:** $\dfrac{5^3/_4 \times 12}{7^5/_6 \times 12} = \dfrac{69}{94}$

SLATE EXERCISES.

What part of

3. 42 is $^3/_7$? 14 is $^4/_7$? 30 is $^1/_7$? 38 is $^1/_2$? 45 is $^5/_9$?

4. 48 is $^5/_6$? 33 is $^4/_{11}$? 52 is $^5/_{13}$? 48 is $^3/_4$? 55 is $^{11}/_{12}$?

5. $^4/_9$ is $^1/_3$? 5 is $2^1/_5$? $4^1/_{12}$ is $3^1/_6$? $1^5/_9$ is $1^1/_7$? $5^1/_2$ is $4^1/_3$?

6. $4^4/_5$ is $2^2/_{10}$? $7^1/_5$ is $4^1/_3$? $8^1/_9$ is $5^1/_7$? $8^2/_3$ is $7^1/_6$? $10^{10}/_{15}$ is $5^1/_3$?

7. 12 is $7^1/_4$? $12^1/_3$ is $6^1/_3$? $11^2/_5$ is $^1/_5$? $^1/_5$ is $^1/_{35}$? $^1/_{35}$ is $^1/_{70}$?

8. $^1/_3$ is $^1/_{12}$? $^1/_{24}$ is $^1/_{72}$? $^1/_{11}$ is $^1/_{121}$? $1^1/_{11}$ is $^3/_{77}$?

160. From a known fractional part of a number to find the number.

Example.—1. 6 is $^3/_4$ of what number?

Analysis.—If 6 is $^3/_4$ of a number, one fourth of the number is $^1/_3$ of $6 = 2$, and four fourths is 4 times $2 = 8$.

Example.—2. $^{18}/_{19}$ is $^6/_7$ of what number?

Analysis.—If $^{18}/_{19}$ is $^6/_7$ of any number, $^1/_7$ of the number is $^1/_6$ of $^{18}/_{19} = {^3/_{19}}$ of the number, and $^7/_7$ or the entire number is $7 \times {^3/_{19}} = {^{21}/_{19}} = 1\,^2/_{19}$.

3. $^5/_7$ is $^5/_8$ of what number?
4. 7 is $^3/_8$ of what number?
5. $4^1/_2$ is $^9/_{10}$ of what number?
6. $^2/_3$ is $^{11}/_{13}$ of what number?
7. $5\,^1/_2$ is $^1/_3$ of what number?
8. $6\,^2/_3$ is $^5/_6$ of what number?

Aliquot Parts.

161. An ***aliquot part*** of a number is any integer or mixed number which will exactly divide it without a remainder.

Aliquot Parts of a Dollar.

50¢ = \$$^1/_2$	$12\,^1/_2$¢ = \$$^1/_8$	$87\,^1/_2$¢ = \$$^7/_8$
$33\,^1/_3$¢ = \$$^1/_3$	10¢ = \$$^1/_{10}$	75¢ = \$$^3/_4$
25¢ = \$$^1/_4$	$8\,^1/_3$¢ = \$$^1/_{12}$	$62\,^1/_2$¢ = \$$^5/_8$
20¢ = \$$^1/_5$	$6\,^1/_4$¢ = \$$^1/_{16}$	60¢ = \$$^3/_5$
$16\,^2/_3$¢ = \$$^1/_6$	5¢ = \$$^1/_{20}$	$37\,^1/_2$¢ = \$$^3/_8$

162. To find the cost of a number of articles when the price is an aliquot part of a dollar.

Example.—1. What will 20 doz. eggs cost @ 25¢ a doz.?

Analysis.—At \$1 a doz., 20 doz. would cost \$20, but at 25¢ (= $^1/_4$ dol.) a doz., 20 doz. will cost $^1/_4$ of \$20, or \$5.

2. What is the cost of 28 readers @ 25¢? @ 50¢?
3. 144 lb. of beef @ $12\,^1/_2$¢? @ $16\,^2/_3$¢? @ $8\,^1/_3$¢?
4. 240 lb. of raisins @ 20¢? @ $16\,^2/_3$¢? @ $12\,^1/_2$¢?
5. 48 pr. of socks @ 50¢? @ $33\,^1/_3$¢? @ $37\,^1/_2$¢?
6. 160 lb. of sugar @ 5¢? @ $6\,^1/_4$¢? @ $8\,^1/_3$¢? @ $12\,^1/_2$¢?

7. 24 pr. of gloves @ $87\frac{1}{2}$¢? @ 75¢? @ $62\frac{1}{2}$¢?

8. 35 handkerchiefs @ 60¢? @ 40¢?

9. 16 baskets @ $37\frac{1}{2}$¢? @ $62\frac{1}{2}$¢? @ $87\frac{1}{2}$¢?

10. 12 pr. of slippers @ \$1.25? @ \$$1.33\frac{1}{3}$? @ \$$1.16\frac{2}{3}$?

11. 230 bu. of wheat @ $62\frac{1}{2}$¢? @ 75¢? @ $87\frac{1}{2}$¢?

12. 84 chairs @ \$1.25? @ \$$1.33\frac{1}{3}$? @ \$$1.37\frac{1}{2}$?

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163. To find the number of articles that can be bought for a given sum, when the price of one is an aliquot part of a dollar.

13. At $33\frac{1}{3}$¢, how many lb. of butter can be bought for \$7?

Analysis.—If $33\frac{1}{3}$¢ will buy 1 lb., \$1 will buy 3 lb., and \$7 will buy 7 × 3 lb. = 21 lb.

Thus we have the convenient

***Rule.*—Multiply the number of articles that can be bought for \$1 by the number of dollars.**

14. At $16\frac{2}{3}$¢, how many books can be bought for \$9? At $33\frac{1}{3}$¢?

15. At 25¢, how many pr. of cuffs can be bought for \$7.50?

16. At $33\frac{1}{3}$¢, how many handkerchiefs can be bought for \$6? @ 50¢? @ 25¢?

17. For \$3.25, how many qt. of milk can be bought @ $6\frac{1}{4}$¢? @ 5¢? @ $8\frac{1}{3}$¢?

18. Find the cost of

7 thermometers	@	.50
9 thermometers	@	.75
5 pr. opera-glasses	@	\$$9.12\frac{1}{2}$
8 pr. roller-skates	@	\$1.75
72 scroll-saw blades	@	.20
18 pr. pincers	@	.35
3 magnets	@	.75
18 pocket compasses	@	\$$1.16\frac{2}{3}$
16 yards silk ribbon	@	$.87\frac{1}{2}$
$8\frac{1}{2}$ lb. copper-wire	@	$.62\frac{1}{2}$
$\frac{1}{8}$ lb. fine copper-wire	@	\$$1.87\frac{1}{2}$

19. Find the cost of

13 printing-presses	@	\$2.75
5 velocipedes	@	\$7.75
7 canoes	@	\$55.75
$8\frac{1}{4}$ bushels potatoes	@	.90
16 cwt. rice	@	\$$6.06\frac{1}{4}$
32 bushels wheat	@	\$$1.16\frac{2}{3}$
12 sacks salt	@	\$$1.33\frac{1}{3}$
9 lb. candy	@	$.87\frac{1}{2}$
8 qt. ice cream	@	.40¢
2 bicycles	@	\$97.50
64 lb. wool	@	$.37\frac{1}{2}$

Miscellaneous Examples.

ORAL EXERCISES.

1. Multiply the numerators of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{3}{8}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{8}$, by 3; by 4; by 5; by 6. How do these multiplications affect the value of the fractions?

Note.—Frequent illustrations should be given with diagrams and counters.

2. Divide the numerators of $\frac{2}{4}$, $\frac{4}{8}$, $\frac{4}{12}$, $\frac{8}{12}$, $\frac{8}{24}$, $\frac{4}{16}$, $\frac{6}{24}$, $\frac{8}{18}$, $\frac{12}{32}$, by 2. How are the values of the fractions affected by these divisions?

3. Multiply the denominators of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{8}$, by 4; by 5; by 10. Are the fractions increased or diminished by these multiplications?

4. Divide the denominators of $\frac{4}{4}$, $\frac{4}{8}$, $\frac{4}{12}$, $\frac{8}{12}$, $\frac{8}{24}$, $\frac{6}{24}$, $\frac{8}{32}$, by 2; by 4. How are the values of the fractions affected by dividing their denominators by integers?

5. State four ways in which the value of a fraction may be changed, and give examples to illustrate each.

6. What fraction is $\frac{1}{4}$ of $\frac{60}{97}$? $\frac{1}{10}$, $\frac{1}{15}$, $\frac{1}{12}$ of the same?

7. By how much is $\frac{1}{3}$ greater than $\frac{1}{12}$? Find the difference with the aid of slips of paper or parts of other objects. Tell what you do, and state the result.

8. Add together $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{1}{15}$ of 60.

9. Which is the greater, $\frac{5}{8}$ of 40, or $\frac{5}{6}$ of 30? $\frac{8}{9}$ of 72 or $\frac{9}{11}$ of 77? (Illustrate with counters.)

10. How many times greater is $\frac{1}{5}$ than $\frac{1}{15}$? $\frac{1}{6}$ than $\frac{1}{24}$? $\frac{2}{5}$ than $\frac{2}{15}$? $\frac{5}{6}$ than $\frac{5}{24}$?

11. Add 3 to each of the terms of $\frac{3}{7}$, and tell how much the value expressed is increased or diminished.

12. Invert $\frac{3}{7}$ thus, $\frac{7}{3}$; add 3 to each term. By how much is the value of this fraction increased or diminished?

13. From 75 subtract $\frac{7}{15}$ of 75, and find $\frac{2}{5}$ of the remainder.

14. What is the difference between $\frac{3}{5}$ of $\frac{5}{6}$ and $\frac{3}{4}$ of $\frac{2}{3}$?

15. Sixteen is $^4/_5$ of what number? 14 is $^2/_3$ of what number?

16. A train starts from Indianapolis at $7^3/_4$ A. M.; it reaches Cincinnati $6^2/_3$ hours later. At what o'clock does it arrive?

17. Add together $^9/_2$ inches, $^3/_4$ foot, and $^1/_8$ yard.

18. I bought 16 oranges for 24¢. How much was that per dozen?

19. Man ordinarily spends $^1/_3$ of his time in sleep. How many hours at that rate does he sleep in a fortnight (14 days)?

20. If I read $^2/_5$ of a book in a day, how long at the same rate shall I be in reading the whole of it?

21. Eight times $^3/_4$ of $18 = ?$ 9 times $^2/_3$ of $16 = ?$ 7 times $^5/_9$ of $15 = ?$

22. I had \$$^3/_4$; spent \$$^1/_{10}$ for ink, \$$^1/_5$ for writing-paper, and \$$^1/_{20}$ for pens. How much money had I left?

23. Read the following fractions in the order of their value, beginning with the smallest: $^7/_4$, $^7/_6$, $^7/_{24}$, $^7/_8$, $^7/_{12}$.

24. Eighteen and three eighths yards are cut from a piece of cloth measuring $27^3/_5$ yd. What fraction of the piece remains?

25. Five ninths is $^2/_3$ of what number? $^7/_{12}$ is $^3/_7$ of what number?

26. How many copper wires $^1/_{30}$ of an inch in diameter must be laid side by side to cover $^1/_2$ inch? $^2/_3$, $^5/_6$, $^9/_{10}$ inch?

27. In an orchard $^1/_6$ of the trees are apple-trees, $^1/_{12}$ pear, $^1/_9$ plum, $^1/_3$ peach, and 22 are cherry-trees. How many in all?

28. Five little girls held a fair for the benefit of the Fresh-Air Fund. After paying out for expenses $^1/_{10}$ of the whole sum received, they contributed \$36 to the Fund. How much did they receive?

29. Early June peas are 18¢ a can at retail. How much do I save per can by buying them at wholesale, \$1.90 per doz.?

30. The sum of two numbers is $2^7/_8$. One of the numbers is $1^2/_3$. What is the other?

31. Johnny weeded $^1/_7$ of his garden on Monday; $^1/_6$ on Tuesday; $^1/_3$ on Wednesday, and in the remaining days of the week finished the task in equal portions. What part did he do each of those days?

32. Edith earns a cent for each extra $^1/_2$ hour she practices her music. Last week she earned \$$^1/_4$. How many extra half-hours did she practice?

33. A woman weaves 6 yards cloth in 2 days, of 12 hours each. What part is the work of 1 hour? If she is paid \$$^5/_8$ a yd., what are her day's wages?

34. A painter bought $18^3/_4$ quarts of turpentine at $^1/_6$ of a dollar per qt. He sold it at $^1/_4$ of a dollar per qt. What did it cost him, and what was his profit?

35. Divide the sum of $2^1/_3$ and $3^1/_2$ by their difference.

36. Twelve yards of goods $^3/_4$ yd. wide will make me a dress. How many yards will I need of silk that is $^1/_2$ yd. wide?

37. On the 4th of July Mr. Brown divided $^3/_5$ of \$4.00 among his children. To the eldest he gave $^1/_4$ of the $^3/_5$, and to each of the others 45¢. How many children had he?

38. John and Will together mow the lawn. John mows $^1/_4$ in 1 hour, and Will $^1/_6$. How long does it take both to mow it?

39. If a man can do $^5/_9$ of a piece of work in 4 days, in what time will he do the entire job?

Analysis.—If he does $^5/_9$ in 4 days, he will do $^1/_9$ in $^1/_5$ of $4 = {}^4/_5$ day, and $^9/_9$, or the whole, in $9 \times {}^4/_5 = {}^{36}/_5 = 7\,^1/_5$ days.

40. If $^4/_{11}$ of an acre of ground yields 40 bushels of tomatoes, how many bushels per acre?

41. If $^3/_8$ of a bushel of Bermuda potatoes costs 90¢, what is the price per bushel?

42. If $^4/_7$ of a yard of velvet costs \$5, what does 1 yard cost?

43. Mr. Jackson sold $^7/_{13}$ interest in his shop for \$5,600. What was the whole business valued at?

44. If a horse can trot $^3/_8$ of a mile in 1 minute, in what time, at the same rate, can he trot 1 mile? $^5/_8$ of a mile?

45. If I had $^1/_3$ and $^1/_4$ more tulips in my garden I should have 57. How many have I now?

46. If a man earns $\$^1/_3$ per hour, a woman $\$^1/_5$, and a boy $\$^1/_{12}$, what do all receive for 1 hour's work?

47. How many hours must the boy work to receive an hour's wages of a man? How many the woman?

48. In what time, working together, can the woman and the boy earn an hour's wages of the man?

49. What number diminished by $^2/_7$ and $^5/_9$ of itself leaves a remainder of 30?

50. Seven tenths of a certain number less $^5/_8$ of it is 15. What is the number?

51. My age is $^3/_5$ of my brother's; his age is $^5/_{12}$ of father's, who is 72 years old. How old am I?

52. A plague carried off $^2/_9$ of a flock of sheep in one week, $^3/_7$ of the remainder the next, and 28 were left. What was the original number of sheep?

53. A contractor was to receive \$60,000 for a building, but forfeited $^1/_{40}$ that amount because it was not finished within the specified time. How much did he lose?

54. If 2 leaps of a dog are equal to 3 leaps of a hare, how many leaps of the dog are equal to 27 of the hare?

55. What number is reduced to 64, when $^8/_9$ of it are taken away?

SLATE EXERCISES.

1. My study measures $14^3/_4$ feet in length, and $12^2/_3$ feet in width. How many square feet in the floor?

2. A money-bag contains 37 half-dollars, 49 quarter-dollars, 37 twenty-cent pieces, 39 dimes, 63 nickels. How much money in all?

3. An employer pays $\$95\frac{1}{5}$ to his workmen, each one receiving $\$13\frac{3}{5}$. How many are there?

4. If 56 laborers earn each $\$\frac{1}{3}$ per hour, how much do they all earn in 6 days and 6 hours, reckoning 8 hours to the day?

5. Twenty-nine and four fifths yards were sold from a piece of cloth measuring $42\frac{3}{4}$ yd. What was the value of the remainder at $\$1\frac{17}{20}$ a yd.?

6. A merchant buys $52\frac{1}{2}$ bushels of beans at $\$1\frac{1}{2}$ a bu.; $35\frac{3}{5}$ bushels of peas at $\$1\frac{3}{5}$ a bu.; $28\frac{5}{8}$ bushels of cranberries at $\$2\frac{1}{3}$ a bu.—(1) Find the cost of each item. (2) Find the total cost, and the whole number of bushels.

7. He made a profit of $1\frac{1}{2}$¢ on every quart of beans, $2\frac{1}{3}$¢ on every qt. of peas, and $2\frac{1}{6}$¢ on every qt. of cranberries.—(1) Find the profit on each item. (2) Find the total profit.

8. A farm of $276\frac{11}{16}$ acres rents for $\$2013\frac{1}{2}$. What is the rent per acre?

9. I bought 45 government bonds at $\$105\frac{1}{2}$, and sold them at $\$106\frac{7}{8}$. Find the gain.

10. A house-painter earns $\$3\frac{3}{4}$ a day of 10 working hours. One week in which he worked extra time his pay amounted to \$25. How many extra hours did he work that week?

11. What will $7\frac{1}{4}$ yards of lace cost at $\$\frac{4}{5}$ per yd.? At $\$\frac{2}{3}$? At $\$\frac{5}{7}$? At $\$\frac{7}{12}$?

12. A person standing exactly under the equator is carried by the rotation of the earth 24,899 miles a day. How many miles is he carried in 1 hour, 2 h., 3 h., 5 h., 6 h., 8 h., 12 h.? (What part of a day is 1 hour? Do the work with as few figures as possible.)

13. If $7\frac{1}{2}$ lb. coffee cost $\$2\frac{1}{10}$, what will $11\frac{2}{7}$ lb. cost? $10\frac{5}{8}$ lb.? $4\frac{8}{9}$ lb.? $12\frac{1}{6}$ lb.?

14. Mr. A. left by will $\frac{3}{8}$ of his estate to his wife, $\frac{2}{5}$ of the remainder to his eldest son, $\frac{1}{3}$ of what was then left to his eldest daughter, and \$20,000 to each of his two other children. What was the value of the estate?

15. A grocer bought two tubs of butter, weighing together $70\,^{11}/_{12}$ lb. One tub when empty weighed $7\,^{1}/_{4}$ lb., and the other $8\,^{1}/_{3}$ lb. How much butter did he buy?

16. Find the sum and the difference of $(3\,^{3}/_{4} \div 5\,^{2}/_{5})$ and $(4\,^{2}/_{3} \div 5\,^{5}/_{8})$.

17. Find the sum and the difference of $(5\,^{4}/_{5} \times 3\,^{6}/_{7})$ and $(7\,^{3}/_{4} \times 3\,^{5}/_{6})$.

18. If it takes a workman $^{1}/_{6}$ of a day to do $^{2}/_{9}$ of a piece of work, how much of it can he do in $^{5}/_{8}$ of a day? How much in $^{3}/_{7}$ of a day?

19. If $^{3}/_{5}$ of an acre of land is sold for $\$45\,^{3}/_{20}$, what is the remainder worth at double the rate?

20. If $^{1}/_{15}$ of a box of merchandise is worth $\$7\,^{1}/_{8}$, what is $^{1}/_{5}$ worth?

21. A grocer mixes $57\,^{1}/_{2}$ lb. of tea, at $\$^{6}/_{10}$ a lb., with $42\,^{1}/_{2}$ lb. of tea at $\$^{7}/_{10}$ a lb. What is the value of a lb. of the mixture?

22. A farmer sold $^{5}/_{8}$ of his wheat at $\$1\,^{1}/_{10}$ a bushel, and received $\$796\,^{2}/_{5}$ for it. How many bushels did he sell, and how many did he have at first?

23. Mr. Hill, having \$500 to pay expenses, made a journey that lasted 6 weeks. On reaching home he had $\$46\,^{4}/_{5}$ left.—(1) How much did he spend? (2) What was the average expenditure per week?

24. I bought a house and paid down $^{1}/_{3}$ of the price, and in one year thereafter I paid $^{2}/_{5}$ of the price. The two payments amounted to \$43,780. What was the price of the house?

25. A clerk has a monthly income of \$75, and spends $\$54\,^{2}/_{5}$ per month. How much does he save a year?

26. By how much would he have to diminish his expenses, per month, to save $\$20\,^{1}/_{2}$ per year more than he now does?

27. A laborer borrowed from his employer $\$66\,^{3}/_{20}$, agreeing to pay it by having $\$2\,^{45}/_{100}$ deducted from his wages every week. How many weeks at that rate did it take him to pay his debt?

28. If $^2/_5$ of 7 lb. of coffee costs $\$^7/_8$, how many lb. can be bought for $\$1^{23}/_{25}$? $\$2^4/_5$? $\$5^3/_5$?

29. What is the sum of the area of 5 fields, containing severally $93^7/_8$, $24^3/_7$, $86^{11}/_{56}$, $56^{17}/_{28}$, and $89^9/_{14}$ acres?

30. What is the cost of $23^1/_2$ lb. flour at $\$^1/_{20}$? $15^3/_4$ lb. oatmeal at $\$^2/_{25}$? $3^2/_5$ lb. raisins at $\$^4/_{25}$? $17^1/_2$ lb. nails at 4¢? 1 doz. fire-shovels at $12^1/_2$¢ apiece?

31. What number multiplied by $3^7/_8$ will give 2; what number divided by it will give $^2/_3$?

32. What number multiplied by $^2/_3$ of $11^3/_4$ will produce 1.

33. In a school of 100 pupils, of whom $^3/_5$ are boys, 7 boys and 4 girls are absent. What part of the boys are present? What part of the girls?

34. One third of the eighth part of what number is equal to $9^1/_3$?

35. How many cubic feet in a box $4^1/_2$ ft. long, $3^1/_3$ ft. wide, $7^1/_2$ ft. deep.? (See problems, page 103.)

36. If one faucet empties a cistern in 6 hours, and another in 9 h., in what time will both together empty it? What part of the contents will the two faucets discharge in 1 hour?

37. In what time will both empty it if the first begins to run after the second has run for 2 hours?

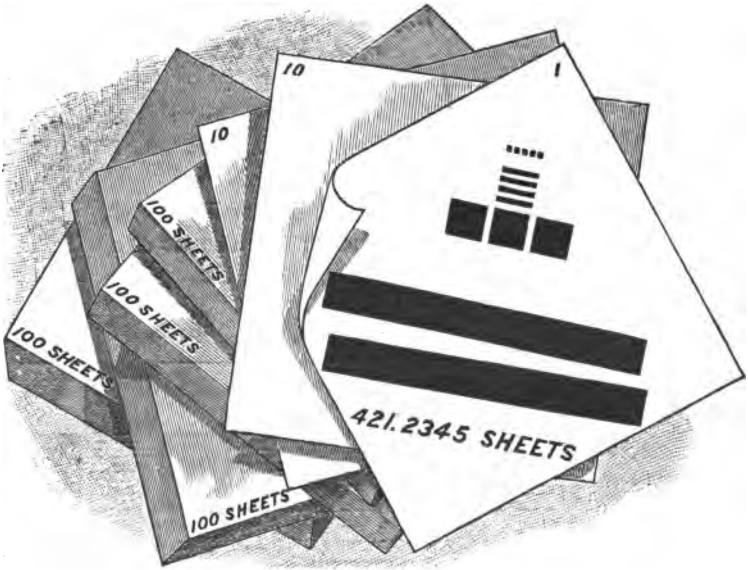
38. A can set the type for a certain book in 6 days, B in 8, C in 9, and D in 12 days. In what time can they do it working together? (What part will they all do in a day?)

39. How long must a room $4^2/_5$ yards wide be to contain as many square yards in the ceiling as a room $7^1/_5$ yards long and $5^1/_2$ yards wide?

40. I can walk 20 miles in 5 hours, and my friend can do it in 6 hours. Starting at the same time from points 20 miles apart and walking toward each other, how far are we apart in 1 hour, and in what time from starting would we meet?

CHAPTER X.

DECIMAL FRACTIONS.



164. The last chapter presented a mode of writing fractions in which the number of parts are indicated by one number and their names by another. This chapter shows how both the number and name of certain fractional parts may be represented by the decimal system.

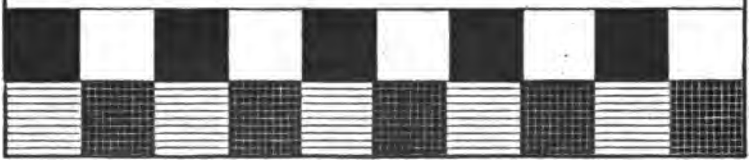
Note.—Exercises on the following diagram are designed to familiarize the pupil with the relations of such parts. Bundles of jackstraws will also serve for illustration.

Illustration.—If a square sheet of paper were ruled into 10 *long slips*, and each of these were subdivided into 10 *small squares*, and the small squares into 10 *short slips*, and each short slip into 10 *tiny squares*, as shown in the two slips below:

1. How many *long slips* would there be? How many *small squares?* How many *small slips?* How many *tiny squares?*

2. What part of the whole diagram is a *long slip?* A *small square?* A *small slip?* A *tiny square?*

3. What part of a *long slip* is a small square? A short slip? A tiny square? What part of a *small square* is a tiny square? etc., etc.



Note.—The questions given above are only suggestive of exercises designed to make the pupil familiar with decimal parts and their relations.

165. The division of anything into ten equal parts, and the subdivision of these into ten smaller equal parts, and so on, are ***Decimal Divisions***, and the parts are ***Decimal Parts.***

Note.—The dime is a decimal part of a dollar, the cent a decimal part of a dime, the mill a decimal part of a cent.

166. A ***Decimal Fraction*** is one or more of the decimal parts of a unit.

Decimals expressed in Figures.

167. The first illustration (page 173) represents 421 sheets of paper, and 2 tenths, 3 hundredths, 4 thousandths, 5 ten thousandths of a sheet; and as each figure of 421 indicates by its place whether it represents units, tens, or hundreds, so the figures 2, 3, 4, and 5 may be made to indicate by their places whether they represent tenths, hundredths, thousandths, or ten-thousandths. But to show that they represent parts and not wholes, that they are *decimals* not *integers*, a point, called the ***decimal point*** (.), is placed before them, and the number is written thus: 421.2345.

168. The cipher is used in decimals, as in integers, to mark vacant places. Thus, if the two long slips were omitted in the illustration, the number represented would be expressed by 421.0345. If there were no long slips nor small squares it would be written 421.0045, etc., etc.

EXERCISES ON DIAGRAM.

1. How many sheets and how many and what parts of a sheet are represented by 4.2? 2.05? 3.82? .35? .23? 1.01? 2.71? .182? .19? 41.41? 3.00? .4321? 10.1? 7.15? 6.01? .101? 17.208? 15.001? 21.0021? .0053?

Give first the descriptive names of the parts, as long slips, small squares, etc., then use the proper arithmetical terms, tenths, hundredths, etc., thus: 4 sheets and 2 long slips, or, 4 sheets and 2 tenths of a sheet.

2. Illustrate by diagram, on slate or blackboard, what is meant by .01, by .25, by .35, by 3.7, by 1.3, by 2.004, etc.

3. Is there any difference in value between 6.7 and 6.70? Between 3.7 and 3.07? Between 5.16 and 50.16? Between .81 and .8100? **(In stating the differences, tell what parts of the diagram are represented in each case.)**

4. Tell how many long slips, small squares, etc., must be cut from a sheet of paper to have .357 of a sheet? To have .5642? To have .045? etc.

Without the aid of words, express in figures the number of sheets and parts of sheets described below, and read, using the proper decimal terms:

1. 207 sheets 7 long slips and 8 tiny squares; 3 small squares 5 short slips and 5 tiny squares.

2. 10 sheets and 1 tiny square; 75 sheets and 1 short slip; 6 sheets and 6 small squares.

3. 17 sheets 7 hundredths and 9 thousandths of a sheet; 13 sheets and 3 ten thousandths.

4. 24 sheets 3 tenths and 6 thousandths; 8 thousandths and 7 ten thousandths.

Note.—We can represent $^1/_3$, $^5/_6$, $^3/_7$, or other common fraction of a decimal part, by writing the common fraction after the decimal, thus: $.2\,^1/_3$ is read $2\,^1/_3$ tenths. $5\,^3/_7$ small squares would be expressed by $.05\,^3/_7$, which is read $5\,^3/_7$ hundredths. $.6\,^3/_7$ is 6 tenths and $^3/_7$ of a tenth.

Without the aid of words, express in figures:

1. 3 sheets $8\,^1/_2$ short slips; 7 sheets $6\,^1/_3$ small squares.
2. 13 sheets $8\,^2/_7$ slips; 23468 sheets $5\,^4/_9$ small squares.
3. 25 sheets $4\,^2/_3$ hundredths; 81 sheets $9\,^1/_9$ thousandths.
4. 86 sheets $5\,^5/_6$ tenths; 4000 and $7\,^3/_7$ ten thousandths.
5. Which has the greatest value, the $^1/_2$ in $2\,^1/_2$, $.2\,^1/_2$, or $.02\,^1/_2$?

Note.—We may represent entire decimal parts by fractions written in the common fractional form, thus: 3 small squares may be represented by $^3/_{100}$.

6. Tell how many whole sheets, long slips, etc., are represented by the following figures: $1\,^5/_{10}$, $2\,^7/_{100}$, $^8/_{10}$, $3\,^8/_{1000}$, $^3/_{100}$, $^7/_{1000}$, $20\,^9/_{10}$, $120\,^1/_{10}$, $99\,^9/_{100}$, $37\,^7/_{100}$, $4\,^3/_{10}$.

7. Write the following fractions in decimal form: $^{17}/_{10}$ $(= 1.7)$, $^{131}/_{100}$, $^{3468}/_{100}$, $^{2426}/_{1000}$, $^{1769}/_{1000}$, $^{4432}/_{10}$, $^{1286}/_{1000}$, $^{316}/_{100}$, $5\,^7/_{10}$, $28\,^{29}/_{10000}$, $34\,^{36}/_{100}$, $5\,^{46}/_{1000}$, $^{71243}/_{10000}$, $104\,^5/_{10}$, $^{111}/_{1000}$, $^{19}/_{10}$.

Note.—Any fraction having for a denominator 10, 100, 1000, etc., is properly a decimal fraction, because it represents parts obtained by the division of the unit into tenths, tenths of tenths, etc., etc. But the term *decimal* is used alone only when there is no denominator expressed.

Definitions.

169. A ***Decimal point,*** or sign (.), is a period prefixed to a decimal to distinguish it from an integer.

170. A ***Pure Decimal*** consists of decimals only.

171. A ***Mixed Decimal*** is one that consists of an integer and a decimal.

172. A ***Complex Decimal*** is one consisting of a decimal with a common fraction annexed.

Decimal Table.

173. The following table will facilitate the learning of the several orders. The correspondence between the names of the places to the right and left of *units* should be noticed.

Table.

NAMES.	*Hundred-millions.*	*Ten-millions.*	***Millions.***	*Hundred-thousands.*	*Ten-thousands.*	***Thousands.***	*Hundreds.*	*Tens.*	***Units.***	*Tenths.*	*Hundredths.*	***Thousandths.***	*Ten-thousandths.*	*Hundred-thousandths.*	***Millionths.***	*Ten-millionths.*	*Hundred-millionths.*
ORDERS.	9th.	8th.	7th.	6th.	5th.	4th.	3d.	2d.	1st.	2d.	3d.	4th.	5th.	6th.	7th.	8th.	9th.
	5	8	7	2	9	0	3	2	7.	0	3	2	1	4	5	1	6
	INTEGERS.									DECIMALS.							

Reading Decimals in Terms of the Lowest Order.

1. 2 long slips and 3 small squares, make how many small squares ?

2. 2 tenths and 3 hundredths, make how many hundredths ?

3. 5 long and 4 short slips, make how many short slips ?

4. 2 tenths, 3 hundredths, and 4 thousandths, make how many thousandths?

5. 2 long slips, 3 small squares, 4 short slips, and 5 tiny squares = how many tiny squares?

6. 2 tenths, 3 hundredths, 4 thousandths, and 5 ten-thousandths = how many ten-thousandths?

Hence for reading decimals we have the

174. *Rule.*—Read the decimal as if it were a whole number, and give it the name of the right hand order.

Thus, .3567 is read 3567 ten-thousandths; .169 as 169 thousandths; .354789 as 354789 millionths.

ORAL EXERCISES.

1. Read, .1; .6; .9; .45; 11.4; 13.47; 51.67; 6.15; 8.24; 98.34; 100.1; 345693.71.

2. Read, 100.73; 27.02; 50.57; 6.67; 41.01; 120.03; 200.01.

3. Read, 1.111; .567; .004; 75.123; 3.004; 1.012; 6.953.

4. Read, 92.009; 9.00012; 13.8947; 57.625341; 1.06777893.

---

5. Read, pronouncing separately the order of each digit in the fractional parts: 61.43 (61 and 4 tenths, 3 hundredths).

10.9; 738.5423; 4.02; 5.063; 31.02803; 39.417356; 10.1324; 12.11; 26.103; 17.1101; 29.922; 30.87203456; 9.39485762.

---

6. Read the following as mixed decimals, that is, the units first then the decimal: (Read 6.12 thus, six and 12 hundredths).

14.013; 6.57; 3.0154; 46; 1044; 9.999; 20.02; 35.04; 46.34256; 50.148735; 83.4283; 87.87328; 7.5983.

7. Read the following as improper fractions, that is, read integer and fraction together as one number, giving to the whole the name of the lowest decimal order. (Read 7.04 as 704 hundredths.)

18.164; 516.2; 5.005; 29.092; 5.79; 13.579; 1357.9; 1.010; 263.4501; 63.4; 63.04; 63.004; 63.0004; 5.00013. (The last is read, five hundred thousand thirteen hundred thousandths.)

Suggestion.—Ask yourself whether it is true that 7.04 is equal to 704 hundredths. Turn to the illustration, on page 173, and study this out for yourself. How many small squares in 7 sheets of card-board? How many in 7 sheets and 4 small squares?

8. If .04 were written in the form of a common fraction, what would the numerator be? What the denominator? Answer like questions with regard to the decimals in exercises 1 and 2.

Note 1.—Observe that when the denominator is written, the decimal point and the ciphers preceding the first significant figure are omitted in the numerator: thus, $.03 = {}^3/_{100}$, not ${}^{.03}/_{100}$. (${}^{.03}/_{100}$ is equivalent to the complex fraction $\frac{{}^3/_{100}}{100}$.)

Note 2.—Observe, also, that the denominator of a decimal, when written, contains as many 0's as there are figures in the decimal.

Writing Decimals.

For the writing of decimals, the following rule will be found serviceable. Skill is to be obtained only by practice.

175. ***Rule.*****—Place the decimal point, then, after considering how many places are needed to give the last figure of the decimal its proper order; write each figure in the order to which it belongs.**

Example.—Write 375 hundred thousandths.

Remembering that hundred thousandths is the fifth decimal order, and observing that 375 contains only three figures, we perceive that two orders must be filled with ciphers, thus: .00375.

SLATE EXERCISES.

Write in figures:

1. Three and fifteen hundredths; thirty-one thousandths.
2. One and one thousandth; twelve and fifteen hundredths.
3. One hundred twenty eight and seventeen thousandths.
4. Seventy-eight ten thousandths; seven hundredths.
5. Sixty-one hundred thousandths; ten and one ten thousandth.
6. Fifty-four thousand and fifty-four ten thousandths.
7. Five thousand seventy-five millionths.

Addition of Decimals.

176. ***Rule.*—Write the numbers to be added so that figures of the same order shall stand in the same column.**

Add as in integers, and place the decimal point in the sum directly under the decimal points in the numbers added.

Examples.—

1.	2.	3.	4.
3.523	.9874	.12	5.678934
23.42	13.21	5.2	2.16674
6.006	45.135	134.56	.00374
4.734	1.0006	42.03	17.00003

5. 6.6+77.77+888.888+26.742+1.2+5.401+.002=

6. 4.1535+.92+12.3472+.006+11.3+2.00046+9.07=

7. 100.2+59.012+8.+3.1205+69.+63.109+934563.4=

8. 604.1+.012+18.069+9.232+8.01+2.10004+3.05=

9. 10.901+12.+43.321986+.79342+4283.4132+6.7=

	10.	11.		12.	13.	
14.	14.3	+2.348	+4.56	+17.01	+384.9000	=
15.	9.58	+8.71	+6.54	+ .004	+ 15.401	=
16.	73.374	+9.234	+3.042	+ 9.345	+ 3.789346	=
17.	1.583	+5.006	+7.1	+ 7.2003	+100.007384	=

Test the accuracy of your results. (See note, page 32.)

Applications.—1. Add the following sums of money: $28.36, $108.09, $27.50, $1.30, $38.742, $387.655, $998.999, $3.27.

2. Six marble blocks weigh respectively 5.73 cwt., 4.834 cwt., 7.938 cwt., 6.4 cwt., 15 cwt., and 387.1 cwt. Find the total weight.

3. A train on the Pennsylvania R. R. ran 56.3 miles in the first hour, 62.34 miles in the second, 59.247 in the third, 60.7304 in the fourth. How many miles altogether?

4. A draper bought 2 pieces of buckskin, each containing 56.34 yards; 2 pieces of rep, each containing 96.05 yards; and 1 piece of broadcloth, containing 27.2 yards. Find the number of yards in the 5 pieces.

Subtraction of Decimals.

177. ***Rule.*—Write the subtrahend under the minuend, so that figures of the same order shall stand in the same column.**

Subtract, as in integers, and place the decimal point in the remainder directly under the decimal points of the minuend and subtrahend.

Examples.—

1. 94.324	**2.** 73.6	**3.** 5.4	**4.** 9.7	**5.** 6.01
7.86	19.79	4.38	6.543	3.4

6. 7384.02	**7.** 9.004	**8.** 3.28764	**9.** 15.60003004
56.934	7.2043	1.00009	.794569376

10. 1.01	**11.** 4.003	**12.** 15.	**13.** 70.	**14.** 50009.
.09	2.006	6.3785	16.7345	5.0009

15. 8.452—3.1052=

16. 92.8245—9.86543=

17. .0052—.0041=

18. 3.004—.0097=

19. 121.12—8.943=

20. 423.4567382—413.05=

21. 73845.009—1.23456=

22. 9384.708—2.3457=

23. 342.5703—.1994=

24. 6534.70045—3.7634=

25. 897.809—3.1073=

26. 828.00019—6.0004=

Applications.—**1.** From a lot containing 10,000 □ yards, 437.296 □ yds. are sold. How large is the remaining part? (The sign □ is used for the word "square.")

2. From 17.256 tons of coal 5.625 tons were used. How much was left?

3. Mr. Smith's property amounted to $47,300.75 when he died. Accounts, to the amount of $340.95, were presented and paid. How much was left to the heirs?

4. The French meter is 39.37079 inches. How much longer than a yard is the meter?

5. Find the difference in height of two flag-staffs, the one measuring 38.75 ft., the other 53.9 ft.

6. Find the difference between .57 and .7; between eight hundred fifty-two ten-thousandths and 1.

Multiplication of Decimals.

Example.—1. Multiply .75 by 3. (Find 3 times .75.)

Process.

$$\begin{array}{r} .75 \\ 3 \\ \hline 2.25 \end{array}$$

Analysis.—3 times 5 hundredths = 15 hundredths = 1 tenth and 5 hundredths; 3 times 7 tenths = 21 tenths; 21 tenths + 1 tenth = 22 tenths = 2 units and 2 tenths.

Repeat the analysis, using the terms *small squares*, *long slips*, and *sheets*, respectively, for hundredths, tenths, and units.

The Multiplier a Decimal.

Example.—2. Multiply .75 by .3. (Find 3 tenths *of* 75 hundredths.)

Process.

$$\begin{array}{r} .75 \\ .3 \\ \hline .225 \end{array}$$

Analysis.—3 tenths of 5 hundredths = 15 thousandths = 1 hundredth and 5 thousandths; 3 tenths of 7 tenths = 21 hundredths; 21 hundredths + 1 hundredth = 22 hundredths = 2 tenths and 2 hundredths.

Repeat the analysis, using the descriptive terms *short slips*, etc. Thus, $^3/_{10}$ of 5 small squares = 15 short slips = 1 small square and 5 short slips, etc.

Example.—3. Multiply .75 by .03. (Find 3 hundredths of .75.)

Process.

$$\begin{array}{r} .75 \\ .03 \\ \hline .0225 \end{array}$$

Analysis.—3 hundredths of 5 hundredths = 15 ten thousandths (see diagram, page 174); 15 ten thousandths (tiny squares) = 1 thousandth and 5 ten thousandths; 3 hundredths of 7 tenths = 21 thousandths; 21 thousandths + 1 thousandth = 22 thousandths = 2 hundredths and 2 thousandths.

Repeat the analysis, using the descriptive terms *tiny squares*, etc.

178. Thus, we find that if the order of the multiplier is units, the order of the product is the same as that of the multiplicand. If the multiplier is tenths, the order of the product is *one degree* lower; if it is hundredths, the order of the product is *two degrees* lower, etc.

179. *Hence, in the product of two decimals there are as many decimal places as there are in the multiplicand, plus the number of decimal places in the multiplier.*

180. *Rule.*—**Multiply as in whole numbers, and from the right of the product point off as many figures for decimals as there are decimal figures in the multiplier and multiplicand together. If there be not so many figures in the product, supply the deficiency by prefixing ciphers.**

ORAL EXERCISES.

4.	5.	6.	7.	8.
.2×3=	14×.6=	.42×3=	.2×.3=	.2×.003=
.4×6=	13×.5=	.26×5=	.3×.6=	.9×.005=
.5×5=	15×.4=	.31×7=	.4×.5=	.5×.008=
.7×2=	17×.3=	.02×9=	.5×.2=	.7×.004=
.8×4=	16×.2=	.63×8=	.6×.4=	.3×.007=

SLATE EXERCISES.

Note.—It is well for pupils to accustom themselves to *estimate results;* for instance, if it is required to multiply 5.65 by 7.001, they should be able to say at a glance that the product will be *about* 39, that is, a little more than $5\,^1/_2$ times 7.

9.	10.	11.	12.
.3×.093=	.5×.934=	.52×.213=	1.5×.3=
.8×.075=	.7×.825=	.73×.332=	2.4×.5=
.5×.084=	.9×.738=	.84×.252=	1.5×.7=
.9×.063=	.6×.225=	.95×.163=	3.2×.6=
.7×.052=	.3×.367=	.62×.421=	1.6×.9=

13. 736.045×.843
14. 93×.0067
15. 4.709×.7635
16. 84.008×100.001
17. 17827.032×8.754
18. .0009×.0543
19. 9.00134×8.004
20. .195×.00027
21. .1825×18.24
22. .75×.30052
23. 84.008×1000.4
24. 258.01×3030.1
25. .98×36.0007
26. .357×88345.4
27. 28.601×3.425

28. Multiply .004; .71; .70014; 1.04 by .0091.
29. " .05; .17; .999; .7534 by .0008.
30. " 1000; 100; .001; .64; .01 by 2.847.

Applications.—1. My age is 1.075 times my brother's; if he is 30, how old am I? If he is 25, how old am I?

2. What is the area of a lot which is 9.34 yd. wide and 48.5 yd. deep? (How many square yards in it?)

3. Find the area of a field .876 miles by .0056 miles?

4. At $5.87 per acre, what is the rent of a farm of 47.9 acres?

5. If I buy 2 cwt. 66 lb. sugar at $13.09 per cwt., and sell it at $.12 per lb., what do I gain or lose on the whole?

Division of Decimals.

Example.—1. How many times .18 in 54?

Process.

$$.18\overline{)54.00}$$
$$300.$$

Analysis.—In 54 units there are 5400 hundredths, and 18 hundredths are contained 300 times in 5400.

Illustration.—18 of the small paper squares represented on page 174 can be taken 300 times from 54 sheets.

2. How many times .18 in 5.4?

Process.

$$.18\overline{)5.40}$$
$$30.$$

Analysis.—In 5.4 there are 54 tenths = 540 hundredths. In 540 hundredths, 18 hundredths is contained 30 times.

Illustration.—Show that 18 small squares are contained 30 times in 5 sheets 4 long slips.

3. How many times .18 in .54? *Ans.*, 3.

4. How many times .018 in 54? In 5.4? In .54?

181. Let it be observed that in every case the dividend must be reduced to an order at least as low as that of the divisor. Evidently, if we are to ascertain how many times 18 short slips there are in any number of sheets, long slips, or small squares, we must first ascertain how many short slips there are. Hence, in division of decimals, there must always be as many decimal places in the dividend as in the divisor.

5. How many times 1.08 in .05778?

Process.

$$\begin{array}{r} .035 \\ 1.08\overline{).05|778} \\ \underline{540} \\ 378 \\ \underline{324} \\ 540 \\ 540 \end{array}$$

Explanation.—Beginning with tenths, we count off as many decimal places in the dividend as there are in the divisor, and separate them from the places to the right by a short vertical line. This marks the point below which no integer can be obtained in the quotient (no quantity can be contained any whole number of times in a quantity less than itself). Here also the decimal places must begin, for, though one tenth of the divisor be not contained in the next partial dividend, the place must be marked by a cipher in order that figures of lower orders may have their proper places.

182. ***Rule*** **—1. Annex ciphers to the dividend, if necessary, till the right hand order is the same as that of the right hand figure of the divisor.**

2. Divide as in simple division. Place the decimal point immediately before the quotient figure that is obtained from the order of the dividend next lower than the lowest order of the divisor.

Note.—There must always be as many decimal places in the quotient as there are in the dividend more than in the divisor.

ORAL EXERCISES.

1.	2.	3.	4.
.4÷8=	2÷.5=	.64÷.08=	.16÷.8=
.6÷5=	5÷.3=	.49÷.07=	.14÷.7=
.8÷4=	4÷.6=	.36÷.03=	.08÷.4=
.2÷2=	5÷.7=	.84÷.04=	.09÷.3=

SLATE EXERCISES.

1. 7.32÷6	**6.** .96÷32	**11.** 1÷.0037	**16.** 1000÷.09
2. 123÷6	**7.** .16÷.4	**12.** 10÷.001	**17.** .0045÷9
3. 127÷6	**8.** .58÷3.1	**13.** .5÷1000	**18.** .03÷1.004
4. 4÷.008	**9.** .6368÷8	**14.** 45.98÷10	**19.** .0375÷.03
5. 4.5÷67.8	**10.** 1÷.0025	**15.** 1000÷.5	**20.** 7986÷3.75

21. 1.6875÷25	**26.** 789.7÷1000	**31.** 604.56÷1000
22. 134.25÷7.5	**27.** 1.6875÷6.75	**32.** 1220.674÷19
23. .0456÷.04	**28.** 2.0005÷7.24	**33.** 144.6955÷8.5
24. 733.26÷33	**29.** 128.175÷7.5	**34.** 12.345÷.00015
25. 1139÷9250	**30.** 7.024÷2.0005	**35.** 15.63386÷4.367

36. 549.9025÷2.345	**41.** .0013409÷.583	**46.** 245.8677645÷405
37. 994.8015÷22.33	**42.** .0000026÷.004	**47.** 20.34407403÷.21
38. 600.2623÷66.77	**43.** 5941.8623÷66.77	**48.** 12345.4321÷111.11
39. 7.006652÷1.234	**44.** 37.873565÷8.765	**49.** 1.33709774÷.11111
40. 1220.674÷64.246	**45.** .0897688÷.0202	**50.** 72.01440072÷8.0008

Applications.—1. The circumference of a circle is 3.14 times the length of the diameter. Find the diameter of a circle whose circumference is 51.339 yd.?

2. The area of a rectangle is 3414.012 □ yd., its width is 125.7 yd. What is its length?

3. 56.325 cwt. of certain goods cost $49.45335; what is the cost of 1 cwt.? Of 1 pound?

4. 36.35 yd. of cloth cost $117.95; what does 1 yd. cost at the same rate?

Reducing Common Fractions to Decimals and Decimals to Common Fractions.

Exercises on Diagram, page 174.

Express in decimals and also in lowest terms of common fractions the parts of the diagram

1. In 2, 3, 4, etc., long slips.
2. In 8, 25, 32, 20, 75 small squares.
3. In 2, 8, 14, 25, 125, 175 short slips.
4. In 8, 16, 32, 125, 1875, 625, 3125 tiny squares.

183. Changing Decimals to Common Fractions.—5. Express .6 in the lowest terms of a common fraction.

Process.—$.6 = {}^6/_{10} = {}^3/_5$.

6. Express .4, .8, .16, .72, .75, .375, .875, .4375, .04, .0016 in lowest terms of common fractions.

Note.—The learner will be able to write out his own rule for the foregoing process.

7. Express in integers and common fractions 1.2, 15.25, 8.6.

8. Express $.4^2/_3$ in the lowest terms of a common fraction.

Process.—$.4^2/_3 = \frac{4^2/_3}{10} = {}^{14}/_{30} = {}^7/_{15}$. (See Art. 156.)

9. In like manner find the equivalents of $.3^4/_5$, $.23^3/_8$, $.7^9/_{16}$, $.324^1/_7$ in common fractions.

184. Changing Common Fractions to Decimals.—Any fractional part of an object must contain a like part of the decimal divisions of the object.

Thus, $^1/_2$ the diagram, page 174, contains $^1/_2$ of ten long slips = 5 long slips = .5; 50 small squares = 50 hundredths, etc., etc.

$^1/_4$ of the diagram contains $^1/_4$ of the decimal divisions, as: $2\,^1/_2$ long slips $= .2\,^1/_2$; or, 25 small squares = .25 of the diagram.

$^3/_8$ of the diagram contains $^3/_8$ of the long slips. $^3/_8$ of 10 long slips $= 3\,^3/_4$ long slips $= .3\,^3/_4$. $^3/_8$ of 100 small squares $= 37\,^1/_2$ small squares $= .37\,^1/_2$; and $^3/_8$ of 1000 short slips = 375 short slips = .375 of the diagram.

185. Hence, to convert a common into a decimal fraction, we take such part of the decimal divisions of the unit as is indicated by the common fraction.

10. Find decimals equivalent to the common fractions, $^2/_5$, $^3/_5$, $^3/_8$, $^9/_{16}$, $^{11}/_{32}$, $^{12}/_{25}$, $^{21}/_{32}$, $^8/_{125}$, $^{27}/_{64}$, $^{101}/_{125}$, $^{333}/_{625}$.

11. Write in integers and decimals equivalents for $3\,^1/_2$, $563\,^3/_4$, $5\,^1/_4$, $7\,^1/_8$, $9\,^{17}/_{25}$, $16\,^4/_5$.

12. Find equivalents for $^1/_3$, $^2/_3$, $^5/_6$, $^8/_9$, $^3/_7$, $^5/_{12}$, $^9/_{14}$, $^{16}/_{21}$, $^{29}/_{30}$, $3\,^2/_3$, $7\,^5/_6$, $8\,^5/_9$, $4\,^2/_7$, $81\,^3/_{14}$, $9\,^1/_9$ in pure or mixed decimals.

Suggestion.—The question should be raised here, why it is that in Examples 10 and 11 all the common fractions are exactly reducible to decimals, while those in 12 are not. Thus the learner may discover for himself the condition under which exact decimal results are possible.

186. ***Rule.*—1. To reduce common fractions to decimals, annex ciphers to the numerator of the common fraction, divide by the denominator. Continue the process till the division is complete, or until the result is sufficiently exact.**

2. Point off as many decimal places in the quotient as there are decimal ciphers annexed to the numerator of the common fraction. If there be not so many places, ciphers must be prefixed to the significant figures to supply the deficiency.

Note.—The further the division is carried, the more exact is the result. In most cases sufficient accuracy is reached in the third or fourth place of decimals.

Repetends.—In the process of division, if a remainder is repeated, the figures of the quotient will be repeated in the same order as after its first occurrence.

187. A figure or set of figures thus repeated is called a ***Repeating*** or ***Circulating Decimal,*** or simply a ***Repetend.***

188. The sign of a repetend is a dot (˙) written over the repeating figure, or a dot over the first and last figure, if it contains more than one.

Note.—At this point the pupil needs to learn no more of this subject than how to indicate a repetend when it occurs, and that he may discontinue the work of division on the first recurrence of any particular remainder. (See Appendix.)

Examples.—1–7. Reduce the following common fractions and indicate the repetends: $^2/_7$, $^2/_9$, $^2/_{11}$, $^1/_3$, $^7/_{13}$, $^8/_{15}$, $^2/_{13}$.

SLATE EXERCISES.

Express equivalents in pure and mixed decimals:

1. $9.7\frac{1}{4}$, $7.7\frac{3}{5}$, $1.6\frac{1}{16}$.	5. $.0000\frac{2}{5} \times .9\frac{7}{8}$.
2. $\$28\frac{13}{25}$, $\$17.07\frac{4}{5}$, $.053\frac{31}{32}$.	6. $10.111\frac{1}{3} \times .033$.
3. $15\frac{7}{20}$, $9.60\frac{7}{8}$, $105.00\frac{17}{25}$.	7. $5.009 \times .08\frac{1}{5}$.
4. $\dfrac{33\frac{1}{3}}{4\frac{1}{5}}$, $\dfrac{17\frac{1}{8}}{7}$, $20.0\frac{3}{5}$.	8. $108\frac{1}{4} \times \frac{1}{3}$ of $\frac{9}{16}$.

9. Find the sum of $\frac{3}{4}$ and .54; the difference of $\frac{3}{4}$ and .54; the product of $\frac{3}{4}$ and .54; the quotient of $\frac{3}{4}$ divided by .54.

Find the sums of

10. $4\frac{3}{7}$, $524.2\frac{1}{7}$, $6.2\frac{3}{4}$, 7, and $.573\frac{3}{5}$.

11. $3\frac{4}{5}$ miles, $5\frac{5}{8}$ miles, 4.7 miles, 7.11 miles, and $99.9\frac{4}{5}$ miles.

12. 4.79 lb., $9\frac{4}{5}$ lb., $10\frac{9}{20}$ lb., $38.59\frac{1}{8}$ lb., 141.1 lb.

13. .125 rod, .1875 rod, $\frac{5}{8}$ rod, $.5\frac{5}{16}$ rod, $1.8\frac{3}{4}$ rod.

14. $1927.96\frac{13}{25}$ acres, $.00\frac{4}{25}$ a., 50.267 a., 1.709 a.

Find the differences between

15. $1.79\frac{1}{2}$ and $.777\frac{1}{8}$; $11.111\frac{1}{16}$ and $11.110\frac{1}{16}$.

16. $1.001\frac{1}{8}$ and $10.100\frac{1}{4}$; $7.9753\frac{2}{3}$ and $6.428104\frac{1}{5}$.

---

17. What number divided by 1.25 will give the product $11 \times 1.1 \times .001\frac{1}{10}$?

18. What was paid for 100 bbls. flour, each 196 lb., at $\$6.66\frac{2}{3}$ per 100 lb.? For 100 bbls. pork, each 200 lb., at $\$.08\frac{1}{4}$ a pound?

19. How many wagon loads in a freight car containing $2\frac{3}{16}$ tons sheet copper, 3.75 tons sheet lead, $5\frac{7}{8}$ tons sheet iron, 7.9375 tons tin plate, $1\frac{3}{8}$ tons being a wagon load?

20. From a sheet of lead weighing 1560.625 lb., circular discs were cut, weighing, respectively, $13\frac{1}{4}$ lb., $17\frac{1}{3}$ lb., 98.875 lb., 59.625 lb., $137\frac{1}{16}$ lb., $122\frac{5}{12}$ lb., $121\frac{1}{4}$ lb. What was the weight of the remnants (scraps)?

189. To find cost when number and price per hundred or thousand are given.

Per *C* is used for *per hundred* and per *M* for *per thousand.* (See page 18.)

Example.—1. What is the cost of 480 lemons at \$3.60 a hundred?

Written Work.

```
  $3.60
   4.80
  28800
 1440
$17.28ØØ
```

Explanation.—In 480 there are 4 hundred and 80 hundredths of a hundred; therefore we find 4 and 80 hundredths times the price of 1 hundred.

Note.—Ciphers at the right of a multiplicand or multiplier may be omitted in computation, inasmuch as they do not affect the value of the result. Hence the work may stand as at the right.

```
  $3.6
   4.8
   288
  144
$17.28
```

2. What must be paid for 17300 bricks at \$7.35 per M?

Written Work.

```
  $7.35
   17.3
   2205
  5145
  735
$127.155
```

Explanation.—17300 = 17.3 thousand; hence, to find the cost, at \$7.35 per thousand, we multiply \$7.35 by 17.3.

3. Find the cost of 7854 railroad ties at \$95.50 a thousand.

4. Find the cost of 1478 feet of lumber at \$45 per M.

5. Mr. Smith bought 50000 shingles at 70¢ a bundle of 250, and 38750 ft. of pine flooring at \$18.75 a thousand. What did they cost?

6. We need 45350 bricks; the price being \$6.90 a thousand, how much will they cost?

7. Mr. Wick bought 280 melons at \$7.40 a hundred. What did they cost him?

8. Find the cost of 2750 laths at 45¢ per C; of 1950 pickets at \$12 per M.

9. What is the cost of 1500 ft. of copper wire at \$2.85 per hundred yards?

10. How much will the steel rails necessary to lay one mile of road cost at the rate of \$49.30 for 100 ft. of rail? (5280 ft. = 1 mile.)

***Rule.*—Find the number of hundreds by pointing off two figures, and of thousands by pointing off three figures, on the right of the given number (representing the quantity), and by this multiply the price per hundred or thousand, as the case may be.**

---

190. To find the cost when the number of pounds and the price per ton (2000 lb.) are given.

Example.—1. What will a load of hay weighing 2386 pounds cost at $19.75 per ton?

Written Work.

2)2.386
1.193
19.75
5965
8351
10737
1193
23.56175

Explanation.—There are 2 thousand and 386 thousandths of a thousand pounds in the load, and one half as many, or 1.193, times 2000 pounds, or tons. Hence the value of the hay is 1.193 times $19.75, the price of 1 ton.

11–14. *Find the cost of*

3500 lb. of hay at $16 a ton.
4835 lb. of salt at $25 a ton.
9350 lb. of silver ore at $43.50 a ton.
380 lb. of straw at $9 a ton.

15. Find total freight charges on machinery shipped from New York to Buffalo in the following quantities, @ $\frac{7}{8}$¢ a ton per mile (see table, page 48):

15000 lb. locomotive castings. 31750 lb. flour-mill machinery.
17570 lb. pumping machinery. 49975 lb. saw- and planing-mill machinery.

16. What is the cost of 47.77 tons of iron rails @ $$29\frac{1}{2}$ a ton. What would be the freight charges from Cleveland to Buffalo @ $1\frac{1}{5}$¢ per ton for a mile?

17. Find cost of 978 tons Bessemer steel rails @ $$40.33\frac{1}{3}$, freight being $1\frac{5}{6}$¢ per ton for a mile, ordered in Cleveland and delivered in Jacksonville? (For distance, see page 48.)

18. What did I pay for 2975 pineapples at $$11.87\frac{1}{2}$ per C?

19. What is the value of 9775 lb. ice at $6.75 a ton?

20. How many thousand cartridges can be bought for $855, there being 5000 in case, the cost of a case being $47.50?

***Rule.*—Multiply the price per ton by one half of the number of thousands of pounds** (number of tons).

Miscellaneous Examples.

1. A bricklayer earned $121.22 in 29 days; how much in 1 day?

2. 38 bales of cotton cost $3213.28; what is the cost of 1 bale?

3. The area of a garden in the form of a rectangle is 4133.64 sq. yd., its length is 76 yd. How wide is the garden? (To *find* the area we multiply the length by the width.)

4. 158 logs measure 3105.648 feet. What do they average?

5. What is the 87th part of 53.244 gallons? Of 53.244 qt.?

6. Divide $^4/_5$ of 8.236 by .138 of $^9/_{10}$.

7. What decimal of $2^1/_2$ yd. is $^1/_4$ yd.?

8. What part of 3 miles is $^1/_8$ of a mile? (Express the answer in decimals.)

9. Bought 3.75 cwt. beef at $.125 per lb.; find the total cost.

10. What number multiplied by 12 will produce .1728?

11. Divide the average of 3.079, 4.276, 5.60554 by .006.

12. If I walk 3.789 miles an hour, how far will my friend walk in 5 hours, if he walks only $^4/_5$ as fast as I do?

13. The French standard of measure, the meter, is 39.37 inches long; how many meters in 1132.134 yards?

14. How much carpet 1.5 yd. wide will cover a floor 22.5 by 19.5 ft.? (How many widths, the carpet being laid from end to end of the room?)

15. A regiment of 550 men has on its sick-list .02 of the number. How many men are fit for service?

16. What decimal fraction, multiplied by $^2/_5$ of $7^4/_5$ gives $^1/_4$ of $^2/_5$ of $^7/_8$?

17. The difference between two numbers is $17^{11}/_{24}$; the greater number is $25^1/_9$, what is the smaller number? (Answer in decimals.)

18. A bar of iron 8 inches square and 1 foot long weighs 216.336 lb., what is the total weight of 5 pieces respectively 3, 4, 5, 6, and 7 ft. long? (Only one multiplication necessary.)

19. What must the dividend be if the divisor is 38.125 and the quotient 5.25 ?

20. What number must be multiplied by 7 to make $^3/_4$? To make $^{11}/_{12}$? To make $2\,^1/_2$? (Solve each by decimals.)

21. Find the price of an ounce avoirdupois, if a lb. costs \$.176 ; \$.2475.

22. The water that will fill a can which is exactly one foot long, wide, and deep is 997.7 oz., or 62.356 pounds ; hammered silver is 10.511 times heavier than an equal bulk of water. What is the weight of $2\,^1/_3$ cubic feet of the silver ?

23. After selling 78.38 acres of his land, a farmer had $198.6\,^3/_4$ acres remaining. How many acres did he have at first ?

24. Increase $^1/_4$ of 7.2 by $^3/_5$ of 6.5, and subtract from the sum the product of 5×1.14.

25. Mr. Smith has loaned \$62848 to different parties for $\$4\,^1/_2$ per year for every hundred. What is his income per year from these loans ?

26. If one yd. of calico is sold for \$.08, how many yd. can be had for $\$6\,^2/_5$, $\$4\,^2/_5$, $\$5\,^1/_5$, $\$7\,^3/_5$, $\$9\,^3/_5$?

27. An agent collected \$347.35, and received for the service 5¢ on every dollar collected. How much did he get ?

28. In a city of 240768 inhabitants, it was found that .125 of the number could not read, and only .875 of those able to read could write. How many were there who could not read ? Who could not write ?

29. How much must be paid for the use of \$750 per year at $\$5\,^1/_2$ a hundred ? For $^1/_2$, $^1/_3$, $^1/_4$, $^1/_5$, $^1/_6$, $^1/_{10}$, $^1/_{12}$ year ? (Express results in decimals.)

30. The use of \$750 cost me \$37.50 a year. What did I pay per hundred ?

31. The use of \$1200 for 10 years cost Mr. Lund \$630. What did the use of \$100 cost him per year ?

32. Mr. Smith paid \$45 a year at \$4.50 per hundred for the use of a certain sum. What was that sum?

33. Mr. Cain borrowed a sum of money at \$3.25 a hundred per year, and in 5 years paid \$162.50 for the use of it. How great a sum was it?

34. Find the cost of 6 gal. 3 qt. vinegar, at \$.125 a gal. (3 qt. = what part of a gal.?)

35. Find the cost of 16 gross 6 doz. lead-pencils, at 55¢ a doz. (A gross is 12 dozen.)

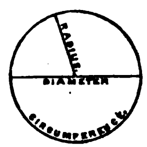
36. Find the cost of $13\,^1/_4$ yd. ribbon, at \$.2325 a yd.

37. Find the cost of 6.25 doz. cabbage-heads at 3¢ apiece.

38. Find the cost of 4 gross 10.5 doz. eggs, at \$.$22\,^1/_2$ a dozen.

39. If a railroad train runs 27.125 miles an hour, in what time will it run 303.6 miles?

Note.—The distance around a circle (the *circumference*) is very nearly 3.1416 times the distance across it through the center (the *diameter*). Let the pupil carefully measure the distance around and across a bushel or peck measure, or around and across a wagon wheel, or any other circle, and see if the distance around is not about $3\,^1/_7$ times the distance across it. ($^1/_7$ is very little greater than .1416.)



40. Find the circumference of a circle, the diameter of which is 4.2 yards.

41. The diameter of a wagon wheel is 40 in. How many yards will the wheel progress in turning 150 times?

42. Find the circumference of a circle, if the length of a radius is 3.75 in. If the diameter is 91.5 in. (Radius = $^1/_2$ Diameter.)

43. A block of gold measuring 1 in. long, wide, and thick (a cubic in.) weighs .7003 of a lb., how much does a cubic foot weigh? (See note, page 103.)

44. If I add the product of 11.111 by 22.02 to 33.033, and from this sum subtract 277.69721, what will the remainder be?

45. If you subtract the product of 5 by 31.565 from the sum of the two products 15 × .178 and 50.05 × 3.1, what will remain?

Bills and Accounts.

191. An ***Account*** is a record which may include services rendered, goods sold, or money paid by one person to another.

192. A ***Debtor*** is a person from whom a debt is due; a ***Creditor*** is a person to whom a debt is due.

193. A ***Bill*** is the creditor's written statement of the items in his account with the debtor.

194. Each item charged is called a ***debit;*** each item acknowledged as received is called a ***credit.***

195. The ***Balance*** of an account is the difference between the footing of the debits and the footing of the credits.

196. To ***receipt*** a bill is to write the creditor's name on the bill under the words ***"Received payment"*** or ***"Paid."***

A bill can be receipted only by the creditor or by a person authorized by him. In the latter case, the person receipting should write under the creditor's name "by" or "per," followed by his own name or initials. When a bill is paid by a promissory note or a due-bill, the fact may be stated after the words "Received payment."

197. To ***extend*** the items of an account is to write in the dollar and cent columns the cost of each article named at the price specified. To ***foot*** the several items is to write their sum at the bottom.

198. An ***Invoice*** is a detailed statement of the quantity, price, and description of goods sent to a purchaser or agent at one time. It includes also all charges, as for packing, cartage, insurance, etc.

The following signs and abbreviations are commonly used in business:

a/c, account.	Co., company.	Inst., this month.
Acc't, account.	C. O. D., collect on delivery.	Int., interest.
Am't, amount.	Cr., credit, creditor.	Mdse., merchandise.
@, at.	Do. or ", the same.	Pay't, payment.
Bal., balance.	Dr., debit, debtor.	P'd, paid.
Bo't, bought.	Fr't, freight.	Rec'd, received.

Let the following bills be neatly and carefully copied, extended, and footed, with pen and ink. They may also be used as materials for dictation exercises.

1. *New York, March 31, 1885.*

MR. GEORGE N. BELL,

Bought of PRINCE & MORTON.

Feb.	7	*3 1/2 lb. Butter*	@ $.32		
"	"	*2 doz. Eggs*	" .35		
"	"	*1 gal. Molasses*			76
"	"	*2 lb. Mixed Coffee*	" .28		
"	21	*25 lb. Lump Sugar*	" .07		
"	"	*1/2 bu. Potatoes*	" 1.50		
Mar.	2	*3 lb. Cheese*	" .18		
"	3	*2 lb. Raisins*	" .15		
				$	

Rec'd Pay't,

PRINCE & MORTON.

2. *Chicago, Aug. 1, 1884.*

MR. JAMES S. COOLEY,

To EDWARD WILLIS, *Dr.*

July	7	*To 1 Cheese-Dish*			50
"	"	" *1/2 doz. Butter-Plates*	@ $1.50		
"	"	" *1 1/2 doz. Dinner-Plates*	" 2.50		
"	9	" *3 Candlesticks*	" .25		
"	10	" *5 Pitchers*	" .75		
"	"	" *1 1/2 doz. Cups and Saucers*	" 1.25		
"	"	" *1 1/2 doz. Forks*	" 4.50		
"	"	" *1 1/2 doz. Knives*	" 4.75		
				$	

Rec'd Pay't,

EDWARD WILLIS.

3. *Cleveland, Jan. 11, 1885.*

MR. J. P. KINGSLEY,

1884. *Bo't of* ADAM JOHNSON.

Nov.	*3*	*9 yd. Cashmere*	@ $.75		
"	"	*1/4 yd. Velvet*	" 1.50		
"	5	*6 yd. Lawn*	" .12 1/2		
"	7	*1 1/2 yd. Silesia*	" .20		
"	"	*3/4 yd. Cashmere*	" .65		
				$	

Rec'd Pay't,

ADAM JOHNSON,

By W. WRIGHT.

4. *Knoxville, Tenn., Aug. 15, 1885.*

MR. GEORGE CURTISS,

1885. *In acc't with* JAMES ARDEN AND COMPANY.

		DR.			
July	*3*	*To 1 Plow*		$15	00
"	"	" *2 Extra Plowshares*	@ $1.50		
"	"	" *1 1/2 lb. Powder*	" .85		
"	"	" *3 lb. Shot*	" .11		
"	6	" *3 Hoes*	" .90		
"	"	" *5 Rakes*	" .95		
		CR.		$	
July	15	*By 20 lb. Butter*	@ $.13		
"	"	" *3 doz. Eggs*	" .18		
"	"	" *10 bu. Potatoes*	" .80		
		Balance,		$	

Rec'd pay't,

JAMES ARDEN & Co.

5. *Atlanta, Ga., Apr. 14, 1882.*

MR. GEORGE READE,

To J. V. CAMP, *Dr.*

Apr.	6	*Repairing House as per contract*		$50	
"	7	*1 1/2 hours' labor*	@ $.32 1/2		
"	"	*12 ft. Clear Lumber*	" .04		
"	"	*2 lb. Nails*	" .05		
"	8	*10 hours' labor*	" .32 1/2		
"	"	*11 ft. Lumber*	" .03 1/2		
"	"	*Cartage*		.	25
"	"	*3 lb. Nails*	" .05		
				$	

Paid,

J. V. CAMP.

6. *Omaha, Neb., Sep. 3, 1882.*

MR. GEORGE HURLBURT,

1882. *In account with* WM. POWERS.

Aug.	17	*To 16 yd. Frieze with Inlay*	@ $.25		
"	"	" *Hanging 14 yd. Border*	" .02		
"	"	" *11 Rolls Paper*	" .37		
"	"	" *Hanging 11 Rolls Paper*	" .25		
"	19	" *Painting Kitchen*		$13	00
"	"	" *26 1/2 yd. Painting*	" .20		
"	21	" *27 yd. Painting in Hall*	" .25		
"	"	" *18 yd. Paper*	" .12		
"	"	" *6 hours Kalsomining*	" .30		
"	"	" *3 yd. Velvet Paper*	" .40		
				$	

Rec'd Pay't by note at 30 d.,

WM. POWERS.

Rule paper in proper form, and make out bills for the following transactions:

7. Mrs. Cole bought of E. P. Dale, of Boston, Feb. 5, 1884, 2 cans of String Beans, @ 10¢; $^1/_2$ bu. Potatoes, @ \$1.00; 2 lb. Tea, @ 60¢; Feb. 9, 1 lb. Crackers, 20¢; 1 doz. Eggs, 32¢; 7 lb. Graham Flour, @ 4¢; Feb. 16, 3 cans Tomatoes, @ 12¢; 4 lb. Prunes, @ 16¢; 1 doz. Oranges, 50¢. Receipt the bill as clerk for Mr. Dale.

8. Charles Martin bought of Joseph A. Snow, of Pittsburg, Pa., Feb. 2, 1884, $2^3/_4$ lb. Mutton Chops, @ 22¢; $^1/_2$ pk. Apples, @ 40¢; 6 lb. Beef, @ 20¢; Feb. 6, $^1/_2$ pk. Sweet Potatoes, @ 30¢; 2 bunches Lettuce, @ 12¢; 2 qt. Turnips, @ 5¢; Chicken, $4^1/_2$ lb., @ 20¢; Feb. 9, 2 lb. Steak, @ 25¢; $^1/_2$ pk. Apples, @ 70¢; 1 qt. Onions, 10¢; Feb. 16, $7^3/_4$ lb. Beef, @ 20¢; 2 cans of Peas, @ 18¢; $^1/_2$ doz. Oranges, @ 50¢; Feb. 20, $9^1/_2$ lb. Ham, @ 18¢; Feb. 26, $2^1/_2$ lb. Lamb Chops, @ 22¢; 1 doz. Oranges, 50¢. Mr. Snow had bought of Mr. Martin 3 pt. of Cream, @ 12¢ a qt., daily through the month. Make out a receipted bill, using Bill 4 (page 196) as a model.

9. Alfred E. Robie bought of John Turner, of New Haven, Conn., April 2, 1885, $2^1/_2$ lb. Sausage, @ 14¢; $^1/_2$ doz. Lemons, @ 25¢; 2 lb. Dried Apples, @ 10¢; Apr. 4, $3^1/_4$ lb. Veal Chops, @ 20¢; Apr. 9, $^1/_2$ pk. Spinach, @ 70¢; 2 lb. Mutton, @ 14¢; Apr. 14, $^1/_2$ pk. Apples, @ 70¢; 2 qt. Sweet Potatoes, @ 10¢; Apr. 18, $6^3/_4$ lb. Beef, @ 20¢; $^1/_2$ doz. Bananas, @ 40¢; 2 doz. Pickles, @ 7¢; 2 qt. Bermuda Onions, @ 20¢; Apr. 23, $3^1/_2$ lb. Steak, @ 22¢; Apr. 28, 2 lb. Rhubarb, @ 10¢; 3 bunches Radishes, @ 7¢.

10. Mrs. James Bird bought of John Burns, of New Orleans, La., the following articles: Feb. 17, 1883, $^3/_4$ doz. Linen Napkins, @ \$1.75; $2^1/_4$ doz. Damask Towels, @ \$4.50; 3 Bath Towels, @ \$2.40 a doz.; Feb. 21, 1883, 2 Table-cloths, @ \$5.50; 1 Piano-cover, @ \$5.00; 7 yd. Cambric, @ \$.12$^1/_2$; 2 pr. Lace Curtains, @ \$2.50 a pair.

11. Robert M. Miles bought of Lane & Bowers, of Philadelphia, Pa., Nov. 21, 1885, 1 Suit for \$28; 3 Shirts, @ \$1.25; 1 pr. Shoes, \$5.50; 6 pr. Socks, @ 35¢; 1 Umbrella, \$2.50; 2 pr. Gloves, @ \$1.75; and 4 pr. Cuffs, @ 35¢. Payment was made by note at 3 months.

12. Mr. George Ross bought of Robert James, of Albany, N. Y., on Mar. 13, 1881, 60 yd. Brussels Carpet, @ \$.85; 40 yd. Moquette Carpet, @ \$1.55; 35 yd. Canton Matting, @ \$.55; 3 Curtain-poles, @ \$4.50; 3 pr. Nottingham Lace Curtains, @ \$5.50.

13. Albert Halsted, in ℅ with George Reese: Aug. 7, 1881, $1\frac{1}{4}$ days' work, @ \$3.25; 44 ft. Pine Lumber, @ \$.06$\frac{1}{2}$; 1 lb. Nails, \$.07; work on Bookcases as per contract, \$13.00; 65 ft. Pine Lumber, @ \$.06$\frac{1}{2}$; $\frac{3}{4}$ lb. Nails, @ \$.07. Cr. by cash, \$5.00.

14. Mr. Robert Holden, of Brooklyn, New York, bo't of Stanley, White & Co., of New York city, Mar. 11, 1884, 3 doz. 8 in. Thermometers on polished walnut, @ \$10; $1\frac{1}{2}$ doz. 8 in. Parlor Thermometers, @ \$4 apiece; 5 doz. tin-case Thermometers, @ \$5; 9 Aneroid Barometers, @ \$5; 15 pr. Opera-glasses, @ \$4.25; 3 Microscopes, @ \$15; 1 large first-class Microscope, \$350; 2 Amateur Photographic Cameras, @ \$25. Paid by note at 3 mo.

15. Mrs. H. R. Otis bo't of Richard Hayes, June 11, 1880, 1 pr. Ladies' Kid Button Shoes, \$6; June 13, 2 pr. Ladies' Patent-Leather Oxford Ties, @ \$4.50; 1 pr. Misses' Kid Button Shoes, \$3.50; 2 pr. Infants' Black Kid Button Shoes, soft soles, @ \$.45; 1 pr. Child's Pebble Spring-heel Button Shoes, \$2.

16. James R. Baldwin bo't of Robert Price, Dec. 19, 1884, 1 copy "Little Men," \$1.35; 1 "Modern Explorers," \$10; 1 "Three Vassar Girls in South America," \$1.30; 1 "Rose in Bloom" and "Eight Cousins," \$2; 13 vol. Shakespeare, @ \$1; 3 vol. "Diamond Edition Poetry," @ \$.90; 5 vol. "Companion Edition Poetry," @ \$1.25; 6 vol. Hawthorne, @ \$1.35; 1 vol. "Sports and Pastimes for American Boys," \$1.25.

Suggestions for Original Problems.

1. Pupils will find suggestions for original problems in the Miscellaneous Exercises; or, it may be required that they construct problems of their own after models dictated by the teacher.

2. Having obtained reliable information from parents and others in regard to prices, trade customs, etc., they can make out bills, and furnish items for bills to be made by the class.

3. They may draw diagrams showing the forms and dimensions of lots to be fenced, dictate the kinds of fences to be built, prices of boards, posts, labor, nails, etc., and require the whole cost. They may give, in like manner, the information necessary to reckon the cost of digging cellars, building walls, laying board, stone, or brick walks, etc., etc. Pupils may often obtain from each other such information as may be needed.

4. Let illustrations, like the one on page 174, be required, showing .33, 1.27, etc., etc., of given squares.

5. Let pupils obtain where they can, the data necessary to enable them to calculate the cost of papering, carpeting, plastering, the schoolroom.

6. Pupils who have a little constructive skill may make paper boxes, and require their classmates to calculate their contents—how many quarts of blackberries or vinegar they will contain, etc.

7. Try the experiment of ascertaining the height of some tall tree or steeple, by measuring the length of its shadow, and the length of the shadow cast at the same moment by a stick or post, the length of which above ground can be easily measured.

8. Give the dimensions of a pile or load of wood, and ask, How many cords? or of a wood-shed, and ask, How many cords can be piled in it? or the length of a pile of wood, and ask how high it must be to contain some required number of cords.

9. Give the dimensions of a box containing a gross of such crayons as are used at the blackboard, and ask the length and width of a case which will exactly contain a gross of such boxes.

CHAPTER XI.

MEASURES.

199. The length, breadth, and height of objects are their ***dimensions.***

A ***line*** has only one dimension—***length.***

A ***surface*** has two dimensions—***length and breadth.***

A ***solid or space*** has three dimensions—***length, breadth, and height or thickness.***

Measures of Extension.

200. Measures used to ascertain how long a line is, or in calculating the size (extent) of a surface or solid, are called ***Measures of Extension.*** These are the ***Linear, Square,*** and ***Cubic Measures.***

Linear or Line Measure.

201. In measuring length or distance, linear or line measure is used. The standard unit is the ***yard.***

Table.

12 Inches (in.)	=	**1 Foot (ft.).**
3 Feet	=	**1 Yard (yd.).**
$16^1/_2$ Feet (or $5^1/_2$ Yards)	=	**1 Rod (rd.).**
320 Rods	=	**1 Mile (mi.).**

Equivalents.

1 mile = 320 rods = 1760 yards = 5280 feet = 63360 inches.

Notes.—1. For measuring cloth the yard is divided into halves, fourths, eighths, and sixteenths. In the United States custom-houses it is divided decimally.

2. A *Furlong* = $^1/_8$ mile.—The rod is also called a *Pole* or *Perch.*

3. A *Pace* is variously estimated from 3 to 3.3 feet.

4. A *Line* = $^1/_{12}$ inch.

202. The mile given in the table is the mile used in land measurements. Its length is fixed by law, and is called the ***statute mile.*** It is thus distinguished from the ***geographical mile*** of the following table, used on shipboard and at sea.

Table.

6 Feet	**= 1 Fathom.**
120 Fathoms	**= 1 Cable Length.**
1.15 + Common Miles	**= 1 Geographical or Nautical Mile.**
3 Geographical Miles or 3.45 + Statute "	**= 1 League** (at sea).

A ***Knot*** corresponds to one geographical or nautical mile, and is used to estimate the speed of vessels at sea.

Note.—In the absence of a more exact instrument the ***hand*** was formerly used as a measure. From this we have the *Palm* (breadth of four fingers) = about 3 inches; the *Hand* (the breadth of palm and thumb, used in measuring the height of horses at the shoulder) = 4 inches; the *Span* (the distance between the tips of the thumb and the little finger, when the hand is extended against a flat surface) = about 9 inches, or $^1/_4$ of a yard.

ORAL EXERCISES.

How many

1. Feet in $3^2/_3$, $4^1/_2$, $7^3/_4$, 4.4, $11^5/_6$, $33^1/_3$ yd.?
2. Feet in 25, 16, 30, 39, $14^2/_5$ in.?
3. Yards in $1^7/_{11}$, $2^1/_2$, 5, $8^2/_{11}$ rods.?
4. Rods in $^1/_2$, $^1/_4$, $^1/_8$ mi.; in 121, $49^1/_2$ yd.?
5. Inches in $1^3/_4$, $6^5/_6$, $3^1/_2$, $5^3/_5$, $7^7/_{12}$ ft.?
6. Feet in $2^1/_2$, $3^1/_4$, $10^1/_4$, $6^1/_2$ fathoms?

Surveyors' Measure.

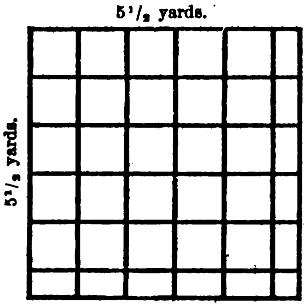
203. ***Gunter's Chain,*** used in measuring roads and the boundary lines of land, is 4 rods (= 66 ft.) in length. It has 100 links, each 7.92 inches long.

Table.

7.92 Inches = 1 Link (li.).
100 Links = 1 Chain (ch.).
80 Chains = 1 Mile (mi.).

Square or Surface Measure.

204. There is no measure which is directly applied to a surface to find its extent. Even if there were such a measure, it would be difficult to apply it. Suppose, for instance, that we wished to ascertain how many square yards there are in a plot of ground $5\,^1/_2$ yards long and $5\,^1/_2$ yards wide. If we had a square-yard measure we might perhaps mark off 25 square yards and the fractions of a yard, as in the diagram. But it would be much easier to measure the length and breadth with a yard-stick, and then compute the number of square yards in the surface.



205. The square inch, foot, yard, rod, and mile are derived from corresponding linear measure.

Table.

144 sq. Inches = 1 sq. Foot. **$30^1/_4$ sq. Yards = 1 sq. Rod.**
9 sq. Feet = 1 sq. Yard. **160 sq. Rods = 1 Acre.**
640 Acres = 1 sq. Mile (or Section of Land).

Equivalents.

□ mile.	acres.	□ rods.	□ yards.	□ feet.	□ inches.
1 =	**640 =**	**102400 =**	**3097600 =**	**27878400 =**	**4014489600**

The sign □ is used for the abbreviation "sq." In written exercises, either can be used.

Note.—The acre has no corresponding denomination in linear measure. A square, measuring 208.71 + feet on each side, contains 1 acre.

ORAL EXERCISES.

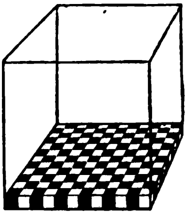
How many

1. Square yards in 12, 1881, 26, 100, 66 □ ft.?
2. Acres in $^1/_{16}$, $^1/_4$, $^5/_8$, $^1/_2$ □ mi.?
3. Square feet in a board 6 ft. 6 in. long, $^2/_{13}$ ft. wide?
4. A board 18 in. long contains half a □ ft.; how wide is it?
5. How many □ rods in $^1/_2$, $^1/_4$, $^5/_8$, $^3/_{16}$ of an acre?
6. How many acres in a half section of land? In a quarter?

Cubic Measure.

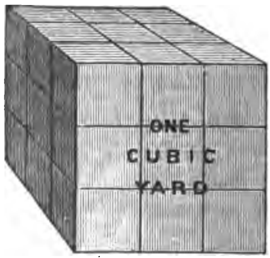
206. To measure a block of marble, or to find how much a box, a bin, or a room will contain, we have to ascertain its length, breadth, and height or thickness, by a *linear measure*, as a foot-rule, a yard-stick, or a tape-line; and, with the aid of the dimensions thus found, to calculate the contents of the block, or bin, or room, in ***Cubic Measure***, that is, we *calculate* how many times the room, or the space occupied by the block, would contain some known cubic unit, such as a cubic inch, cubic foot, etc.

207. A ***Rectangular Solid*** is a solid having six rectangular faces.



208. A ***Cube*** is a rectangular solid having six equal square faces. (See also page 103.)

The figure at the left represents the outlines of a cubic foot, with a layer or course of cubic inches at the bottom. With this figure before the pupil let him answer the following questions: 1. How many cubic inches in the course represented? 2. How many such courses are needed to complete the foot? 3. How many cubic inches in a cubic foot? In $^1/_{12}$? $^3/_4$? $^1/_{24}$? etc.



On inspection of the figure at the right, answer the following questions: 1. How many cubic feet in a cubic yard? 2. What is the length of each edge of a cubic foot? 3. Can you lift a cubic foot of granite?

4. How many cubic feet in $^1/_3$ of a cubic yard? 5. How many cubic feet in $^1/_9$ of a cubic yard?

6. How many cubic inches in a cubic foot? 7. How many cubic inches in a cubic yard? In $^1/_3$ of a cubic yard?

209. Thus the cubic inch, foot, and yard are derived from the corresponding linear measures.

Table.

1728 Cubic Inches	=	**1 Cubic Foot.**
27 Cubic Feet	=	**1 Cubic Yard.**

Equivalents.

1 Cubic Yard = 27 Cubic Feet = 46656 Cubic Inches.

Note.—Higher denominations than these are seldom referred to.

Wood Measure.

210. Wood cut in "lengths" of 4 feet is called "cord wood." A pile of cord wood four feet high and eight feet long, or equal bulk of other material, is called a ***Cord.***

211. One foot in length of such a pile is called ***a cord foot.***

Table.

16 Cubic Feet	**= 1 Cord Foot.**
8 Cord Ft. or 128 Cubic Ft.	**= 1 Cord.**

ORAL EXERCISES.

How many

1. Cubic feet in $^1/_3$, $2^1/_2$, $1^5/_9$, $3^2/_3$ cu. yd.?
2. Cubic feet in $^1/_2$, $^1/_8$, $^1/_4$, 2.625 cords?
3. Cubic inches in an iron bar $13^1/_2$ in. long, $3^1/_3$ in. wide, $^1/_2$ in. thick?
4. Cubic inches in a brick 8 by 4 by $2^1/_2$ inches?
5. Cubic yards in a wall 6 ft. high, 9 in. thick, and 20 yd. long? (6 ft. = 2 yd., 9 in. = $^1/_4$ yd.)
6. Cord feet in $3^5/_8$, 7.125, 4.375 cords?

Measures of Capacity.

212. For measuring fruits, berries, roots, grains, and other dry commodities, we use ***Dry Measure.*** The standard unit is the ***Bushel*** = 2150.42 cubic inches.

Dry Measure.

Table.

2 Pints (pt.)	**= 1 Quart (qt.).**
8 Quarts	**= 1 Peck (pk.).**
4 Pecks	**= 1 Bushel (bu.).**

Equivalents.

1 Bushel = 4 Pecks = 32 Quarts = 64 Pints.

Charcoal and coke are frequently measured by the *chaldron*, of 36 bushels.

213. For measuring liquids, such as water, wine, vinegar, milk, etc., we use ***Liquid Measure.*** The standard unit is the ***Gallon*** = 231 cubic inches.

Liquid Measure.

Table.

4 Gills (gi.)	=	**1 Pint (pt.).**
2 Pints	=	**1 Quart (qt.).**
4 Quarts	=	**1 Gallon (gal.).**

Equivalents.

1 Gallon = 4 Quarts = 8 Pints = 32 Gills.

214. Comparison of Dry and Liquid Measures.

The Dry Quart contains 67.2 cubic inches.
The Liquid Quart contains 57.75 cubic inches.

Notes.—1. Thus it will be seen that the retailer who uses the liquid instead of the dry quart, in measuring berries and small fruits, cheats his customers out of a little more than one quart in seven.

2. Barrels, tierces, hogsheads, puncheons, pipes, butts, tuns, etc., have no standard capacity. The quantity of liquid contained in them is usually found by actual measurement, called gauging.

3. When the barrel is spoken of as a measure of the capacity of vats, cisterns, etc., $31\frac{1}{2}$ gallons are meant. In measuring beer, the barrel has 36 gallons, and $1\frac{1}{2}$ barrels (or 54 gal.) make a hogshead.

ORAL EXERCISES.

How many

1. Quarts in $2\frac{1}{4}$, $3\frac{1}{2}$, $6\frac{5}{8}$, 4.25, 5.5 gal.?
2. Pints in 2, 10, 15, 180 gi.?
3. Gallons in 9, 14, 27, 17, 30, 111, 63 pt.?
4. Quarts in 7, 20, 31, 15, 50, 25, 35, 45 gi.?
5. Quarts in $\frac{1}{2}$, $2\frac{1}{4}$, $1\frac{7}{8}$ bu.?
6. Pints in $3\frac{1}{2}$, $8\frac{3}{4}$, 6.25, $9\frac{5}{8}$, 10.125 qt.?
7. Bushels in 10, 1.6, 23, 2.8, 17 pk.?
8. Pecks in $2\frac{1}{2}$, $4\frac{1}{4}$, $3\frac{3}{4}$, 5.75 bu.?
9. Quarts in 5, $8\frac{1}{2}$, $10\frac{1}{4}$, $33\frac{1}{3}$, 13.825 pt.?

Measures of Weight.

215. For weighing gold, silver, the precious stones, etc., ***Troy Weight*** is used. The standard unit is the ***Troy pound*** = 5760 grains.

Troy Weight.

Table.

24 Grains (gr.)	=	**1 Pennyweight (pwt.).**
20 Pennyweights	=	**1 Ounce (oz.).**
12 Ounces	=	**1 Pound (lb.).**

Equivalents.

1 Pound = 12 Ounces = 240 Pennyweights = 5760 Grains.

Practical illustrations of Troy weight are to be found in the United States coins: The gold dollar weighs 25.8 grains; the silver dollar, $412^1/_2$ grains; the small silver coins, 385.8 grains to a dollar (that is, 10 single dimes, or 4 quarters, or 2 half-dollars, weigh 385.8 grains). The nickel 5¢ piece weighs 77.16 grains; the 3¢ piece, 30 grains, and the bronze 1¢ piece, 48 grains.

Gold and silver are bought and sold by the ounce, weights of these metals never being expressed in pounds. The ***carat,*** very nearly equal to $3^1/_5$ Troy grains, is used in weighing diamonds and other precious stones.

The word *carat* is also used in expressing the number of parts of pure gold in articles of jewelry, etc. If 18 parts out of 24 are pure gold, and the remaining 6 parts are alloy, the metal is said to be of 18 carats, etc.

ORAL EXERCISES.

How many

1. Pennyweights in $3^1/_4$, 5.3, $6^3/_4$, 9.2, $4^3/_8$ oz.?
2. Ounces in $1^1/_2$, $3^1/_4$, $4^1/_3$, $7^1/_8$ lb.?
3. Pounds in 16, 30, 27, 9, $9^1/_3$, 23.3 oz.?
4. Grains in $^1/_6$, $^5/_8$, $1^2/_3$ pwt.? In 1.5, $2^1/_4$ oz.?
5. Ounces in 45, 56, 90, 18, 50 pwt.?
6. Ounces of pure gold in 44 oz. of watch-cases, 18 carats fine?

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7. Of how many carats is a mixture of 27 oz. gold and $13^1/_2$ oz. alloy?
8. How many pwt. of alloy must be put with 25 pwt. of pure gold to make a mixture of 20 carats?

Apothecaries' Weight.

216. ***Apothecaries' Weight*** is used only by physicians in prescribing and by apothecaries in compounding medicines. When sold by weight, avoirdupois weight is used.

Table.

20 Grains (gr.)	=	**1 Scruple ℈.**
3 Scruples	=	**1 Dram ʒ.**
8 Drams	=	**1 Ounce ℥.**
12 Ounces	=	**1 Pound ℔.**

Equivalents.

℔ 1 = ℥ 12 = ʒ 96 = ℈ 288 = gr. 5760.

Note.—1. It should be observed that in this weight the signs precede the numbers to which they belong. 2. The grain, the ounce, and the pound are of the same value as the corresponding denominations in Troy weight.

ORAL EXERCISES.

How many

1. Pounds in ℥ 36? ℥ 40? ℥ 27? ℥ 75?
2. Grains in ℈ $1^1/_2$? ℈ $^1/_4$? ℈ $^1/_2$? ℈ 3.75? ℈ .1?

Avoirdupois Weight.

217. For the common purposes of trade, ***Avoirdupois Weight*** is used. The standard unit is the pound of 7000 grains.

Table.

16 Ounces (oz.)	=	**1 Pound (lb.).**
100 Pounds	=	**1 Hundredweight (cwt.).**
20 Hundredweight	=	**1 Ton (T.).**

The term *cental* is beginning to be used for hundredweight.

Equivalents.

1 Ton = 20 Hundredweight = 2000 Pounds = 32000 Ounces.

Formerly 112 lb. were reckoned a hundredweight, and 2240 lb. a ton. This weight is still used in weighing iron, coal at the mines, ores, and goods on which duties are paid at the United States custom-houses.

218. **Comparison of Troy with Avoirdupois Weight.**

Avoirdupois:	**1 lb. = 7000 grains.**	**1 oz. = $437^1/_2$ grains.**
Troy:	**1 lb. = 5760 grains.**	**1 oz. = 480 grains.**

ORAL EXERCISES.

How many

1. Ounces in $1\frac{1}{2}$, $2\frac{3}{4}$, $5\frac{3}{8}$, $4\frac{5}{16}$, $8\frac{7}{8}$, $6\frac{3}{4}$ lb.?
2. Pounds in $\frac{1}{2}$, $1\frac{3}{4}$, 7.1, $8\frac{2}{5}$, $6\frac{1}{4}$, $12\frac{6}{10}$ cwt.?
3. Pounds in 12, 46, 22, 33.6, 29, 176 oz.?
4. Cwt. in $2\frac{1}{5}$, $3\frac{1}{4}$, $4\frac{3}{4}$, 11.2, 9.9 T.?
5. Pounds in 1.3, $2\frac{1}{4}$, $3\frac{2}{5}$, $5\frac{3}{4}$, $7\frac{17}{20}$ T.?

219. Weight being very commonly employed in estimating quantities of grains, roots, etc., the weight of the bushel, as fixed by law in many States, for some of the more important commodities, is given below.

The general usage is found in the second column. In the third, exceptions are noted so far as known. (See Haswell, Ed. 1885, and Report No. 14, H. R., 46th Congress, 1st Session.)

Commodities.	Lb. per bu.	Exceptions.
Barley	48	Ariz. and Wash., 45; Cal., 50; Md. and Penn., 47; N. H. and Del., not reported.
Shelled corn	56	Ariz., 54; Cal., 52; N. Y., 58.
Oats	32	Iowa, Mont., and Mo., 35; Md., 26; Neb. and Ore., 34; Me., N. H., and N. J., 30; Wash., 36; Ky., $33\frac{1}{3}$; Del., not reported.
Rye	56	Cal., 54; La., 60; Del. and Me., not reported.
Potatoes	60	Ohio, 58; Ariz., Cal., Del., La., Md., Penn., not reported.
Wheat	60	Conn., 56; R. I., not reported.
Pease	60	No exceptions reported.

Usage in regard to the following articles is not so uniform as in case of those given in the foregoing list:

Corn in the ear. Variously estimated from 68 to 70 lb.

Corn meal. Del., 44 lb.; Ill., 48 lb.; most other States, 50 lb.

Beans. Me., 64 lb.; N. Y., 62 lb.; many others, 60 lb.

Clover seed. Mont., 45 lb.; N. J., 64 lb.; Penn., 62 lb.; in almost all others, 60 lb.

Timothy seed. Wis., 46 lb.; N. Y. and Mont., 44 lb.; Dakota, 42 lb.; in many others, 45 lb.

Mineral coal. Ky. and Penn., 76 lb.; Ind., 70 lb.; in most others, 80 lb.

The following standards are generally accepted:

100 lb of grain or flour	= 1 cental.	196 lb. of flour	= 1 barrel.
100 lb. of dry fish	= 1 quintal.	200 lb. of beef or pork	= 1 "

100 lb. of nails = 1 keg.

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220. Measures of Value.

United States or Federal Money.

For the United States or Federal Money table, see Art. 89.

The *gold coins* are the \$1, \$2 $^1/_2$ (quarter-eagle), \$3, \$5 (half-eagle), \$10 (eagle), and \$20 (double-eagle) pieces. The *silver coins* are the \$1, 50¢, 25¢, and 10¢ pieces. The 5¢ and 3¢ pieces are made of *nickel;* the cent of *bronze.* Other coins are occasionally found in circulation, but are no longer coined, such as the trade-dollar, the 20¢, the 5¢ and the 3¢ silver pieces, and the 2¢ piece of bronze.

Canadian Money.

221. The unit of Canadian currency, like that of the United States, is called a dollar. It is divided into 100 cents, and the cent into 10 mills.

The legal coins are—*Gold:* the British sovereign, worth \$4.8665, and the British half-sovereign; *Silver:* the 50¢, 25¢, 10¢, and 5¢ pieces; *Bronze:* 1¢.

The silver and bronze coins have the same values as the corresponding coins of the United States.

English or Sterling Money.

Table.

4 Farthings (far.) = 1 Penny (d.).
12 Pence = 1 Shilling (s.).
20 Shillings = 1 Pound (£).

Equivalents.

1 Pound = 20 Shilling = 240 Pence = 960 Farthings.

The coins of Great Britain are—*Gold:* the sovereign = \$4.8665, and the half-sovereign; *Silver:* the crown (5 shillings) = \$1.216+, and the half-crown; the florin (2 shillings) = \$.486; the shilling = \$.243; the sixpenny, fourpenny, and threepenny pieces; *Copper:* the penny, half-penny, and the farthing ($^1/_4$ penny). The guinea = 21 shillings, though no longer coined, is frequently mentioned as if in common use.

222. Money of Other Countries.

a. French money : 1 franc (fr.) = 100 centimes (c.) = 19.3¢ in United States money.

The ***Gold*** coins of France are the 100, 50, 20, 10, and 5 franc pieces; the ***Silver*** coins are the 5, 2, 1, $^1/_2$, and $^1/_5$ franc pieces; the ***Bronze***, 10, 5, 2, and 1 centime (pronounced *sonteem*) pieces; ***Copper***, 10 and 5 centimes. The values of these coins are indicated by their relations to the franc.

b. German money : 1 mark (reichsmark) (*M*, m.) = 100 pfennigs (pf.) = 23.8¢ in U. S. money.

The ***Gold*** coins of the German Empire are the 20 and 10 mark pieces; the ***Silver*** coins are the 5, 3, 2, 1, and $^1/_2$ mark and 20 pfennig pieces; the ***Nickel***, 10 and 5 pfennig; the ***Copper***, 2 and 1 pfennig. The ***Thaler*** (silver) = \$.746, and the ***Groschen*** (silver) = $2\,^1/_2$¢, are also in common use.

For the values of other foreign coins, see Appendix.

223. The following approximations are sufficiently exact for general estimates : One U. S. Dollar may be counted as equal to 5 Francs (France, Belgium, and Switzerland), or to 5 Lire (Italy), or to 5 Peseta (Spain), or to 4 Shillings (England), or to 4 Marks (Germany).

ORAL EXERCISES.

How many

1. Pence in 2, $3^1/_3$, $9^5/_6$, $14^1/_6$, 21 s.? In $^1/_2$ crown?
2. Farthings in $8^1/_2$, $14^1/_4$, $23^3/_4$, 3 pence?
3. Pounds in 50, 15, 75, 105, 130, 244 shillings?
4. Shillings in 30, 6, 45, 33, 81, 108 pence?
5. Shillings in £$7^1/_5$? £$15^3/_4$? £$22^1/_2$? $2^2/_3$ guineas?
6. Centimes in $^1/_2$, $1^3/_5$, $5^1/_4$, $17^3/_4$, $28^2/_5$ francs?
7. Marks in 175, 210, 1728, 3042 pfennigs?
8. Dollars may be counted as equal to 18 roubles? 42 roubles? 166 roubles?
9. Dollars may be counted as equal to 34, 78, 92, 118 s.? To 1 guinea? To 75, 130, 195 peseta?
10. Dollars may be counted as equal to 250000 francs? To £340000? To 3000 marks? To 1500 roubles?

Definitions.

224. A ***point*** has position, without length, breadth, or thickness.

225. A ***line*** is the path of a point in motion. If the point moves without change of direction, the path is a ***straight line.*** If the point changes its direction continually while moving, the path is a ***curved line.***

226. If the moving point passes around a fixed point, so that its distance from the fixed point does not vary, the path of the moving point is the ***circumference of a circle,*** and the fixed point within is the ***center*** of the circle.

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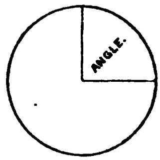
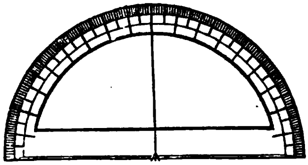
227. For the measurement of angles (see Art. 56), the circumference of the circle is conceived to be divided into 360 equal parts, called degrees. The angle in this figure is an angle of 90 degrees—one fourth of the 360 equal parts into which the circumference is supposed to be divided.



228. An angle of 90 degrees (written 90°) is a ***right angle.*** An angle of less than 90° is an ***acute angle.*** An angle greater than 90° is an ***obtuse angle.***

229. Two lines which form an angle of 90° are said to be ***perpendicular*** to each other. (See also Art. 57.)



Note.—A degree, being $^1/_{360}$ part of any circumference, is very minute, if the circle is a small one; but a degree of the circumference of the earth is 69.16 miles in length. A degree of the sun's circumference is about 7444 miles long. Compare these with one degree on the protractor, as here represented. A *protractor* is an instrument used for the measurement of angles.

230. For more exact measurements, the *degree* (°) is divided into *minutes* (′), and the minutes into *seconds* (″), according to the following table:

Circular Measure.

Table.

60 Seconds (″) = 1 Minute (′).
60 Minutes = 1 Degree (°).
360 Degrees = 1 Circumference.

ORAL EXERCISES.

How many

1. Degrees in $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{12}$, $\frac{1}{6}$, $\frac{1}{30}$, $\frac{1}{24}$ circumference?
2. Minutes in $3\frac{1}{4}°$? $2\frac{1}{2}°$? $5\frac{5}{6}°$? $4\frac{1}{5}°$? $7\frac{1}{3}°$?
3. Degrees in 180′? 150′? 3600′? 420″? 1800″? 6000″?
4. Minutes in 900″? 720″? 342″? 1275″? 3333″?

231. Time Measure.

Table.

60 Seconds (sec.) = 1 Minute (min.).
60 Minutes = 1 Hour (h.).
24 Hours = 1 Day (d.).
7 Days = 1 Week (wk.).
365 Days 5 Hours 48 Min. 46.4 Sec. = 1 Solar Year (yr.).

The year given in the table, which is a little less than $365\frac{1}{4}$ days, is the time it takes the earth to go around the sun, and hence the time required for a complete change of seasons. But to count this quarter of a day with every year would be extremely inconvenient. It is much easier to count one additional day every fourth year, and hence this is generally done, but not always, for in a hundred years we should thus gain nearly a day too much; so the hundredth years (centennial years) are commonly counted as ordinary years; but here again we have to say not always, for we should thus lose nearly a day in 400 years. Hencc the centennial years divisible by 400 are counted as leap years.

The following is the rule by which leap years may be known for several thousands of years to come:

232. All years divisible by 4, except centennial years not divisible by 400, are leap years.

233. There are 12 months in a year. The number of days in each is given in the following

Table.

	Months.		Days.		Months.		Days.
1st.	January	(Jan.)	31	7th.	July	(July)	31
2d.	February	(Feb.)	28 or 29	8th.	August	(Aug.)	31
3d.	March	(Mar.)	31	9th.	September	(Sept.)	30
4th.	April	(Apr.)	30	10th.	October	(Oct.)	31
5th.	May	(May)	31	11th.	November	(Nov.)	30
6th.	June	(June)	30	12th.	December	(Dec.)	31

The 29th day of February is the day added to make a leap year.

The following lines are used to aid the memory in recalling the number of days in the several months:

"Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Except February alone,
Which has but 28 in fine,
Till leap year gives it 29."

The long months may be distinguished by observing that their names are the only ones that contain the letter *c*, or that have *a* for their second letter, and, except *June*, the only ones that have *u* for their second letter. (See whether this is true.)

ORAL EXERCISES.

How many

1. Days in $2^1/_2$, $4^3/_7$, 14 wk.?
2. Hours in $2^1/_6$, $4^1/_5$, $3^3/_4$ d.? 1 wk.?
3. Minutes in $1^1/_2$, $3^1/_3$, $2^1/_4$, $4^3/_5$, 12, 24 h.?
4. Weeks in 9, 30, 23, 17, 60, 90, 365 d.?
5. Days in Aug.? Apr.? Dec.? Jan.? Sept.? Feb.? July? Nov.? Mar.? Oct.? June? May?
6. Days in the year 1886? 1894? 1896? 1900? 1800? 2000?

234. Miscellaneous Measures.

Counting.		Paper.	
12 Units	= 1 Dozen (doz.).	24 Sheets	= 1 Quire (qu.).
12 Dozen	= 1 Gross (gro.).	20 Quires	= 1 Ream (r.).
20 Units	= 1 Score.	2 Reams	= 1 Bundle.
5 Score	= 1 Hundred.	5 Bundles	= 1 Bale.

MISCELLANEOUS ORAL EXERCISES.

How many

1. Inches in $2\frac{1}{2}$, 3.75 ft.?
2. □ ft. in $3\frac{1}{3}$, 5.75 □ yd.?
3. Cu. in. in $1\frac{1}{2}$, 1.5 cu. ft.?
4. Pints in 11.75, $9\frac{1}{4}$ qt.?
5. Pecks in $3\frac{3}{4}$, 5.125 bu.?
6. Quarts in 15, 20.5 pk.?
7. Grains in 7, $7.66\frac{2}{3}$ pwt.?
8. Seconds in $3\frac{1}{4}$, $5\frac{1}{8}$ min.?
9. Minutes in .125, $\frac{5}{6}$ h.?
10. Degrees in $.16\frac{2}{3}$ circum.?
11. Dozen in $3\frac{1}{4}$, 4.375 gross?
12. Pwt. in 11.5, $9\frac{3}{4}$ oz.?
13. □ ft. in $1\frac{1}{8}$, 2.5 □ yd.?
14. □ in. in .375, $1\frac{3}{8}$ □ ft.
15. Cu. ft. in $1.33\frac{1}{3}$ cu. yd.?
16. Feet in $7\frac{1}{2}$, 7.5 fathoms?
17. Feet in $3\frac{1}{4}$ cable lengths?
18. Yards in 1.5, 2.6 rd.?
19. Rods in 7.7, 11.5 yd.?
20. Feet in 9.6, 15.6 in.?

How many

21. Lb. pork in $2\frac{1}{2}$, 3.7 bl.?
22. Pence in $14\frac{5}{6}$ s.?
23. Pounds in 32 oz. avoir.?
24. Pounds in 8.4 oz. Troy?
25. Gallons in 2.6, 17 qt.?
26. Gills in 27, 1.3 pt.?
27. Ounces in 36, 96 pwt.?
28. Lb. Troy in 3, 18 oz.
29. □ ft. in 288, 72 □ in.?
30. Feet in 10.8, 8.4 in.?
31. Feet in 7.2, $10\frac{4}{5}$ in.?
32. Cu. yd. in 10.8 cu. ft.?
33. Bushels in 23, 97 pk.?
34. Pecks in 23, 97 qt.?
35. Pints in 7.2, 39 gills?
36. Fath. in $1\frac{1}{2}$ cable lengths?
37. Feet in $15\frac{1}{2}$, 14 hands?
38. Quarts in 73, 95 pt.?
39. Pecks in 3.2, .96 qt.?
40. Lb. pork in 5, $1\frac{1}{4}$ bl.?

Definitions.

Compound Denominate Numbers.

235. Number when applied to specified objects is said to be ***concrete.***

236. Number when not applied to specified objects is said to be ***abstract.***

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237. To measure a quantity is to find how many times it contains some known quantity used as a standard of comparison.

238. A known quantity, fixed by law or custom as a standard of comparison, is called a *Unit of Measure.*

Note.—A yard is a standard fixed by law for measuring length or distance. A hand is a standard fixed by custom for estimating the height of horses.

239. Units of measure have special ***denominations*** or names by which they are designated, and hence they are called *Denominate Units.* (Denomination means name.)

240. A ***Denominate Number*** is a number of denominate units.

241. A ***Simple Denominate Number*** is one that consists of units of only one denomination.

242. A ***Compound Denominate Number*** is one that consists of units of two or more denominations.

243. Changing the denomination in which a quantity is expressed is called ***Reduction.***

244. ***Reduction Ascending*** is changing an expression of quantity from a less to a greater unit of measure.

245. ***Reduction Descending*** is changing an expression of quantity from a greater to a less unit of measure.

SLATE EXERCISES.

Example.—1. Reduce 3 gal. 2 qt. 1 pt. 3 gi. to gills.

Process.

3 gal. 2 qt. 1 pt. 3 gi.
4
14 qt.
2
29 pt.
4
119 gi.

Caution.—Say *12, 14*; not 4 times 3 = 12, and 2 are 14, etc.

Analysis.—Since there are 4 qt. in one gallon, there must be 3 times 4 qt. = 12 qt. in 3 gal.; 12 qt. + 2 qt. = 14 qt. Since there are 2 pt. in 1 qt., there must be 14 times 2 pt. = 28 pt. in 14 qt.; 28 pt. + 1 pt. = 29 pt. Since there are 4 gi. in 1 pt., there must be 29 times 4 gi. in 29 pt. = 116 gi.; 116 gi. + 3 gi. = 119 gi. Hence, in 3 gal. 2 qt. 1 pt. 3 gi. there are 119 gi.

Example.—2. Reduce 119 gi. to higher denominations.

Analysis.—Since there is 1 pt. in 4 gills, there are as many pints in 119 gi. as there are times 4 gi. = 29 times, with 3 gills remaining. Since there is 1 qt. in 2 pt., there are as many quarts in 29 pt. as there are times 2 pt. = 14 times, with 1 pt. remaining. Since there is 1 gal. in 4 qt., there are as many gallons in 14 qt. as there are times 4 qt. = 3 times, with 2 qt. remaining. Hence, in 119 gi. there are 3 gal. 2 qt. 1 pt. 3 gi.

Process.

4)119 gi.
2)29 pt. + 3 gi.
4)14 qt. + 1 pt.
3 gal. + 2 qt.

Note.—Analysis here supersedes the necessity for any rule.

Reduce

1. 375.96 inches to yards.
2. 2480 oz. to hundredweight.
3. 23 h. .48 min. to seconds.
4. 29738.7 inches to rods.
5. $5.33\frac{1}{3}$ days to minutes.
6. 96 oz. of lead to pounds.
7. 96 cu. ft. to cu. yd.
8. 1 gro. 10 doz. to units.
9. 3 mi. 173 yd. 2 ft. to in.
10. 19.3 pecks to bushels.
11. 5238 far. to shillings.
12. 29 wk. 6 d. to hours.
13. 495 sheets to quires.
14. 69472 lb. to hundredweight.
15. 13 mi. 1.537 yd. to yards.
16. £6 17 s. 10 d. to farthings.
17. 5620 hours to weeks.
18. 593 yd. 1.8 in. to inches.
19. 27 gro. 11 doz. to units.
20. 25 T. 7 cwt. to ounces.
21. 157 quires to reams.
22. 187.6 quarts to pecks.
23. 5 bu. 3 pk. 7 qt. to pints.
24. 10 wk. 3 d. 10 h. to sec.
25. 27 lb. 9 oz. Troy to grains.
26. 705 quarts to bushels.
27. 5.934 feet to yards.
28. 5638 d. to pounds sterling.
29. 14 h. 36 min. .5 sec. to sec.
30. 893 units to gross.
31. 14 mi. 18 rods .6 yd. to yd.
32. 19 cwt. 46 lb. 9 oz. to oz.
33. 3 wk. 5 d. .19 h. to min.
34. 484 pecks to bushels.
35. 276457 ounces to tons.
36. 4563 shillings to pounds.
37. 654 dozens to gro.
38. 2 s. 6 d. 3 far. to farthings.
39. 8349250 seconds to days.
40. 5 gal. 3.5 qt. to pints.
41. 14 pk. 1 qt. to pints.
42. 628 pints to pecks.
43. 12 bu. 1.75 pk. to pints.
44. 1 mi. 58 yd. .8 in. to in.

Reduce

45. 945.6 ounces to lb. avoir.
46. 25 yr. 79 d. to days.
47. 2572 gills to quarts.
48. 1 mi. 13.62 yd. 2 ft. to in.
49. 2500 inches to feet.
50. 17 cwt. 95 lb. to pounds.
51. 93 reams 2 quires to quires.
52. 976.3 far. to pounds.
53. 42345 ounces to tons.
54. 2 cwt. 75 lb. to ounces.
55. 2 qt. 1 pt. .3 gi. to gills.
56. 7892.8 minutes to days.
57. 29650 seconds to hours.
58. 19 gr. 8 doz. 10 units to units.
59. 71 bu. 3 pecks to quarts.
60. 593 pints to gallons.
61. 25971 yards to miles.
62. 3000 gills to gallons.
63. 79 tons 2 cwt. to pounds.
64. 930780 minutes to weeks.
65. 738 reams to sheets.
66. £345 18 s. 8 d. to pence.
67. 7453 sheets to reams.
68. 284 gal. 3 qt. 1 pt. to gills.
69. 7 gr. 6 doz. 11 units to units.
70. 127 T. 15 cwt. 25 lb. to lb.
71. 4 reams 22 sheets to sheets.
72. 9256.35 feet to miles.
73. 13 gal. 1 pt. 2 gi. to gills.
74. 57289 hundredweight to T.

Reduction of Denominate Fractions.

Reduction Descending.

ORAL EXERCISES.

1. How many ounces in 7 pounds avoirdupois?

Analysis.—7 lb. = 7×16 ounces, or 112 ounces.

2. How many cents in $^1/_2$, $^1/_4$, $^2/_5$, $^9/_{10}$, $^7/_{20}$, $^{11}/_{50}$, $^3/_{10}$ dollar?

Analysis.—\$1 = 100¢; $^1/_2$ dol. = $^1/_2$ of 100¢ = 50¢.

3. How many inches in $^2/_3$, $^5/_6$, $^7/_{10}$, $^2/_5$, $^7/_{12}$, $^1/_2$, $^3/_4$ foot?

Analysis.—1 foot = 12 in.; $^2/_3$ foot = $^2/_3$ of 12 in. = 8 in.

4. How many inches in $^1/_3$, $^2/_5$, $^5/_6$, $^2/_9$, $^3/_8$, $^7/_{10}$ yard?
5. How many gills in $^1/_2$, $^2/_3$, $^5/_6$, $^1/_9$, $^7/_{12}$, $^3/_4$ quart?
6. How many pints in $^3/_4$, $^5/_6$, $^2/_3$, $^7/_8$, $^3/_{10}$, $^6/_7$ gallon?
7. How many grains in $^1/_2$, $^1/_5$, $^1/_4$, $^1/_3$, $^3/_4$, $^7/_8$ oz. Troy?
8. How many ounces in $^1/_2$, $^1/_3$, $^3/_4$, $^4/_5$, $^5/_6$, $^7/_8$ lb. Troy?

SLATE EXERCISES.

On pages 216 and 217 the pupil learned to change integral numbers from higher to lower and from lower to higher denominations. Here he will find the same principles applied to the reduction of fractional expressions from one denomination to another.

Examples.—(1.) Reduce 3 bushels to pints. (2.) Reduce $^3/_4$ bu. to pints. (3.) Reduce .75 bu. to pints.

These problems differ from each other only in this, that in the first the number of bushels to be reduced is expressed by an integer, in the second by a common fraction, and in the third by a decimal fraction. They are all solved by multiplication, and the reasons for multiplication are the same.

(1.)	(2.)	(3.)
3 bu.	$^3/_4$ bu.	.75 bu.
4	4	4
12 pk.	$^{12}/_4 = 3$ pk.	3 pk.
8	8	8
96 qt.	24 qt.	24 qt.
2	2	2
192 pt.	48 pt.	48 pt.

---

4. How many units in $2^2/_3$ gross?

Analysis.—2 gr. = 2 × 144 units = 288 units. $^2/_3$ gr. = $^2/_3$ of 144 units = 96 units. 288 + 96 = 384 units.

5. How many □ inches in $^1/_4$, $^1/_8$, $^2/_5$, $2^1/_2$, $3^3/_4$ □ yards?
6. How many □ feet in $^1/_3$, $^1/_6$, $1^1/_4$, $2^3/_8$, 7, 6 □ rods?
7. How many inches in $8^1/_2$, $3^3/_4$, $6^5/_7$, $9^1/_6$ feet?
8. How many cu. inches in $2^1/_4$, $6^3/_7$, $5^5/_{12}$, $3^7/_{23}$ cu. feet?
9. How many pints in $3^3/_4$, $3^1/_8$, $4^5/_7$, $6^3/_5$ bushels?
10. How many pennyweights in $3^5/_8$, $5^7/_{12}$, $9^3/_5$ pounds?
11. How many feet in $^1/_4$, $^1/_3$, $^1/_8$, $1^1/_4$, $^3/_4$, $^5/_6$ rods?
12. How many grains in $^7/_8$, $3^4/_9$, $^{11}/_{12}$, $5^7/_{16}$ pounds avoir.?
13. How many hours in $6^5/_9$, $3^7/_8$, $2^7/_{16}$, $5^{11}/_{12}$ months?
14. How many feet in $^7/_8$, $^4/_9$, $^5/_{12}$, $^{11}/_{16}$ mile?
15. How many gills in $^4/_7$, $1^5/_9$, $3^4/_5$, $2^5/_{48}$ gallons?

Reduction Ascending.

ORAL EXERCISES.

1. What part of a lb. is 2, 4, 5, 10, 12, 15 ounces avoir.?

Analysis.—2 oz. are $^2/_{16}$ of a pound avoirdupois because they are 2 of the 16 equal parts (ounces) into which a pound can be divided.

2. What part of a bushel is 6, 9, 13, 17, 21 quarts?

3. What part of a month is $2^1/_2$ days?

Analysis.—1 d. $= {}^1/_{30}$ mo., $2^1/_2$ d. $= \frac{2^1/_2}{30}$ mo. $= {}^5/_{60}$, or $^1/_{12}$ month.

4. What part of a shilling is $^3/_4$ penny?

SLATE EXERCISES.

Examples.—(1.) Reduce 1 pt. to the fraction of a bushel. (2.) Reduce $^1/_5$ pt. to the fraction of a bushel. (3.) Reduce .2 pt. to the decimal of a bushel.

These are similar problems, and are all solved by division, the divisors being the same in each case.

(1.)	(2.)	(3.)
2)1 pt.	2)$^1/_5$ pt.	2)0.2 pt.
8)$^1/_2$ qt.	8)$^1/_{10}$ qt.	8)0.1 qt.
4)$^1/_{16}$ pk.	4)$^1/_{80}$ pk.	4)0.0125 pk.
$^1/_{64}$ bu.	$^1/_{320}$ bu.	0.003125 bu.

246. *From the above illustrative examples it may be seen that the process of reducing fractions is the same as that of reducing integers from one denomination to another.*

4. How many days in $34^1/_2$, $37^5/_6$, $48^1/_{10}$, $50^3/_4$ hours?

This question if given in full would be, "How many days and what fraction of a day in," etc.

5. How many gallons in $75^1/_3$ pints? In $83^7/_8$, $92^3/_4$, $102^{11}/_{12}$ quarts? In $56^1/_4$, $48^2/_3$ gills?

6. What part of a hundredweight is $^3/_5$, $^5/_8$, $^7/_{10}$, $2^3/_7$, $4^5/_6$, $7^7/_{12}$, $19^3/_8$ pounds?

7. Reduce to acres: $^7/_{20}$, $1^5/_8$, $845^6/_7$, $98374^1/_2$ □ yd.

To Integers of Lower Denominations.

SLATE EXERCISES.

Example.—1. Find the value of the fractional part of $2^{17}/_{36}$ lb. Troy in integers of lower denominations.

Common Fractions.

$$
\begin{array}{l}
2|^{17}/_{36} \text{ lb. Troy.} \\
\underline{\;12\;} \\
204/_{36} = 5|^{24}/_{36} \text{ oz.} \\
\qquad \underline{\;20\;} \\
\qquad 480/_{36} = 13|^{12}/_{36} \text{ pwt.} \\
\qquad\qquad \underline{\;24\;} \\
\qquad\qquad 288/_{36} = 8 \text{ gr.}
\end{array}
$$

Decimal Fractions.

$$
\begin{array}{r}
2^{17}/_{36} \text{ lb.} = 2.47^{2}/_{9} \text{ lb.} \\
2|.47^{2}/_{9} \text{ lb.} \\
\underline{12} \\
5.|66^{2}/_{3} \text{ oz.} \\
\underline{20} \\
13.|33^{1}/_{3} \text{ pwt.} \\
\underline{24} \\
8 \text{ gr.}
\end{array}
$$

Answer to both.—2 lb. 5 oz. 13 pwt. 8 gr.

2. How many pounds and ounces in $^2/_3$, $^5/_6$, $^3/_8$, $^5/_9$, $^7/_{10}$, $^{11}/_{12}$, $^4/_{15}$, $^{17}/_{33}$, $^{90}/_{91}$ cwt.?

Analysis.—$^2/_3$ cwt. = $^2/_3$ of 100 lb. = $66\,^2/_3$ lb.
$^2/_3$ lb. = $^2/_3$ of 16 oz. = $10\,^2/_3$ oz. Hence $^2/_3$ cwt. = 66 lb. $1\,^2/_3$ oz.

3. How many □ yards and □ feet in $^3/_8$, $^2/_3$, $^5/_6$, $^4/_9$ acre?

4. How many hours, minutes, and seconds in $^1/_{21}$, $^4/_7$, $^5/_9$, $^3/_{10}$, $^5/_{16}$, $^7/_{19}$, $^{20}/_{27}$ day?

5. How many feet and inches in $^7/_8$, $^4/_7$, $^5/_{11}$, $^7/_{18}$ yard?

6. How many pence and farthings in $^5/_6$, $^7/_8$, $^9/_{10}$, $^{11}/_{12}$, $^{12}/_{13}$, $^{14}/_{17}$, $6\,^5/_8$ shillings?

7. How many weeks, days, and hours in $2^3/_4$, $5^6/_7$, $^{11}/_{19}$, $^{21}/_{31}$, $^3/_{17}$ common years?

8. How many cubic yards and feet in $1^1/_8$, $9^5/_7$, $7^1/_{12}$, $3^1/_2$, $11^{13}/_{120}$ cords?

9. How many pennyweights and ounces in $11^1/_9$, $7^1/_8$, $5^2/_{13}$, $67^1/_{13}$, $33^5/_{32}$, $25^1/_{16}$ pounds Troy?

10. How many grains and scruples in ʒ $7^1/_9$, ʒ $9^1/_8$, ʒ $3^1/_6$, ʒ $7^1/_{480}$? In ℥ $1^5/_{18}$, ℥ $^{15}/_2$, ℥ $^9/_{33}$? In ℔ $7^1/_{52}$, ℔ $1^1/_{11}$, ℔ $5^1/_{21}$?

SLATE EXERCISES.

Change to integers of lower denomination:

1. $^7/_8$ ton.
2. .1125 ton.
3. .1325 pint.
4. $^3/_9$ cwt.
5. .125 hhd.
6. .99 bl. pork.
7. .38675 oz. Troy.
8. .9975 ft.
9. $^1/_{12}$ □ yd.
10. .7846 acre.
11. $^4/_5$ mile.
12. $^3/_{10}$ day.
13. .8 cord of wood.
14. .98974 qt.
15. .7859 cwt.
16. .7775 oz. Troy.
17. $^5/_{11}$ ton.
18. $^9/_{11}$ bl. of flour.
19. .375 ream.
20. $^1/_5$ cu. yd.
21. $^1/_8$ cu. ft.
22. $^3/_7$ □ mile.
23. $^9/_{17}$ acre.
24. $^3/_{13}$ rod.
25. $^7/_{256}$ lb. avoir.
26. $^5/_6$ degree.
27. $^3/_5$ of $^5/_7$ lb. avoir.
28. $^3/_5$ of 1 cwt. 56 lb.
29. $^1/_7$ of 1 mi. 160 rd.
30. $^5/_7$ of $^3/_5$ lb. Troy.
31. .735 bl. of beef.
32. .9 × .87 lb. Troy.
33. .1755 yd.

To Fractions of Higher Denominations.

SLATE EXERCISES.

Reduce 5 d. 14 h. 24 min. to a fraction of a week.

Explanation.—Reducing the two periods of time to be compared, to the same denomination, we have

5 d. 14 h. 24 min.	1 wk.
24	7
134 h.	7 d.
60	24
8064 min.	168 h.
	60
	10080 min.

Analysis.—5 d. 14 h. 24 min. = 8064 min. One week = 10080 min. 8064 min. is $\frac{8064}{10080}$ of 10080 min. Hence,

$$5 \text{ d. } 14 \text{ h. } 24 \text{ min. is } \frac{8064}{10080} \text{ or } ^4/_5 \text{ of a week.}$$

247. *Second Method.*—The lowest denomination may be reduced to a fraction of a unit of the next higher; then this fraction, together with the integer to which it now belongs, may be reduced to a fraction of the next higher, and so on till the entire compound number is reduced to the required fraction, as follows:

Suggestion.—The work is conveniently arranged by writing the several denominations in a column, beginning with the lowest, and writing the resulting fractional quotients on the right of a light vertical line, as below. In the progress of the work this line is disregarded. It is useful only to prevent mistakes.

Process.

$$\begin{array}{r|l} 60)\underline{24 \text{ min.}} & \\ 24)\underline{14 \text{ h.}} & \underline{^2/_5} \\ 7)\underline{5 \text{ d.}} & \underline{^3/_5} \end{array}$$

5 d. 14 h. 24 min. = $^4/_5$ wk.

Analysis.—1 min. = $^1/_{60}$ h.

24 min. = $^{24}/_{60} = {}^2/_5$ h.

$^2/_5$ h. + 14 h. = $14\,^2/_5$ h.

1 h. = $^1/_{24}$ d.

$14\,^2/_5$ h. = $\dfrac{14\,^2/_5}{24}$ d.

$\dfrac{14\,^2/_5}{24}$ d. = $^{72}/_{120}$ d. = $^3/_5$ d.

$^3/_5$ d. + 5 d. = $5\,^3/_5$ d.

1 d. = $^1/_7$ wk.

$5\,^3/_5$ d. = $\dfrac{5\,^3/_5}{7}$ wk.

$\dfrac{5\,^3/_5}{7}$ wk. = $^{28}/_{35}$ = $^4/_5$ wk.

The operation by decimals is as follows:

Process.

$$\begin{array}{r} 60)\underline{24 \text{ min.}} \\ 24)\underline{14.4 \text{ h.}} \\ 7)\underline{\ 5.6 \text{ d.}} \\ 0.8 \text{ wk.} \end{array}$$

Explanation.—Here the process of reasoning is precisely the same as for similar reductions in integers, thus: Since there is 1 h. in 60 min., there are as many h. in 24 min. as there are times 60 min. in 24 min. = .4 times, etc. etc. But this process differs from the process of reduction in integers in the addition of the higher denominations as we come to them, while in integers there are no such additions to make.

SLATE EXERCISES.

What part of

1. 1 ton is 7 cwt. 79 lb. 11 oz.?
2. £5 is 1100 d.?
3. 1 cwt. is 79 lb. $9\,^1/_4$ oz.?
4. 3 acres is 1700 □ ft.?
5. 1 yr. is 89 d. 1 h. 12 min.?
6. £1.7835 is £1 15 s. 8.04 d.?
7. 6.75 bu. is 3 pk. 3 qt.?
8. 5 □ ft. is 289 □ in.?
9. 4 □ miles is 347 acres?
10. 59° is 13° 13′ 13″?
11. 13 cu. ft. is 578 cu. in.?
12. 3 oz. is $5\,^1/_{10}$ pwt.?
13. 7 days is 37 min. 37 sec.?
14. 1 yr. is 89 d. 17 h. 8 min.?
15. 52 d. 16 h. is 49 d. 9 h.?
16. 1 cwt. is 13 lb. 16 oz.?

What part of

17. 25 cu. ft. is 864 cu. in.?
18. 6 d. 1 hr. is 4 d. 20 h.?
19. 13 cords is 13 cu. ft.?
20. 1.25 ton is $17\frac{3}{5}$ oz.?
21. 13 yd. is 13 in.?
22. 13 gal. is 3 qu. 1 pt. $1\frac{1}{4}$ gi.?
23. 36 gal. is 27 gal. 2 qt. 1 pt.?
24. 1728 cu. in. is 445 cu. in.?
25. 1 lb. Troy is 7 oz. 6 pwt.?
26. 1 lb. Troy is 11 oz. 7 pwt.?
27. 1 ton is 47.73 lb.?
28. $\frac{3}{4}$ mi. is 527.3994 yd.?
29. $\frac{2}{3}$ acre is 420.1883 □ yd.?
30. 1 oz. Troy is 1 oz. avoir.?
31. 1770 oz. is $\frac{1}{3}$ cwt.?
32. 1 ton is 6 cwt. 7 lb.?
33. $\frac{3}{4}$ lb. Troy is 12 gr.?
34. 17 h. is 19 min. 13 sec.?
35. 17 h. is .1175 d.?
36. 777 oz. Troy is 3 lb. 11 oz.?
37. $\frac{4}{5}$ lb. avoir. is 21 gr.?
38. $\frac{3}{5}$ lb. avoir. is $\frac{3}{5}$ lb. Troy?
39. 3 cwt. 99 lb. is $\frac{1}{8}$ ton 33 lb.?
40. 69 cwt. is 69 lb.
41. 2 □ mi. is 345 a. 17 □ rd.?
42. 5 cords is 7.125 cord feet?
43. ℔ 3 is ℥ 3 ʒ 1 ℈ 2 gr. 16?
44. 180° is 5° 18′ 22″?
45. 1 ch. is 3 rd. 3 li. 5 in.?
46. 1 yr. is 5 h. 46.4 sec.?

Addition.

(Compound Denominate Numbers.)

Example.—1. What is the sum of 13 gal. 2 qt. 1 pt. 3 gi.; 14 gal. 2 qt. 2 gi.; 7 gal. 3 qt. 3 gi.; 9 gal. 1 qt. 1 pt. 2 gi.; 6 qt. 1 pt. 1 gi.?

Written Work.

Gal.	qt.	pt.	gi.
13	2	1	3
14	2	0	2
7	3	0	3
9	1	1	2
	6	1	1
47	0	1	3

Explanation.—Numbers of the same denomination are written in the same column for convenience of addition. The sum of the column of gills is 11 = 2 pt. 3 gi. The 3 gi. is written under the column added, and the 2 pt. are added with the column of pints. The sum of the pints is 5 = 2 qt. 1 pt. The 1 pt. is written under the column of pints, and the 2 qt. are added to the quarts, and so on.

Note.—The pupil will perceive that the process at the left is like that of simple addition, except that instead of the divisor being always 10, as in simple numbers, it varies with the denomination. It is always as many units of the denomination of the column added as are required to make a unit of the next higher.

SLATE EXERCISES.

2.

T.	cwt.	lb.	oz.
11	18	77	11
32	11	31	10
43	17	63	13

3.

Mi.	yd.	ft.	in.
4	1678	2	11
2	1123	1	10
3	1456	2	9

4.

Rd.	yd.	ft.	in.
5	3	2	8
8	0	1	9
15	4	1	10
10	1	2	3
39	4	2	6
		1	6
39	5	1	0

5.

A.	□ rd.	□ yd.	□ ft.	□ in.
2	115	20	3	31
7	218	32	6	15
1	25	31	8	25
3	34	27	7	100
15	75	21	7	27
			2	36
15	75	22	0	63

Notes.—1. In Ex. 4, the sum of the yards is 10, or 1 rod $4\frac{1}{2}$ yd.; we write the 4, and for the $\frac{1}{2}$ yd. we add 1 ft. 6 in.

6.

Oz.	pwt.	gr.	24
5	16	$15\frac{2}{3}$	16
1	14	$23\frac{5}{8}$	15
0	17	$0\frac{7}{12}$	14
2	4	$21\frac{5}{6}$	20
3	19	$8\frac{3}{4}$	18
14	12	$22\frac{11}{24}$	83

2. In Ex. 5, the sum of the □ yards is 112, or 3 □ rods $21\frac{1}{4}$ □ yd.; write the 21 □ yd., and for the $\frac{1}{4}$ □ yd. add 2 □ ft. 36 □ in.

3. The fractions of the lowest denomination being added together, and reduced, the resulting integer, if any, is added to the given integers of that denomination.

Examples.—7. Find the sum of 13 cwt. 21 lb. $13\frac{5}{6}$ oz.; 3 cwt. 18 lb. $9\frac{7}{10}$ oz.; 25 cwt. 31 lb. $15\frac{3}{8}$ oz.

8. Add 58 gal. 3 qt. 1 pt. $3\frac{1}{2}$ gi.; 45 gal. 3 qt. 1 pt. $1\frac{1}{5}$ gi.; 38 gal. 1 qt. 1 pt. $3\frac{1}{4}$ gi.; 26 gal. 3 qt. $3\frac{1}{3}$ gi.

9. Add £7305 14 s. $8\frac{1}{2}$ d.; £8737 13 s. $4\frac{3}{4}$ d.; £513 6 s. 5 d.; £67 5 s. $10\frac{1}{6}$ d.

10. Add 37 cu. yd. 15 cu. ft. 1084 cu. in.; 9 cu. yd. 13 cu. ft. 1556 cu. in.; 86 cu. yd. 22 cu. ft. 695 cu. in.; 24 cu. yd. 8 cu. ft. 924 cu. in.

11. Add 17 tons 11 cwt. 99 lb. 15 oz.; 7 cwt. 97 lb. 13 oz.; 7 tons 7 cwt. 7 lb. $7\frac{7}{10}$ oz.; 11 tons 11 cwt. 11 lb. 11 oz.; 179 cwt. 1780 lb. 11797 oz. ; 137 tons 19 cwt. 89 lb. 15 oz.

Subtraction.

Example.—1. Mr. Jones had £4 4 s. 2 d., out of which he paid Mr. Smith £1 3 s. 6 d. How much did Mr. Jones have left?

Explanation.—To pay the 6 d. Mr. Jones obtains change (12 d.) for a shilling, and putting this with the 2 d. he has 14 d. 14 d. — 6 d. = 8 d. Having taken 1 s. from 4 s. there remain but 3 shillings, which he pays to Mr. Smith, and has no shillings left. He then pays £1 out of the £4, and has £3 0 s. 8 d. left of the £4 4 s. and 2 d. which he had at first.

Written Work.

£	s.	d.
4	4	2
1	3	6
3	0	8

On comparing this process with the one represented on page 42, the pupil will see in what they are alike and in what respect they differ.

SLATE EXERCISES.

Find the differences

2.

£	s.	d.	far.
173	8	5	0
75	9	7	3

3.

Bu.	pk.	qt.	pt.
324	2	3	0
235	3	7	1

4.

Mi.	yd.	ft.
17	1375	2
7	938	2

5.

Rd.	yd.	ft.	in.
15	3	2	3
8	4	1	9
6	4	0	6
		1	6
6	4	2	0

6.

□ rd.	□ yd.	□ ft.	□ in.
35	14	6	81
13	25	7	108
21	18	7	117
		2	36
21	19	1	9

Notes.—1. In Ex. 5, when we come to the yards, we say, 4 from $8\,^1/_2$ yd. leaves $4\,^1/_2$ yd.; set down 4, and for the $^1/_2$ yd. add 1 ft. 6 in. to the feet and inches respectively.

2. In Ex. 6, when we come to the sq. yd., we say, 26 from $44\,^1/_4$ sq. yd. leaves $18\,^1/_4$; set down 18, and for the $^1/_4$ sq. yd. add 2 sq. ft. and 36 sq. in. to the remainder.

7.

Cwt.	lb.	oz.
15	33	$11\,^1/_4$
8	98	$14\,^1/_2$
6	34	$12\,^3/_4$

8.

Yr.	wk.	d.	h.	min.	sec.
14	0	2	20	31	52
9	1	6	23	56	58

9.

Bu.	pk.	qt.	pt.
169	2	1	$1\,^1/_4$
71	3	7	$1\,^7/_9$

10.

Gal.	qt.	pt.	gi.
15	3	1	$3\,^1/_6$
7	3	1	$3\,^5/_8$

11.

Mi.	rd.	yd.	ft.	in.
26	230	4	2	10
19	309	5	2	$11\,^7/_9$

Multiplication.

Example.—1. Seven bins of equal dimensions are full of wheat. On careful measurement one of them is found to contain 12 bu. 3 pk. 5 qt. How much is there in the 7 bins?

Explanation.—Seven times 5 qt. = 35 qt. = 4 pk. 3 qt. 3 qt. being written under quarts in the multiplicand, the 4 pk. are added to seven times 3 pk. Seven times 3 pk. = 21 pk. 21 pk. + 4 pk. = 25 pk. = 6 bu. 1 pk. Seven times 12 bu. = 84 bu. 84 bu. + 6 bu. = 90 bu. Hence seven times 12 bu. 3 pk. 5 qt. = 90 bu. 1 pk. 3 qt.

Written Work.

Bu.	pk.	qt.
12	3	5
		7
90	1	3

Note.—The pupil should obtain the result also by addition, and thus the relation of addition and multiplication will be more deeply impressed on his mind. (See also page 55.)

2.

Yd.	ft.	in.
5	2	$7\frac{5}{8}$
		12
70	1	$7\frac{1}{2}$

Explanation.—12 times $\frac{5}{8}$ in. = $\frac{60}{8}$ = $7\frac{4}{8}$ or $7\frac{1}{2}$ in. 12 × 7 in. = 84 in. 84 in. + $7\frac{1}{2}$ in. = $91\frac{1}{2}$ in. = 7 ft. $7\frac{1}{2}$ in. $7\frac{1}{2}$ inches being written under the inches of the multiplicand, the rest of the process is similar to that explained above.

3. Multiply 7 gal. 3 qt. 1 pt. 3 gi. by 156.

If the multiplier is large, it is sometimes convenient to multiply by its factors, as in this case by 13 and 12. An advantage of this method is that all the written work is preserved as a part of the process. One process may be used to test the other.

Gal.	qt.	pt.	gi.
7	3	1	3
			13
103	2	0	3
			12
1243	0	1	0

Examples.—4. Multiply 3 lb. 8 oz. 18 pwt. 8 gr. by 35. (Employ factors of 35.)

5. Multiply 27 gal. 3 qt. 1 pt. 3 gi. by 36; by 236.

6. Multiply 17 wk. 4 d. 13 h. 27 min. 36 sec. by 9; by 79.

7. Multiply 23 cu. yd. 6 cu. ft. 459 cu. in. by 8; by 72.

8. Multiply 6 lb. 8 oz. 15 pwt. $19\frac{8}{13}$ gr. by 42; by 84.

9. Multiply ℔ 9, ℥ 11, ʒ 7, ℈ 2, gr. 17, by 17; by 36.

10. Multiply 1 ton 13 cwt. 73 lb. 9 oz. by 65; by 47.

11. Multiply 17 h. 47 min. 39 sec. by 25; by 124.

Division.

Example.—1. If £13 8 s. 7 d. is equally distributed among 12 boys, how much does each receive?

Written Work.

12)£13	8 s.	7 d.
1	2	$4^7/_{12}$

Explanation.—This example is taken from an old English work. It is accompanied with a happy illustration, from which the following is extracted:

"Suppose a gentleman leaves with a schoolmaster £13 8 s. 7 d. in 13 one pound notes, 8 shillings, and 7 penny pieces, which he orders to be equally divided among 12 good boys. In compliance with this kind direction, the master calls the 12 boys up to his desk, and, in the first place, gives to each boy a one pound note. Having thus disposed of 12 notes, he changes the one which remains for 20 shillings, and adding these to the 8 s. given him by the gentleman, he has now 28 shillings. Out of these he gives 2 shillings to each boy, and has 4 shillings left. These he changes for 48 penny pieces, and putting them to the 7 which he had at first, has now 55," etc.

Suggestion.—Let the pupil complete the explanation, exchanging the 7 d. which will be left (of the 55 d.) into farthings, and distributing the farthings. He will find at the end that there are 4 farthings left. Since there were no smaller coins the distribution could practically go no further, although the exact fraction is stated mathematically in the solution.—The author referred to gives the remainder to a poor woman who is going by at the moment.

SLATE EXERCISES

2.

Bu.	pk.	qt.	pt.
5)25	3	7	1
5 bu.	0 pk.	6 qt.	$^3/_5$ pt.

3.

	Lb.	oz.	pwt.	gr.
63)	15	8	9	12

If *the divisor is a compound quantity* of the same kind as the dividend (that is, if we wish to see how many times one quantity is contained in another of the same kind, or what part one is of the other), we first reduce dividend and divisor to the lowest denomination in either, and then proceed as in simple division.

4. How many times does 86 bu. 3 pk. 7 qt. 1 pt. contain 14 bu. 1 pk. 7 qt. $1^5/_6$ pt.?

FIRST STEP—REDUCTION.

86 bu. 3 pk. 7 qt. 1 pt.	14 bu. 1 pk. 7 qt. $1^5/_6$ pt.
347 pk.	57 pk.
2783 qt.	463 qt.
5567 pt. (dividend)	$927^5/_6$ pt. (divisor)

SECOND STEP—DIVISION.

$927\frac{5}{6})5567$

$5567)33402$

Answer.—6 times.

Explanation.—When seemingly prepared for the division it is perceived that the lowest denomination is *sixths* of a pint. Hence divisor and dividend are both reduced to sixths of a pint before the division is attempted.

Examples.—**5.** Divide 878 wk. 4 d. 15 h. 37 min. 36 sec. by 9.

6. Divide 4285 cu. yd. 6 cu. ft. 1689 cu. in. by 23; by 85.

7. Divide 3964 lb. 9 oz. 15 pwt. 18 gr. by 12; by 97.

8. Divide 5863 gal. 3 qt. 1 pt. 3 gi. by 8; by 75.

Find

9. $^1/_7$ of 3 tons 17 cwt.; $^1/_9$ of 72 tons 13 cwt. 50 lb.

10. $^1/_{12}$ of 17 □ ft. 72 □ in.; $^1/_{144}$ of 7 cu. ft. 576 cu. in.

11. $^5/_7$ of 91 bu. 4 qt. 1 pt.; $^3/_5$ of 37 gal. 2 qt.

12. $^7/_{11}$ of 975 lb. 13 oz. avoir.; $^1/_3$ of 8 lb. 7 oz.

13. $.66\frac{2}{3}$ of 4 cu. ft. 14 cu. in.; .875 of 1 cu. yd. 15 cu. ft. 1 cu. in.

14. .125 of 126 cwt. 7 lb. 11 oz.; $^7/_9$ of 83 gal. 3 qt. 1 pt.

15. How many bars of gold, each weighing 5 oz. 13 pwt. 21 gr. can be made of a bar weighing 1064 oz. 14 pwt. 15 gr.?

16. How many pieces of cord, each $5\frac{3}{4}$ yd. long, can be cut off a length of 100 yards, and what length will remain?

17. How many jars, each containing 2 gal. 3 qt. 1 pt. 3 gi., can be filled out of a cask containing 285 gal.?

18. How many portions of time, each equal to 1 day 7 h. 45 min. 56 sec., are contained in 346 d. 18 h. 34 min. 32 sec.?

19. A silver ingot (of coin standard) weighing 175 oz. 13 pwt. 9 gr. contains silver enough for how many silver dollars? (P. 207.)

20. From the same ingot how many silver quarters could be made? How many dimes?

21. How many packages, each weighing ℥ 6, ʒ 4, can be made from a quantity of medicine weighing ℔ 17.25?

Adding and Subtracting Denominate Fractions.

1. Add $^5/_8$ gal. and $^1/_2$ qt. 2. Subtract $^5/_6$ h. from $^2/_9$ d.

First reduce the fractions to integers, then proceed as above.

Operation.	Operation.
$^5/_8$ gal. = 2 qt. 1 pt.	$^2/_9$ d. = 5 h. 20 min.
$^1/_2$ qt. = 1 pt.	$^5/_6$ h. = 50 min.
3 qt. 0 pt.	4 h. 30 min.

Examples.—3. Add $^1/_6$ wk., $^2/_3$ d., and $^5/_6$ h.

4. Add $^1/_2$ cwt., $^3/_4$ lb., and $^2/_3$ oz.
5. Add $2^3/_8$ bu., $^6/_7$ pk., and $^1/_3$ qt.
6. Add $^7/_9$ gal. and $^1/_{10}$ qt.
7. Add £$^4/_5$, $^2/_3$ s., and $^8/_{10}$ d.
8. Subtract $^1/_{16}$ h. from $^6/_7$ d.
9. Subtract $2^3/_4$ sq. rd. from $1^1/_4$ acre.
10. Subtract $^5/_8$ oz. from $^2/_5$ lb. Troy.
11. Subtract $^8/_9$ pwt. from $^2/_3$ oz.
12. Add $^9/_{16}$ cwt. $10^3/_5$ lb. and $7^2/_5$ oz.
13. Add $^3/_5$ of a ton, $^5/_8$ of a cwt., and $^2/_3$ of a lb.
14. $5^1/_2$ miles $-$ $5^2/_3$ fur. $+$ $33^7/_{11}$ rods = ?
15. $^5/_{32}$ □ mile $+$ $^7/_{10}$ acre $+$ 0.75 □ rod = ?
16. $^4/_7$ of a wk. $+$ $^3/_5$ of a day $+$ $^5/_6$ of an h. $+$ $^1/_4$ of a min. = ?
17. Subtract $^4/_7$ lb. avoirdupois from $^4/_5$ lb. Troy.
    (Find the result in grains.)
18. From $2^{17}/_{36}$ lb. Troy take $^{19}/_{96}$ oz. Troy.
19. Take $^{47}/_{64}$ cwt. from $^{1355}/_{1792}$ T.
20. From $11^7/_9$ wk. subtract $8^6/_7$ d.; $8^6/_7$ h.
21. Find the sum of $^4/_7$ cwt., $8^5/_6$ lb., and $3^9/_{10}$ oz.
22. Find the difference between $3^7/_{11}$ miles and $35^{29}/_{33}$ rd.
23. Add $^3/_5$ wk., $^3/_4$ d., $^5/_7$ h., and $^2/_3$ min.
24. Add $^4/_5$ of a pound avoirdupois and $^3/_7$ of a pound Troy.

To find Difference of Time between Dates.

Example.—1. How many years, months, and days from June 12, 1868 to May 7, 1879?

Yr.	mo.	d.
1879	5	7
1868	6	12
10,	10,	25.

Solution.—May 7, 1879, is the 7th day of the 5th month of 1879, and June 12 is the 12th day of the 6th month of 1868. Hence, by subtracting 1868 years 6 months and 12 days from 1879 years 5 months and 7 days, we find the time elapsed from the earlier to the later date. We consider 30 days a month, irrespective of what calendar months may intervene between the two dates.

248. This method, though usually employed in business, does not obtain the *exact* time elapsing from one date to another. A more accurate method is *to find first the number of entire years between the dates, then of entire months, and lastly of the days.*

The difference between the results of these methods may be seen from solutions of the following problem:

2. A sum of money was borrowed Sept. 18, 1867, and paid March 16, 1870. How long was it in the hands of the borrower?

First Method.

1870 yr.	3 mo.	16 d.
1867 "	9 "	18 "
2 yr.	5 mo.	28 d.

Second Method.

From Sept. 18, 1867, to Sept. 18, 1869	= 2 yr.
" Sept. 18, 1869, to Feb. 18, 1870	= 5 mo.
" Feb. 18 to March 16	= 26 d.
Total:	2 yr. 5 mo. 26 d.

249. The *first method* is based on the supposition that there are 360 days in the year, and 30 days in each month.

The *second method* takes no account of the number of days in the several years nor in the entire month, but reckons a year from a given day of one year to the corresponding day of the next, and a month from a given day of one month to the same day of the next. In reckoning the odd days, however, it takes into account the number of days in the month *preceding the last.*

250. To find the *exact number of days* between two dates, we must add together the number of days in the several years, allowing 366 days for a leap year, then the number of days in the odd months, according to the calendar, and finally the number of remaining days.

Third Method.

Whole years.	From Sept. 18, 1867, to Sept. 18, 1868	= 366 days.
	" Sept. 18, 1868, to Sept. 18, 1869	= 365 "
Remaining days.	Sept. 12, Oct. 31, Nov. 30, Dec. 31, Jan. 31, Feb. 28, Mar. 16.	= 179 "
	The exact time in days	= 910 days.

Examples.—Find the interval of time between the following dates by the first method:

1. Feb. 3, 1845, and Dec. 17, 1852.

2. Oct. 19, 1871, and Nov. 1, 1873.

3. Apr. 2, 1876, and Jan. 31, 1881.

4. Sept. 30, 1872, and July 2, 1879.

5. How many years, months, and days from the Declaration of Independence to the surrender of Cornwallis, at Yorktown, 1781, Oct. 19?

6. Find the present age of the American Republic, born 4th of July, 1776.

7. Washington was born Feb. 22, 1732, and lived 67 yr. 9 mo. 22 d. At what date did he die?

8. Find the exact number of days of your own life.

9. A person born Dec. 8, 1845, died, aged 36 years, 1 mo. 18 d. What was the date of his death?

10. General Grant died July 23, 1885, at the age of 63 yrs. 2 mo. 26 d. What was the date of his birth?

11. Abraham Lincoln died Apr. 15, 1865, and General Garfield Sept. 19, 1881. By what length of time did the death of each precede that of General Grant?

Longitude and Time.

251. The line which may be supposed to be drawn from pole to pole through any place on the surface of the earth is called the meridian (mid-day line) of that place. All places having the same meridian have their noon at exactly the same moment.

Since the earth revolves upon its axis from west to east, the sun seems to *come* from the east to each meridian successively, and thus to go around the earth from east to west in 24 hours.

The circumference of the earth is divided into 360 degrees (360°), and inasmuch as it revolves once in 24 hours, 15° must pass under the sun in one hour; $1/60$ of $15° = 1/4° = 15'$ of circumference in 1 minute of time, and $1/60$ of $15' = 1/4' = 15''$ of circumference in 1 second of time. Hence we have the following table, showing the correspondence of longitude and time.

Table of Longitude and Time.

360° of Longitude	correspond	to	**24 hours in time.**
15° of	"	"	**1 hour in time.**
15' of	"	"	**1 min. in time.**
15" of	"	"	**1 sec. in time.**

Note.—If three clocks, all keeping correct time, be placed at the distance of 15° longitude from each other, the one farthest east will, at any moment, be found one hour faster, and the one farthest west one hour slower, than the clock midway between them.

ORAL EXERCISES.

1. If, in traveling, I find my watch, which is a reliable time-keeper, growing faster and faster as compared with the time in the places through which I pass, should I conclude that I am traveling eastward or westward?

2. If I find my watch an hour fast, how many degrees have I gone, and in which direction? If I find it half an hour fast? 15 minutes?

3. How many degrees of the earth's surface pass under the sun's vertical rays in 2, 4, 7, 13, 19, 21 hours?

4. I start from Cincinnati, and, on arriving at another city, compare my watch with a well-regulated clock, and find it faster than my watch; have I traveled east or west? I find it 20 minutes faster; how many degrees have I traveled?

5. When it is 12 o'clock noon at Omaha, what is the time at places lying 15, $7\frac{1}{2}$, $3\frac{3}{4}$ degrees west? East?

6. What difference in longitude makes a difference of 1 hour 30 min. in time? Of $1\frac{1}{2}$ min.? Of $1\frac{1}{2}$ sec.?

7. Suppose the sun is rising at 4 o'clock A. M. on the first meridian (Greenwich), on what meridian is it noon?

8. What is the difference of time between Greenwich (on the first meridian) and a place lying under the 74th meridian?

SLATE EXERCISES.

9. What is the difference of longitude between Washington and Cleveland, the difference in time being 18 min. 32 sec.?

	18 min.	32 sec.
		15
4°	38′	0

Explanation.—One second in time corresponds to a difference of 15″ of longitude, hence 32 sec. in time correspond to 32 times 15″ in long. = 480″ = 8′ in long.

One min. in time corresponds to 15′ in long., hence 18 min. in time correspond to 18 times 15′ in long. = 270′ of longitude. 270′ + 8′ = 278′ = 4° 38′ of longitude.

Note.—Let it be kept in mind that, inasmuch as there are 360° in the circumference of the earth, and only 24 h. in the day, there are *more degrees* in any difference of longitude *than hours* in difference of time; more *minutes of longitude* than *minutes in time*, and more seconds of longitude than seconds in time. (How many times as many?)

10. When it is noon at Washington, it is only 11 o'clock, 17 min. and 44 sec. A. M. at Chicago. Find (*a*) the difference in time; (*b*) the difference in longitude.

To find Difference in Time.

a.	12 h.	0 min.	0 sec.
	11	17	44
		42 min.	16 sec.

To find Difference of Longitude.

b.		42 min.	16 sec.
			15
	10°	34′	0″

252. The names of a few important cities are given below, with the longitude of each *from Greenwich* (see Haswell, ed. 1885):

Cities.	Longitude.	Cities.	Longitude.
Albany, N. Y.	**73° 45′ 24″ W.**	**New Orleans, La.**	**90° 3′ 28″ W.**
Berlin, Germ.	**13° 23′ 45″ E.**	**New York, N. Y.**	**74° 0′ 24″ W.**
Boston, Mass.	**71° 3′ 30″ W.**	**Paris, France.**	**2° 20′ 0″ E.**
Calcutta, India.	**88° 20′ 0″ E.**	**Philadelphia, Pa.**	**75° 9′ 3″ W.**
Chicago, Ill.	**87° 37′ 47″ W.**	**Rome, Italy.**	**12° 27′ 0″ E.**
Cincinnati, O.	**84° 29′ 45″ W.**	**St. Louis, Mo.**	**90° 12′ 4″ W.**
Cleveland, O.	**81° 40′ 30″ W.**	**St. Petersburg, Russ.**	**30° 19′ 0″ E.**
London, Eng. (Greenwich.)	**0° 0′ 0″**	**San Francisco, Cal.**	**122° 23′ 19″ W.**
		Washington, D. C.	**77° 0′ 36″ W.**

11. When it is noon at St. Louis, is it before or after noon at Washington? At San Francisco?

12. When it is noon at San Francisco, is it before or after noon at St. Louis? At Washington?

13. When noon in St. Louis, about what time in Calcutta?

14. When it is midnight at Paris, what is the local time at Cleveland?

Solution.

(1st step.)

Cleveland	81°	40′	30″
Paris	2	20	
	84°	0′	30″

(2d step.)

15)84° 0′ 30″
5 h. 36 min. 2 sec.

(3d step.)

12 h.	0 min.	0 sec.
5	36	2
6	23	58

Ans.—23 min. 58 sec. past 6, P. M.

Explanation.—*First step.*—To find the difference of longitude we add the longitude of Paris to that of Cleveland, since Paris lies east and Cleveland west of Greenwich.

Second step.—Since 15° of longitude produce a difference of 1 hour in time, 84° produce a difference of 5 hours in time and something more, for there are 9° more than 5 times 15° in 84°.

The remainder, $9° = 9 \times 60' = 540'$, which being added to the 2′ in the next term, we have 542′, etc. (The pupil should be able to take the remaining steps of the analysis without aid.)

Third step.—Since Cleveland is west from Paris, the time of Cleveland is earlier than that of Paris, hence we subtract 5 h. 36 m. 2 sec. from 12 h.

Rules for Computations in Longitude and Time.

I. For finding difference of longitude, difference in time being given:

253. ***Rule.*—Multiply the difference in time by 15, and the hours, minutes, and seconds of time will give respectively °s, ′s, and ″s of longitude.**

II. For finding difference in time, difference of longitude being given:

254. ***Rule.*—Divide the difference of longitude by 15, and the °s, ′s, ″s of longitude will give respectively hours, minutes, and seconds of time.**

SLATE EXERCISES.

Examples.—Using the longitudes given in the above table,

Find the difference in time between

1. Albany and Chicago.
2. Berlin and Paris.
3. Greenwich and St. Petersburg.
4. Boston and Cleveland.
5. Boston and St. Louis.
6. St. Louis and San Francisco.
7. Rome and Paris.
8. Washington and Calcutta.
9. Cincinnati and San Francisco.
10. New York and New Orleans.
11. Washington and Philadelphia.
12. San Francisco and Calcutta.

For oral exercises, let the differences in time be estimated. Rough estimates are as frequently required as exact computations.

13. When it is 12 o'clock noon at Greenwich, what is the time at each of the cities mentioned in the table (Art. 252)?

14. Suppose it to be 8 o'clock P. M. (*Post meridian* or after noon) at Berlin, what time is it at the other cities?

15. Suppose it to be 7 o'clock A. M. (*Ante meridian* or before noon) at New York, what time is it at the other cities?

16. The difference in time between two places is 1 h. 15 min. 26 sec.; what is their difference of longitude?

17. Find the distance in geographical miles between two places on the equator that are 3 h. 2 min. 12 sec. apart. (1° = 60 geo. mi.)

Applications and Review.

1. If 1 cwt. costs \$16.16, \$18.25, \$19.50, \$25.25, \$36.36, what is the cost of 328 lb.?

2. If 1 gallon costs \$$2\frac{4}{5}$, \$3.50, \$$1\frac{3}{4}$, \$3.30, \$$4\frac{1}{2}$, \$$2\frac{3}{8}$, what is the cost of $5\frac{1}{4}$ gallons?

3. If 1 pound costs $7\frac{1}{2}$ francs, what is the cost of $3\frac{3}{5}$, 2.5, $1\frac{3}{4}$, 4.6, $5\frac{7}{8}$ pounds?

4. If 1 cwt. costs \$127.64, what is the cost of $3\frac{1}{2}$, $6\frac{3}{4}$, $9\frac{2}{5}$, $17\frac{1}{8}$ lb. avoirdupois?

5. What will a lot, measuring $57\frac{1}{2}\times100$ feet, cost, if the price of 1 □ foot is \$$25\frac{1}{2}$, \$$37\frac{3}{4}$, \$$48\frac{1}{5}$, \$$57\frac{1}{2}$?

6. What will be the cost of $5\frac{1}{2}$, $8\frac{3}{4}$, $15\frac{7}{8}$, $23\frac{6}{7}$ oz., if 1 oz. cost $37\frac{1}{2}$¢?

7. What does a family spend for meat in a month of 30 days, at $43\frac{3}{4}$¢ a day? In a year of 365 days?

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8. If 5 gal. cost \$1.15, \$2.35, \$4.25, \$6.75, \$7.45, \$15.25, \$20.20, what is the cost of $\frac{1}{2}$ gallon? (One step.)

9. If $4\frac{1}{2}$ lb. cost \$2.32, \$3.48, \$4.28, \$5.44, \$6.64, \$7.22, \$9.04, what is the cost of $\frac{1}{2}$ lb.?

10. If $3\frac{1}{2}$ dozen cost \$14.48, \$28.36, \$30, \$16.75, \$27.50, what is the cost of 1 gross?

11. If 6 reams of writing-paper cost \$7.20, \$18.50, \$20.25, \$30.75, \$50.15, what is the cost of 18 quires?

12. If 9 acres cost \$100.75, \$140.25, \$225.50, \$350.40, what is the cost of 67.5 acres?

13. If 7 barrels cost \$48.25, \$36.70, \$64.83, \$94.24, what is the cost of $\frac{1}{2}$ barrel?

14. If 8 cords cost \$24.25, \$25.75, \$26.40, \$38.85, what is the cost of $\frac{1}{4}$ cord?

15. If 1 cwt. costs \$568.25, what will 20 lb. cost? 25 lb.? $33\frac{1}{3}$ lb.? (Pursue the shortest method.)

1. If $^1/_5$ gal. costs 52¢, what will 2, 5, 7, 17, 46 gal. cost?

Analysis.—If $^1/_5$ gal. costs 52¢, 1 gal. will cost $5 \times 52¢ = \$2.60$, and if 1 gal. costs \$2.60, 2 gal. will cost $2 \times \$2.60 = \$5.20$.

2. If $^1/_5$ bu. costs 80¢, what will 5, 9, 13, 23 bu. cost?

3. If $^1/_3$ dol. is paid per hour for labor, what is paid for 19, 23 h.? For $2^1/_2$ days, 10 h. per day? For 3 days?

4. If $^1/_{10}$ gal. costs 48¢, what will 7, 18, 32, 21 gallons cost?

5. If $^1/_{12}$ lb. Troy costs $\$^1/_4$, what will 9, 11, 19, 35 oz. cost?

6. If 8 oz. avoirdupois cost \$28.30, what will 17, 73, 85, 99 lb. cost?

7. If $^1/_6$ quire costs $^1/_5$ fr., how many francs will 8, 13, 23, 45 quires cost?

8. If $^3/_{16}$ lb. costs \$.07, \$.09, \$.11, what is the cost of 6 lb.?

9. If $^2/_3$ doz. pens cost $^1/_{10}$ dol., what will 6, 9, 17, 28 doz. cost?

10. If 1 qt. costs 10¢, 12¢, 18¢, what will 18 bu. cost?

11. If $^1/_6$ doz. costs \$.60, \$.40, \$.30, \$.20, what will 1 gross cost?

12. If $^1/_2$ doz. costs $\$^1/_6$, $\$^1/_3$, $\$^1/_8$, $\$^1/_9$, what will 1 gross cost?

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13. If $1^3/_4$ thousand shingles cost $\$6^2/_3$, what will 28 M cost?

14. If 6 oz. Troy cost $^9/_{10}$ dol., what will 5, 8, 30 lb. cost?

15. If $1^7/_8$ bu. cost $\$2^7/_8$, what will 19, 31, 84, 73 bu. cost?

16. If $4^1/_5$ bl. cost \$105, what will 8, $7^1/_2$, $11^3/_4$ bl. cost?

17. If $3^3/_4$ cords cost $\$24^1/_2$, what will 9, 12, 19 cords cost?

18. If $3^2/_3$ oz. cost \$.70, what is the cost of 5, 8, 17 oz.?

19. If $1^1/_4$ pk. cost $\$1^1/_4$, what is the cost of $1^1/_4$, $2^1/_2$, $3^3/_4$ bu.?

20. If $3^3/_5$ cwt. cost $\$46^4/_5$, what is the cost of $3^3/_4$, $6^1/_7$ cwt.?

21. If $1^2/_3$ doz. cost $\$1^3/_4$, what is the cost of $2^3/_7$, $7^1/_8$ doz.?

22. If 4.5 yd. cost \$12.30, what is the cost of 7.7, 9.13 yd.?

23. If 3.48 lb. cost \$1.24, what is the cost of 2.36, 9.81 lb.?

Square and Cubic Measures.

Examples.—1. How many square yards of oil-cloth will cover a floor 14 ft. long and 12 feet wide?

Analysis.—The area of the floor is 14 × 12 ft. = 168 □ ft., = $18^2/_3$ □ yd., hence $18^2/_3$ □ yd. of oil-cloth will be required to cover the floor.

2. How many acres in a roadway 100 rods long and 18 yards wide?

3. How many bricks will it take to pave a sidewalk 32 ft. long and 6 ft. wide, there being $4^1/_2$ bricks to the □ foot?

4. How many yards of paper will be needed to paper a room 14 ft. long, 12 ft. wide, and 9 ft. high, if the paper is 18 in. wide, no deduction being made for doors and windows?

5. How many rolls of paper of 8 yd. each will be needed to paper a room 18 ft. long, 15 ft. wide, and 10 ft. high, if the paper is 18 in. wide, and one roll is saved by the windows and doors?

Explanation.—Rolls of wall-paper 24 ft. long would make 2 strips each 10 ft. long, the strips not being pieced; but, if the 4 ft. left were used under and over the openings, we would need to know how many times the surface of a roll is contained in the surface of the walls. (For paper-hangers' method, see p. 266.)

6. How many □ yd. of plastering are required for a room 20 ft. long, 15 ft. wide, and 10 ft. 6 inches high?

7. How many shingles will it take to cover both sides of a roof, the rafters of which are 16 ft. long and the ridge-pole is 23 ft. long, if each shingle has an area of 162 □ inches, but $^2/_3$ of it is covered by other shingles?

8. How many cords of wood in a pile 36 ft. long, 10 ft. 6 in. high and 4 ft. wide?

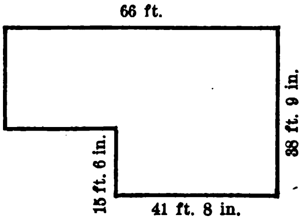
Analysis.—Length × width × height = no. cubic ft., and 128 cubic feet are equal to 1 cord.

9. How many cords of wood in a pile 50 ft. long, 11 ft. 3 in. high, and 5 ft. wide?

10. How many cubic feet in a room 16 ft. long, 14 ft. wide, and 9 feet high?

11. What is the capacity in gallons of a vat 10 ft. long, 3 ft. wide, and 4 feet deep?

12. How many cu. ft. in a square tank, $2\frac{1}{3}$ yd. wide and long, and 8 ft. 6 in. deep?



13. How many qt. of milk can be put into a can containing $1496\frac{1}{2}$ cu. in.?

Remember, a gallon fills the space of 231 cubic inches.

14. How many loads of earth must be removed in digging a cellar to the depth of 6 ft., and of other dimensions as given in the diagram? (A load is estimated to be 1 cubic yard.)

15. How much will it cost to pave the floor of this cellar at 14¢ per □ foot? How may bricks will it require if laid on edge 7 to the □ foot?

16. How many square inches in the largest circle that can be cut from a card-board 2 ft. square? (See note, page 243.)

17. A tank is 5 ft. 6 in. long, 5 ft. 3 in. wide, and 6 ft. 8 in. deep. How many gallons will it hold? How heavy is the water contained in it, if 3 ft. deep? (A cubic foot of pure water at 62° weighs 997.68 oz.)

18. If a horse can draw 1600 lb. on a given road, how many cubic feet of lead can 2 horses draw on the same road, a cubic foot of lead weighing 709.5 lb.? How many men whose average weight is 165 lb. 12 oz.?

19. A cubic foot of ice at the temperature of 32° weighs 57.5 lb. How many tons can be stored in an ice-house that is 80 ft. long, 30 ft. 9 in. wide, and 20 ft. deep?

20. How many paper boxes 3 in. long, 2 in. wide, and 2 in. deep, can be packed in a box 3 ft. long, 2 ft. wide, and $1\frac{1}{2}$ ft. deep?

21. How many cubic inches in a can holding 16 gal. 1 pt.?

22. How many square feet in the floor of your school-room to each pupil present?

23. At $3\frac{1}{2}$¢ a □ yd., how much will it cost to have 3 ceilings kalsomined, each measuring 15 by $14\frac{1}{2}$ ft.?

24. At the same rate, what will it cost for kalsomining the ceiling and walls of a room 16 ft. long, 15 ft. wide, and $10\frac{1}{2}$ ft. high, allowing $8\frac{1}{2}$ □ yd. for doors and windows?

Analysis.—The ceiling contains 16×15 ft. $= 240$ □ ft.
Two walls contain each $16 \times 10\frac{1}{2}$ ft. $= 336$ □ ft.
Two walls contain each $15 \times 10\frac{1}{2}$ ft. $= 315$ □ ft.
891 □ ft. or 99 □ yd.
99 □ yd. $- 8\frac{1}{2}$ □ yd. $= 90\frac{1}{2}$ □ yd.
At $3\frac{1}{2}$¢ what will $90\frac{1}{2}$ □ yd. cost?

25. What will it cost to paint a room 24 ft. long, $20\frac{1}{2}$ ft. wide, and 12 ft. high, at $10\frac{1}{2}$¢ a □ yd., taking out 4 □ yd. for windows?

26. What will it cost to paper a room of the same dimensions as those given in Example 24, if the paper is 18 in. wide, and a roll of it, measuring 8 yd. in length, costs 22¢. The border costs 3¢ a yard.

27. What will the laying of a flag-stone walk, 85 ft. long and 5 ft. wide, cost at \$2.15 a □ yd.?

28. What will it cost to have the roof of a house shingled, the rafters of which are 16 ft. long and the ridge-pole 25 ft. long, if the □ yard costs \$.50?

29. What will it cost to have a tin roof put on my stable, each slope of which measures 20 by 14 ft., at \$5.75 per 100 □ ft.?

30. How many bricks are required to pave a cellar 36 ft. long and $24\frac{1}{2}$ ft. wide, a brick measuring 8 by 4 inches? What will be the cost of the job if I am charged \$1.65 per □ yard?

31. What will it cost to fill a jug, which contains 2310 cubic inches, with vinegar, at 7¢ per quart?

32. The same telegram is sent direct from New York City, at 12 o'clock noon, to Cleveland, Cincinnati, St. Louis, and San Francisco. At what time should it be received in each city?

Miscellaneous Problems.

1. Fred paid $8.50 per week for board from April 3, 1883, to May 8, 1884; how much did his board cost him for that period?

2. Three lb. of sugar are needed for canning 5 qt. strawberries; how many lb. of sugar are required for $3\,^5/_8$ bu. of berries?

3. Five francs are equal to 4 marks; what are 500 francs worth in German money? (Estimate.) What are 600 marks worth in French money? (Estimate.)

4. Of 104.688 lb., 26.9 lb. were sold yesterday, and $^1/_4$ of the remainder to-day. How many lb. and oz. are left?

5. What is the general estimate for 1,000,000 francs, in U. S. money? For £240,000? What are the exact values of these sums?

6. A wall 1690 feet long is to be built in 30 days, and it is found that 7 men in 14 days have completed only 490 feet; how many additional men must be employed that the wall may be completed in the required time?

7. A grocer bought goods done up in pound packages. On weighing them he found each to be one ounce short. How much should be deducted if $45 was the charge for the whole?

8. If nine geese will yield 4 lb. 8 oz. of bed-feathers, how many lb. will 24 geese yield, at the same rate in the same time?

9. The ocean covers .734, the land .266, of the surface of the earth. How many times the surface of the land is the surface of the water?

10. In the northern hemisphere of the earth, the land covers .4 of the surface, while in the southern hemisphere it covers .12. How many times as much land in the northern hemisphere as in the southern?

11. Dividing the surface of the earth into 1000 equal parts, 398.7491 of these parts are in the torrid zone, 259.1555 in each of the temperate zones. How many are in the arctic and antarctic zones?

12. Sound travels 1090 ft. per sec. How far in .16$^2/_3$ min.?

13. Three persons together buy a quantity of butter, weighing 260 lb., for \$92$^1/_2$. The first takes 1 cwt., the second 90 lb., and the third the remainder. How much does each have to pay?

14. Of a certain kind of cloth, 29 in. wide, 12 yd. are required for a dress. How many yd. would be required if the cloth were 35 inches wide, provided the two kinds cut to equal advantage?

15. A lady bought a quantity of butter weighing 24 lb. How many ounces are used per day, if the whole quantity lasts her family from the first of May to the 15th June?

16. A cwt. of salt cost \$4; what is the cost of 1 lb.? Of 4 oz.? Of 500 lb.? Of a ton? Of 200 lb.?

17. If a thousand cigars cost \$85; what is the cost of 3 boxes, each containing 50? How much does a man pay for cigars who smokes 3 per day for one year?

18. How many lb. of butter can be made in 1 week from the milk of 12 cows, giving an average of 12 qt. 1 pt. each daily, if 25 qt. yield 1 lb. 8 oz. butter?

19. How much per hour do I pay the laborer who works 3$^3/_4$ days, 8 hours a day, and receives \$5 for the time?

20. A family agreed to pay rent at the rate of \$190 a year, but after 7$^1/_2$ months left the premises, with consent of the proprietor. How much should they pay?

21. William, walking briskly, goes 6600 paces per hour. He reaches the next village in 40 minutes. How many paces distant is the place? How many miles if each pace is 2.7 ft.?

22. If the distance from one place to another is 16 miles 80 rods, how far is it to an intermediate place, the latter distance being $^5/_6$ of the former?

23. Find the area of a circle in □ yd., if its diameter is 4.05 yards.

Note.—The area of a circle is .7854 of the area of a square whose side is equal to the diameter of the circle.

24. A man saw the flash of a cannon, which was discharged in the distance, $5\frac{3}{4}$ seconds before he heard the report. How far was he from the cannon? (See Ex. 12.)

25. Between the lightning and the thunder I noted $9\frac{5}{8}$ seconds; how far away was the thunder-cloud?

26. Ernest bought 3 yd. of broadcloth for $11, and when he took it to the tailor to have a coat made of it, he found that he had to get $\frac{2}{3}$ yd. more. How much did the cloth cost him?

27. If the knitting of a pair of stockings costs 24¢, and a lb. of worsted costs $1.10, what will a pair of stockings cost for which 4 ounces of worsted are used?

28. A family uses daily $\frac{4}{5}$ lb. of butter at 2¢ an oz., and $1\frac{1}{2}$ qt. of milk at 4¢ a pint. How much do the butter and milk cost the family per month of 30 days?

29. A farmer sold 25 bu. 3 pk. of pears at 45¢ per bu. How many yards of calico at $8\frac{1}{2}$¢ can he obtain for the proceeds?

30. In making strawberry-jam, 1 lb. 2 oz. of sugar are commonly taken to 1 lb. 4 oz. of fruit. Find the amount of sugar required for 38 lb. of berries.

31. In making jelly we commonly take 1 lb. of sugar to 1 pint of juice. Find the amount of sugar required for $17\frac{1}{2}$ qt. juice.

32. A grocer bought a barrel of vinegar containing 123 qt., at 12¢ a gallon, and paid for it with coffee at $37\frac{1}{2}$¢ a lb. How many lb. did it require?

33. What will it cost to have a manuscript of 8 quires 16 sheets copied, at 15¢ a page, one side only of each leaf to be written on?

34. How many cubic feet of masonry in a wall 33 yd. long, 10 ft. high, and 1 ft. 6 in. thick?

35. How many cubic inches in a tank 9 ft. 6 in. long, 5 ft. 3 in. wide, and $3\frac{1}{3}$ ft. deep? How many gallons?

36. A wheel turns 300.5 times in 6 min. 15 sec.; how many times in 1 min.?

37. Our earth completes its circuit around the sun in 365.242199 days, and thereby travels a distance of 129847287.467 geogr. miles. What distance does it travel in 1 day? In 1 sec.?

38. A commission merchant wishes to ship 1384 bu. of grain in sacks, each holding 2 bu. 3 pk.; how many sacks does he need? (1 sack for any remainder.)

39. A railroad track has a grade of 35 ft. 9 in. to the mile. In how many miles will the rise amount to 143 ft.?

40. Mr. M. received 3 shipments of goods, namely, 20 packages, each weighing 6.666 lb., 25 packages, each weighing 3.166 lb., and 30 packages, each weighing 5.4 lb. How many pounds and ounces do the three shipments together weigh?

41. If the circumference of a circle is 10 yd., what is its diameter in feet? (Circumference 3.1416 times the diameter.)

42. If the height of a staircase is 5 yd., and that of each step $7\frac{1}{2}$ inches, how many steps are there in the staircase?

43. An astronomical clock lost 17.63 seconds in 500 days. What was the average loss per day?

44. How many gallons in a tank 11 ft. deep, 7 ft. long, and 6 ft. wide?

45. What will 1 foot 6 inches cost, if $5\frac{1}{2}$ yards cost \$2.10?

46. A dairy-man has 3 large vessels of equal capacity, and a small one, the capacity of which is $\frac{3}{5}$ of that of one of the large ones. The 3 large vessels together hold 450 gal. How much will he pay for milk to fill them all at $3\frac{1}{4}$¢ a qt.?

47. A merchant bought 4 cwt. sugar for \$38; he used 40 lb. himself, and sold the remainder so as to make $1\frac{1}{2}$¢ profit per lb. on the whole quantity. How much per lb. did he sell it for?

48. Three dozen silver spoons weigh 1 lb. 2 oz.; how much do 4 spoons weigh? (What table?)

49. How many □ yds. multiplied by 17 will equal 1530 □ ft.?

50. What is the cost of $2\frac{2}{7}$ oz., if $\frac{1}{8}$ lb. cost 14¢?

51. If multiplying a number of feet and inches by 3 and by 4 and adding the products give 76 ft. 5 in., what is the number?

52. It takes 4 men 75 days of 8 h. each to dig over a certain piece of land; how many hours and minutes will it take 5 men?

53. A stick is placed perpendicularly, so that 1 yd. 27 in. are above ground. It throws a shadow of $2^4/_5$ yards in length. The shadow of a church steeple near by is at the same time of the day 66 yd. 2 ft. long. How tall is the steeple?

54. A man earned \$1.25 every week day, and spent \$5.09 every week. In how many weeks had he saved \$113.27?

55. Upon the circumference of a wheel are 48 teeth, which are exactly 1.294 inches apart (from the center of one tooth to the center of another). What is the diameter of the wheel?

56. If 1870 shingle nails weigh $8^1/_2$ lb., how many such nails in 2 ounces?

57. A locomotive runs $90^5/_8$ miles in 2 h. 21 min.; at the same rate, in what time will it run $1^1/_2$ mile?

58. If I receive \$$432^3/_4$ interest in .5 year, how much do I receive in 4 mo.?

59. If .3 yd. are equal to .9558 of a certain measure, how many such measures are equal to 289 rd. 3 yd. 1 ft. 6 in.?

60. If $^7/_{15}$ cu. yd. of marble costs \$$18^1/_5$, what will 9 cu. ft. cost?

61. To travel on foot from A to B in 6 days, I must walk 2 miles an hour for 6 h. every day. How long will it take me if I make $^3/_4$ of a mile a day more?

62. In $3^1/_3$ years the interest of a sum of money is \$$43^1/_5$; how much is that per month?

63. Mr. A earns \$$123^3/_4$ in 27 days and 4 hours, working 8 hours per day. What is that per day? Per hour?

64. A type-setter can set $^3/_4$ of a tabular statement in $5^3/_4$ ›urs; what part of it can he set in 30 minutes?

65. The light of the sun reaches the earth in 8.22 min., thereby traveling 91242000 miles. What distance does it travel in 1 second?

66. A workman, receiving $\$^1/_8$ per hour, was paid $\$13^1/_2$ for how many days of 9 h. each? How many weeks?

67. A locomotive runs 270 miles in 6.3 hours; how many miles is that per hour? Per minute?

68. A wheel turns $35^3/_4$ times in 33 seconds; how often in 1 minute?

69. We receive the oil used in our lamp in a tin can 8 in. long and wide and 12 in. deep. How much oil in the can when full?

70. A railroad track rises $24^4/_5$ ft. in a distance of 2086 ft. In what distance does the elevation amount to 10 ft.?

71. The wheel of a wagon in turning $26^1/_2$ times advances 319 yd. 1 ft. 10 in. What is the circumference of the wheel? What is its diameter?

72. Mr. A. buys 3 casks of wine, each containing 52.04 gal., and pays $1.85 per pt. What did the wine cost him?

73. How many T., cwt., and lb. in a block of marble measuring 2 cu. yd., if $^3/_5$ cu. yd. weigh 2719.575 lb.?

74. A grocer received a quantity of coffee in 3 bags, the first weighing 108 lb. 12 oz., the second 96 lb. 8 oz., and the third 120 lb. 2 oz. The cost of the whole was $130.15. What was the cost per cwt.?

75. The fore-wheels of a wagon are 6 ft. 7.56 in. in circumference, the hind-wheels 9 ft. 11.34 in. How often do the latter turn while the former turn 108.6 times?

76. To make 25 cwt. of bell-metal, 1.625 of a ton of copper and .875 of a ton of tin are required. How much copper and tin are required for a bell weighing 10 T.?

77. The Thames River tunnel measures 3960 yd. in length. How many meters long is it, 35 yards being equal to 32 meters?

78. In what time does a mason build $6\frac{3}{4}$ cu. yd. of masonry if he builds at the rate of $6.66\frac{2}{3}$ cu. yd. in 5 days?

79. The expenses of a traveling agent amounted to $1012.50 in 180 days; how much were they per day?

80. A druggist was obliged to sell 40.5 lb. of drugs for the same sum for which he had bought 36.75 lb. Receiving only $9.25 per pound, what had he paid per oz.?

81. The Cunard steamer Oregon burns on an average 337 tons of coal per day, the ordinary time of the trip being $6\frac{3}{4}$ days. How many car-loads of coal of 11 tons each are required for one trip, and what is the cost of the coal in United States currency, at 11 shillings per ton?

82. How many chaldrons of charcoal in a bin 15 ft. long, 13 ft. 6 in. wide, and 7 ft. 5 in. deep?

83. How many gallons in a cistern 8 ft. in diameter and 7 ft. 3 in. deep, if the capacity for each 10 inches in depth is 313 gallons?

84. What is the difference between two lots of land, one containing 23 square rods, the other being 23 rods square? (23 rods on each side.)

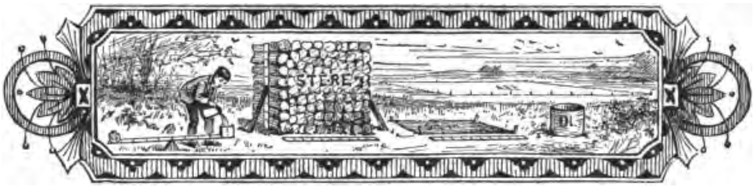
85. How many cubes measuring 3 inches each way can be cut from a cubic yard of marble?

86. In a schoolroom measuring 32 ft. 8 in. long, 28 ft. wide, and 14 ft. 6 in. high, how many cubic feet of space to each one of 56 pupils?

87. If a tank is 5 ft. 6 in. long, and 3 ft. wide, how much does the water in it weigh, when the water is 2 ft. 9 in. deep?

88. A laborer is employed to saw four piles of cord-wood, each 12 ft. long and 5 ft. 8 in. high, at 75¢ per cord. What will he receive for the job?

89. A bin full of wheat and a tank full of water each measure 5 ft. 8 in. long, 4 ft. wide, and 3 ft. 9 in. deep. How many quarts does one contain more than the other?



CHAPTER XII.

METRIC SYSTEM OF WEIGHTS AND MEASURES.

255. The metric standard for the measurement of distance is the ***Meter,*** which is 39.37 inches long (very nearly 3 ft. $3^3/_8$ in.).

256. From the *meter* all other measures of this system are derived, hence the name ***Metric System.***

The rule represented below is *one tenth* of a meter in length. It is subdivided into ten equal parts, or *hundredths*, and these again into *thousandths*.

One decimeter (tenth of a meter) subdivided into centimeters (hundredths) and millimeters (thousandths).

257. The capacity of a box one tenth of a meter long, wide, and deep (see cut) is the standard unit for both dry and liquid measures. Such a measure is called a ***Liter*** (pronounced Lee'ter), and is equal to 1.0567 liquid quarts.

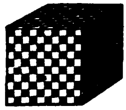
258. The weight of so much pure water as would fill a measure one hundredth of a meter long, wide, and deep is the standard unit of weight, and is called a ***Gram.*** A gram is equal to 15.432 grains.

Note.—To familiarize pupils with the meter, liter, and gram, the school should be supplied with a ***meter-stick,*** a ***liter measure,*** and a ***gram weight.***

In absence of this apparatus, the pupil can make a set for himself. Let him cut a stick 3 ft. $3^3/_8$ in. long, and he will have a ***meter-stick.*** Let this be divided by cross lines into ten equal parts; these will be ***decimeters*** (tenths of a meter—same length as that of the rule represented above). Or, he can make a pocket meter out of a piece of cord, cut long enough to allow for tying knots, by which to divide the whole into 10 equal parts.

A box one decimeter long, wide, and deep, made of pasteboard or tin, will serve very well to represent the *liter*, which is the unit of dry and liquid measures. The large square represented on page 256 is of the same size that the bottom and sides of the box should be.

If the box were made of tin, and filled with water at a certain temperature (39.2° Fahrenheit), the water would weigh a thousand grams, called a kilogram. This is the standard for weighing groceries, etc. A gram would be the weight of the water contained in a little cup that might be molded around a block, the size of which is exactly represented by the cut in the margin.



EXERCISES.

1. With a meter-stick, or string one meter in length, measure the height of your desk; the width of the school-room door; the length and width of the room; the length and width of the school-house; the length and width of the platform; of a window; the width of the nearest street or road.

2. Ascertain how many steps you have to take to go 5 meters; 10 meters; 20 meters.

3. Ascertain how many liters there are in a peck of oats; in a quart of beans (dry measure); in a bushel of wheat. (If a liter were equal to a quart (dry measure), how many liters should a bushel hold?)

4. With the use of a balance, ascertain the common weight in grams of a stick of candy; of a slate pencil; of a primer or a first reader; of your knife; a key; a rule, etc.

To perform most calculations in metric weights and measures nothing more is needed than knowledge of decimals. The following exercises will be readily performed without further explanation:

5. Add 35.6 m., 456.35 m., 93.12 m., 6375.01 m., 0.931 m.

6. Add 46.325 m., 0.56 m., 842.1 m., 3.004 m., 621.583 m.

7. From 563.83 m. take the sum of 98.375 m. and 61.094 m.

8. From 832 liters take the difference between 156.22 l. and 2.345 l.

9. Find the cost of 83.75 m. of cloth at $3.25 per meter.

10. Find the cost of 6.5 liters, at $1.85 per liter.

We express the greater distances in miles and rods, and smaller ones in yards, feet, and inches; so in the metric system the higher and lower denominations are used for different purposes.

259. In the following table the names of the orders given above the line of dots are the same that are found in the decimal numeration table, see page 177. The names below the dots are the corresponding names applied to the metric linear measure:

Names of Orders used in Notation and Numeration.	*10 Thousands*	*Thousands*	*Hundreds*	*Tens*	***Units***	*Tenths*	*Hundredths*	*Thousandths*
Corresponding Names applied to Metric Linear Measure.	*Myria-meter*	*Kilo-meter*	*Hecto-meter*	*Deka-meter*	***Meter***	*Deci-meter*	*Centi-meter*	*Milli-meter*

The ***prime unit*** is the ***meter*** = 39.37 inches. Ten units of any order = 1 of the next higher.

260. By substituting ***liter*** for *meter* we have the table for ***measures of capacity***, and by substituting ***gram*** for *meter* we have the table for ***measures of weight.***

The following are the denominations thus formed:

Linear.		Dry and Liquid.		Weight.	
Mĭl'lĭ-mē'ter	(mm.)	**Milli-lī'ter**	(ml.)	**Milli-grăm**	(mg.)
Cen'tĭ-meter	(cm.)	**Centi-liter**	(cl.)	**Centi-gram**	(cg.)
Deç'ĭ-meter	(dm.)	**Deci-liter**	(dl.)	**Deci-gram**	(dg.)
Mē'ter	(m.)	***Lī'ter***	(l.)	***Grăm***	(g.)
Dek'a-meter	(Dm.)	**Deka-liter**	(Dl.)	**Deka-gram**	(Dg.)
Hec'to-meter	(Hm.)	**Hecto-liter**	(Hl.)	**Hecto-gram**	(Hg.)
Kil'o-meter	(Km.)	**Kilo-liter**	(Kl.)	**Kilo-gram**	(Kg.)
Mўr'ĭ-a-meter	(Mm.)	**Myria-liter**	(Ml.)	**Myria-gram**	(Mg.)
				Quĭnt'-al	(Q.)
				Ton-neau'	(T.)

Notes.—1. Notice that the abbreviations for measures *larger* than the *unit* begin with capital letters, the abbreviations for measures *smaller* than the unit begin with small letters. Let the pupils construct, for class use, oral exercises similar to the following, on all the tables, using the appropriate abbreviations.

2. The names of the denominations may be readily learned by repeating only parts of the names, thus: *milli*, *centi*, *deci*, ***meter***, *deka*, *hekto*, *kilo*, ***myriameter***. As these are repeated, the learner should *think* of the meanings of the prefixes, which are as follows:

The Meanings of the Prefixes.

GREEK.	SIGNIFICATION.	LATIN.	SIGNIFICATION.
Deka-	ten.	Deci-	tenth.
Hekto-	hundred.	Centi-	hundredth.
Kilo-	thousand.	Milli-	thousandth.
Myria-	ten thousand.		

ORAL EXERCISES.

1. How many meters (m.) in a *myria*meter (Mm.)? In a *kilo*meter (Km.)? In a *hekto*meter (Hm.)? In a *deka*meter (Dm.)?

(Myria = ten thousand; kilo = a thousand; hekto = a hundred; deka = ten.)

2. How many *deka*meters (Dm.) in a *hekto*meter (Hm.)? In a *kilo*meter (Km.)? In a *myria*meter (Mm.)?

3. How many *hekto*meters (Hm.) in a *myria*meter (Mm.)? In a *kilo*meter (Km.)?

4. How many Km. in a Mm.?

5. What part of a m. is a *deci*meter (dm.)? A *centi*meter (cm.)? A *milli*meter (mm.)?

(Deci = one tenth; centi = one hundredth; milli = one thousandth. Compare decimal, cent, mill.)

6. How many mm. make one cm.? One dm.? One m.?

7. How many cm. make one dm.? One m.? One Dm.?

8. What part of one Km. is one cm.? One Hm.? One mm.?

9. What part of two cm. are two mm.? Of five dm. are five cm.? Of seven Hm. are seven m.?

10. How many ml. in 1.5 cl.? .75 dl.?

11. How many grams in .25 Mg.? $^1/_8$ Dg.?

SLATE EXERCISES.

1. Write the Linear Measure table, thus:

Table.

10 millimeters = 1 centimeter.
10 centimeters = 1 decimeter, etc.

2. Write the Dry and Liquid Measure table in the same way. (The table of *Liters.*)

3. Also the table of Weights. (The table of *Grams.*)

4. Write in full the denominations indicated by m., cl., Dg., dm., Kl., cm., Mm., Hl., mm., Mg., Dm., cg., ml., Kg., Hm., Dl., mg., Km., Hg., dl., g., dg., l.

5. Read 858.65 m., giving separately the denomination of each figure. *Ans.*, 8 hektometers, 5 dekameters, 8 **meters,** 6 decimeters, 5 centimeters.

6. In the same way read:

295.31 m.	65.12 l.	89367.351 m.	62354.319 g.
30.02 l.	.203 l.	5432.019 g.	3124.009 m.
3.892 g.	18.303 m.	10000.01 l.	85492.88 g.
1.993 m.	83.5 g.	654.321 m.	987.002 l.
384.002 g.	8.654 l.	30.50 g.	124.03 m.
50.023 l.	.009 m.	123.456 g.	98765.432 l.

7. Write 3 Kg. 5 g. 3 cg. 4 mg. in the denomination of the prime unit (grams). *Ans.*, 3005.034 g.

Note.—Be careful to fill all intervening vacant orders with ciphers, so that each digit shall by its position indicate its denomination. In the number given above, there are no hektograms, no dekagrams, no decigrams, hence the ciphers.

8. In the same way write:

5 Hl. 7 Dl. 8 dl. 2 cl. 5 ml.	7 Dg. 3 g. 9 cg. 1 mg.
1 Mm. 6 Hm. 5 m. 3 mm.	9 Kl. 8 Dl. 2 l. 3 dl.
2 Dg. 5 g. 3 cg. 8 mg.	5 Km. 6 Hm. 1 m. 3 dm.
6 Kl. 5 dl. 4 cl. 3 ml.	7 Mg. 2 Dg. 3 cg. 9 mg.
4 Hm. 5 Dm. 3 dm. 8 mm.	7 Ml. 8 Hl. 9 l. 1 cl.
3 Kg. 8 dg. 5 cg. 7 mg.	5 Mm. 9 Km. 5 Hm. 8 Dm.

Reductions.

1. Reduce 25.325 kilometers to decimeters.

1 Km. = 10 Hm.,	hence	25.325 Km.	= 253.25 Hm.
1 Hm. = 10 Dm.,	"	253.25 Hm.	= 2532.5 Dm.
1 Dm. = 10 m.,	"	2532.5 Dm.	= 25325. m.
1 m. = 10 dm.,	"	25325. m.	= 253250. dm.

Thus, in *Reduction Descending,* each step removes the decimal point one place to the right.

261. *Rule.*—To reduce a higher metric denomination to a lower: Remove the decimal point one place to the right for each step of the reduction. Annex ciphers, if necessary.

2. Reduce 435.32 dm. to Hm.

10 dm. = 1 m.,	hence	435.32 dm.	= 43.532 m.
10 m. = 1 Dm.,	"	43.532 m.	= 4.3532 Dm.
10 Dm. = 1 Hm.,	"	4.3532 Dm.	= .43532 Hm.

Thus, in *Reduction Ascending,* each step removes the decimal point one place to the left.

262. *Rule.*—To reduce a lower metric denomination to a higher: Remove the decimal point one place to the left for each step of the reduction. Prefix ciphers if necessary.

The two foregoing rules may be given in one, thus:

263. *Rule.*—To reduce a number from one metric denomination to another: Remove the separatrix from the right of the given denomination to the right of the denomination required, and change the abbreviation accordingly.

EXERCISES.

1. How many cm. in 7 m.? How many dg. in 3 Dg.?

2. How many meters in 3.15 Km.? How many liters in 6.17 Hl.? How many grams in 18.416 mg.?

3. Express the sum of 231 cm., 2859 dm., 354 mm. in meters.

4. Add 28.35 m., 200.03 m., 123.9 m., 456.7 m., and express the answer in hectometers.

5. Express in grams and add 127 dg., 7200 Hg., 8.83 Kg.

6. Find how many grams remain if you take 8 Kg. from 58 Kg.

7. Carl is told to measure the water in a vessel containing 14.31 l., with a cup holding 3 cl. How often will he have filled the cup, if his measurement is correct?

8. The circumference of May's hoop measures 3.8 m.; how many times will it turn in rolling a distance of 53.2 m.?

9. A nickel 5¢ piece weighs 5 g. How many such pieces can be made of a bar of coin metal weighing 5 kilograms.

10. Add 4.97 m., 21 cm., 6.03 m., 9.137 m., 38 dm.

11. Subtract 9 Km. 6 Dm. 7 m. 3 dm. from 1 Mm.

12. Multiply 18.28 Dg. by 29. Express the product in cg.

13. Divide 5238.45 l. by 8. Express the quotient in Dl.

14. Sold 14.23 l. at $.50 a dl. How much did I receive?

15. A train runs 54.5 Km. an hour. How far in 4.5 hours?

16. How many kilometers of telegraph wire are needed to connect two stations, if the distance between two poles is 43 m., and there are 516 poles between the stations? (517 × 43.)

17. How many m. of fence are needed to close in a field 535.5 m. long and 285.5 m. wide?

18. What will be the profit on $1\frac{1}{4}$ Hl. of vinegar, bought at $4 a Hl. and sold at 8¢ per liter?

19. What will be the profit on 10 g. of calomel bought for 50¢, if sold in powders of 5 dg. at 5¢ each?

20. A merchant bought cloth at $1.14 a m.; for how much per m. must he sell it to gain $\frac{1}{3}$ of the cost?

21. If I buy 6.328 Ml. at 5¢ a l., and sell it at 2¢ a dl., do I lose or gain? How much?

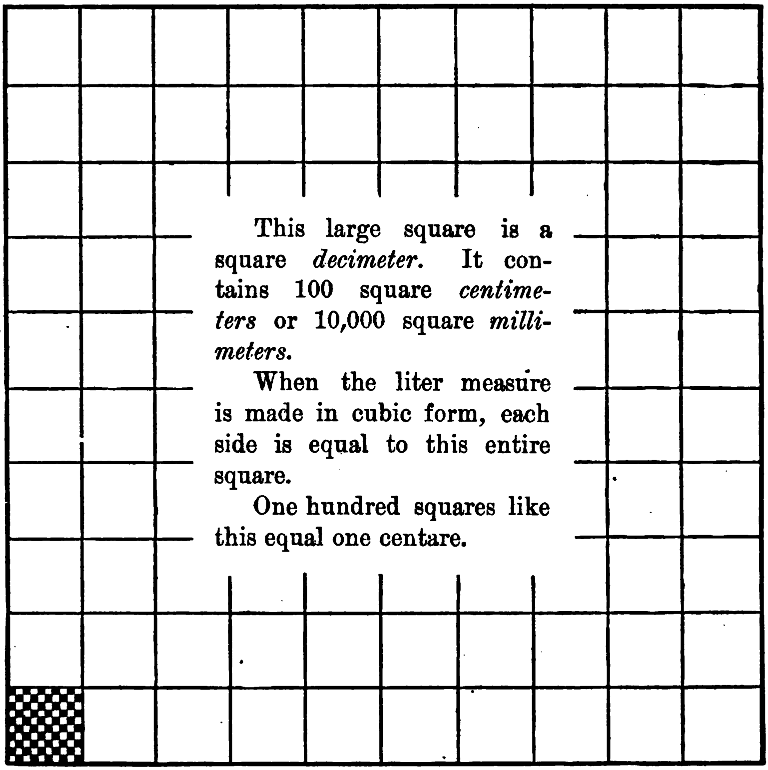
22. How many l. of grain will fill a box 7 m. long, wide, and deep?

Square Measure.

Table.

100 Sq. mm. = 1 Sq. cm.
100 Sq. cm. = 1 Sq. dm.
100 Sq. dm. = 1 Sq. m.
100 Sq. m. = 1 Sq. Dm. = 1 Are.
100 Sq. Dm. = 1 Sq. Hm.
100 Sq. Hm. = 1 Sq. Km.

Notes.—1. For land measure, the square Dm. is called an ***Are*** (pronounced like the verb *are*), from the Latin *area*, which means a level piece of ground.



2. The standard unit of land measure is derived directly from the dekameter, instead of from the meter, because a large unit is more convenient for measuring large surfaces.

The pupil may provide himself with a string, 10 meters long, and with this measure off a square Dm. on the schoolroom floor, or on the play-ground. Thus he will form an idea of the French measure of land, and measuring off 1 □ m. in a corner of an *are*, he will have a *centare.* The table for land measure is

100 Centares (ca.) = 1 Are (a.).
100 Ares (a.) = 1 Hectare (Ha.).

Inasmuch as in square measure 100 units of each order make one unit of the next higher, each denomination must have two places. For the same reason, *cents* have two places in writing dollars and cents.

Compare 5 dollars 7 cents, which is written \$5.07, with 5 □ Hm. 17 □ Dm. 6 □ m. 5 □ dm., written thus: 51706.05 □ m.

EXERCISES.

1. Write the following as square meters:

7 □ Km. 19 □ Hm. 30 □ Dm. 5 □ m. *Ans.*, 7193005 □ m.

5 □ Hm. 5 □ Dm. 5 □ m. 5 □ dm. 5 □ cm. *Ans.*, 50505.0505 □ m.

2. Write the following as square dm.: 6 □ Dm. 3 □ m. 81 □ dm. 53 □ cm.

3. How many centares are 19 hectares 59 ares and 48 centares?

4. Express in ares 58 □ Km. 93 □ Hm. 73 □ Dm. 42 □ dm.

5. How many □ m. in 2509703 □ mm.? In 35020509 □ mm.?

6. Reduce 17.519 □ m. to □ cm.; also to □ mm.

7. If $^1/_2$ of a □ m. costs \$14.90, what will 290 □ cm. cost?

8. Find the area of a floor 5.2 m. long, and 3.6 m. wide.

9. How many bricks, each 20 cm. long and 10 cm. wide, will it take to pave a cellar 10 m. long and 8.5 m. wide?

10. Reduce 2.1736 Ha. to □ m.; 517.3 centares to □ m.

11. Reduce 3872847 □ m. to a.; also to Ha.

12. How many hectares in a lot 59 m. long and 21 m. wide?

13. Reduce 12856 ares to hectares?

14. Find the area of your schoolroom in metric measurement.

15. Supposing your school-lot to be a rectangle 140.5 m. long and 70.5 m. wide, and the buildings to occupy just 400 □ m., what space is left for play-ground ?

16. Express the result of Ex. 15 in □ m.; in centares.

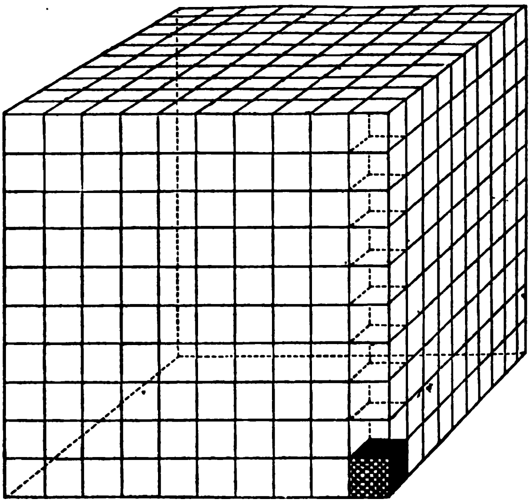
17. Find how many rolls of wall-paper, 10 m. long, $^1/_2$ a meter wide are needed to paper a room, the size of which is 6.5 m. × 4.2 m. × 3.2 m. Deduct 2.47 □ m. for window and door.

18. Mr. Quinn had 5 hectares, 5 ares, 9 centares of land, and sold first 0.5, then 0.3 of it for $384 an are. What did he get for what he sold ? How much was left ?

Cubic Measure.

Table.

1000 Cu. mm. = 1 Cu. cm.
1000 Cu. cm. = 1 Cu. dm. = 1 Liter (capacity).
1000 Cu. dm. = 1 Cu. m. = 1 Stere (pronounced *stair*).



Wood Measure.

10 Decisteres (ds.) = 1 Stere (s.).
10 Steres (s.) = 1 Dekastere (Ds.).

Notes.—1. From the cut on page 258 the pupil may see that a cu. m. (= 1 stere) is equal to 1000 cu. dm., and that 1 cu. dm. (= 1 liter) is equal to 1000 cu. cm. (milliliters). If the cut were twice as long, wide, and high, it would represent a liter of actual size.

2. From the same cut (which is only $^1/_{20}$ of a meter each way) the pupil can see that a cu. m. is 100 times 100 times 100 cu. cm, or, in other words, that a cubic meter contains 1 million cubic centimeters.

264. Since, in cubic measure, 1000 units of each denomination make one unit of the next higher order, each denomination must have three places. For instance: 3 cu. dm. would be written 0.003; 3 cu. cm. would be written 0.000003; 3 cu. millimeters would be written 0.000,000,003.

EXERCISES.

1. Write 319 cu. m. 99 cu. dm. 285 cu. cm. and 4 cu. mm. *Ans.*, 319.099285004 cu. m.

2. Express in steres, 19 dekasteres 6 steres 7 decisteres.

Note.—The units of wood measure form a scale of tens; each denomination, therefore, needs but one place.

3. Reduce 7 Ds. 5 s. and 6 ds. to ds.

4. Reduce 29 cu. m. 312 cu. dm. 703 cu. cm. to cu. dm.

5. Add 3 cu. m. 18 cu. dm. 207 cu. cm.; 385 cu. m. 230 cu. dm. 395 cu. cm. 10 cu. mm.; 831 cu. m. 300 cu. cm. Express the sum in cu. meters, then in cu. dm.

6. How many cu. m. of earth must be removed to build a cistern 3.5 m. deep, and 1.8 m. wide both ways?

7. I had to pay $1.50 per cu. m. for excavating and removing earth. The hole made was 6.5 m. long, 5.2 m. wide, 3.3 m. deep. How much did I have to pay?

8. How long must a pile of wood be so that it may contain 13 steres, if it is 4.5 m. high and 2.3 m. wide?

9. How many loads of earth, each filling 3.25 cu. m., will fill a hole 12.3 m. long, 6.5 m. wide, and 5.1 m. deep?

10. What is the cost of building a wall of masonry 2.3 m. high, 17.65 m. long, and .35 m. thick, at $7.45 a cu. m.?

Tables of Equivalents.

Measures of Length.			Measures of Capacity.			
1 mm. =	0.03937	of an inch.	1 ml. =	0.0010567	of a qt.	(Liquid).
1 cm. =	0.3937	of an inch.	1 cl. =	0.010567	of a qt.	"
1 dm. =	3.937	inches.	1 dl. =	0.10567	of a qt.	"
1 m. =	39.37	inches.	1 l. =	1.0567	quarts	"
1 Dm. =	393.7	inches.	1 Dl. =	.28375	of a bu.	(Dry).
1 Hm. =	3937.	inches.	1 Hl. =	2.8375	bushels	"
1 Km. =	39370.	inches.	1 Kl. =	28.375	bushels	"
1 Mm. =	393700.	inches.	1 Ml. =	283.75	bushels	"

Measures of Weight.

1 mg. =	0.015432	of a grain.	
1 cg. =	0.15432	of a grain.	
1 dg. =	1.5432	grains.	
1 g. =	15.432	grains.	
1 Dg. =	0.022046	of a lb.	(Avoir.).
1 Hg. =	0.22046	of a lb.	"
1 Kg. =	2.2046	lb.	"
1 Mg. =	22.046	lb.	"
1 Quintal =	220.46	lb.	"
1 Tonneau =	2204.6	lb.	"

Square Measure.

1 □ cm. =	0.1550	of a □ inch.
1 □ dm. =	15.50	□ inches.
1 □ m. =	1.196	□ yd.
1 are =	119.6	□ yd.
1 hectare =	2.471	acres.

Cubic Measure.

1 cu. cm. = 0.061 cu. in.

1 cu. dm. = 61.027 cu. in.

1 cu. m., or 1 stere = 1.3079 cu. yd., or 0.2759 of a cord.

SLATE EXERCISES.

1. How many feet in 9 dm.? In 682 mm.?

2. How many pounds av. in 2000 kilos? (Kilo is the commercial name for kilogram.)

3. One lb. av. is what decimal fraction of one kilo?

4. 10 cords equal how many steres?

5. How many gallons in 20 liters?

6. How many hectares in 2471 acres?

7. 5678 bushels equal how many hektoliters?

8. One common ton is what part of a metric ton?

9. I imported 1000 m. silk at a cost of 12 francs per meter, and sold it at $2.50 per yard. How much did I gain?

10. A grocer bought 10 Hl. of potatoes at $1.00 per hektoliter, and sold them at $.50 per bu. Did he gain or lose, and how much?

Original Problems.

1. Ask certain members of the class to bring to school a meter stick, or a cord or tape-line a *meter* long, marked off into *decimeters*, and, if possible, into *centimeters*; ask others to bring a *liter* measure, others a *kilogram* of lead or nails, others a sheet of paper measuring a *centare*. Require that the measures shall be made by the individuals who bring them.

2. Ask for a description of the *are* and of the *stere*.

3. Give the measurements made in feet and inches by yourself of some known object of moderate length, say of the fence on one side of the school-yard, and ask the members of the class to compute the measure in meters. Then test by the use of the most accurate metric measure you can get. Apply this method to measures of capacity, of surface, of solids, and of weight.

4. Ask each member of the class to report his weight in kilograms, having first found it in pounds, and require the others to reduce the kilograms to avoirdupois weight.

CHAPTER XIII.

PRACTICAL MEASUREMENTS.

Lumber.

265. Boards 1 inch or less in thickness are estimated by the square foot.

Thus, a board 16 feet long, 12 inches wide, and 1 inch or less in thickness would contain 16 □ feet. A board 16 feet long, 11 inches wide, 1 inch thick or less, would contain $^{11}/_{12}$ of 16 □ feet $= 14\,^{2}/_{3}$ □ feet, etc.

266. When lumber is more than 1 inch thick the thickness is taken into account, and the ***board foot***, 1 foot square and 1 inch thick, becomes the standard by which it is estimated.

Thus, a piece of lumber 16 ft. long 12 in. wide and $1\,^{1}/_{4}$ in. thick would contain 16 board feet $+ \,^{1}/_{4}$ of $16 = 20$ board feet. If $1\,^{1}/_{2}$ in. thick it would contain 24 board ft., if 2 in. thick it would contain 32 board feet, etc. A plank 2 inches thick is thus reckoned as two boards, each an inch thick.

12 board feet = 1 cubic foot.

In the measurement of the width of a board a fraction greater than a half inch is called a half, and if less than a half it is rejected.

The width of a tapering board is measured at the middle, or half the sum of the end measurements is taken as the mean or average width.

267. Hewn timber is sold either by board or cubic measure.

1. Find the cost of *boards* 16 ft. long, 1 in. thick, of different widths, as below, @ \$31 per M; also their value if 18 ft. long:

12 in.	16 in.	15 in.	$19\,^{1}/_{2}$ in.	19 in.	$9\,^{1}/_{2}$ in.
13 in.	$14\,^{1}/_{2}$ in.	18 in.	$17\,^{1}/_{2}$ in.	$18\,^{1}/_{2}$ in.	$10\,^{1}/_{2}$ in.
11 in.	$12\,^{1}/_{2}$ in.	14 in.	8 in.	17 in.	$11\,^{1}/_{2}$ in.
9 in.	20 in.	10 in.	7 in.	16 in.	$8\,^{1}/_{2}$ in.

Suggestion.—If the boards were laid side by side, how many square feet would they cover? Only one multiplication is needed.

2. How many board feet in a stick of timber 15 in. wide, 14 in. thick and 20 ft. long? How many cubic feet?

Solution.

$20 \times {}^{15}/_{12} = {}^{300}/_{12} = 25$ board ft.

$14 \times 25 = 350$ board ft.

$350 \div 12 = 29\,{}^{1}/_{6}$ cubic ft.

Analysis.—1. A board 1 in. thick, 15 in. wide, and 1 ft. long, would contain ${}^{15}/_{12}$ of a board foot, and if 20 ft. long it would contain $20 \times {}^{15}/_{12} = 25$ board feet. But a piece of timber 14 in. thick contains 14 times as much lumber as a board of the same length and width and only 1 in. thick. $14 \times 25 = 350$ board feet.

2. Twelve board feet being equal to a cubic foot, 350 board feet contain as many cubic feet as there are times 12 board feet in 350, which is $29\,{}^{1}/_{6}$. Hence in 350 board feet there are $29\,{}^{1}/_{6}$ cubic feet. *Ans.*

3. How many board feet in 29 joists, each 28 ft. long, 16 in. wide, and 3 in. thick?

4. At $25 per M, what will be the cost of 8 inch square timbers, measuring respectively 18, 24, 22, 16, 32, and 28 feet long?

Masonry and Brick Work.

268. Masonry is commonly estimated by the perch.

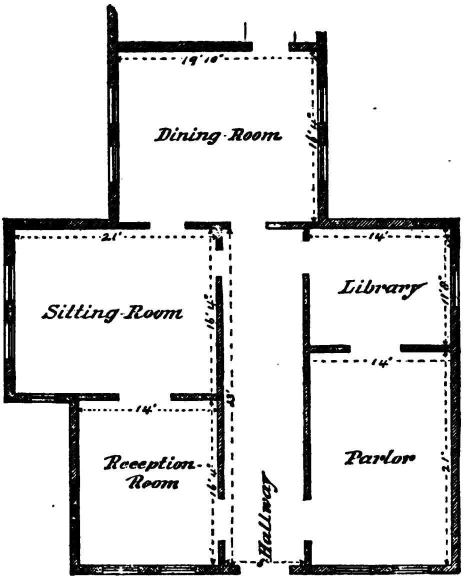
A *perch* of masonry actually contains $24\,{}^{3}/_{4}$ cubic feet, its dimensions being $16\,{}^{1}/_{2} \times 1\,{}^{1}/_{2} \times 1$ ft., but it is variously estimated in different localities, sometimes at only $16\,{}^{1}/_{2}$ cubic feet. It is gradually falling into disuse, and the cubic foot and yard taking its place.

1. Find how many perches of masonry in the walls of a cellar, that is 50 ft. long and 43 ft. wide, the walls being 8 ft. high and 24 in. thick, 112 cubic feet being allowed for openings.

In estimating material, corners are measured once, and allowance is made for doors and windows. *In estimating labor*, the corners are measured twice, and usually only ${}^{1}/_{2}$ is deducted for openings.

2. How much will the bricks for a wall 45 ft. long, 7 ft. high, and 16 in. thick cost at $8.75 per M, one gate-way 4 ft. wide being deducted.

Fourteen common bricks are usually allowed to a □ foot in one face or side of an 8 in. wall, and 7 additional bricks for every 4 in. increase in width. Bricks nominally of the same style and from the same manufacturer vary in size, so that a table of exact dimensions is impracticable.



Flooring.

1. Find the cost of flooring the *parlor, library, sitting and reception rooms* with lumber @ \$35 per M, making no allowance for waste—the cost of laying the floor being \$1.50 per square.

Note.—100 square feet of surface is called a *square*.

Suggestion.—Find first how many feet of lumber will be needed and what it will cost, and then the cost of laying the floor at the given price per square.

2. Find the cost of flooring the *dining-room* with 3 in. ash @ \$45 per M, no allowance for waste, and the cost of laying being \$2.12 1/2 per square.

3. Find the cost, @ 2¢ per board foot, of flooring joists, 8 in. wide, 2 1/2 in. thick, and of various lengths, as follows:

For hallway,	34 joists,	8 ft. 8 in.	For reception-room,	15 joists,	17 ft.
" parlor,	22 "	14 " 8 "	" sitting-room,	22 "	"
" library,	15 "	12 " 4 "	" dining-room,	21 "	"

Plastering.

269. The processes of calculating the cost of plastering and painting are quite simple, but the rules for the measurement of the work vary in different localities, and require experience in their application. It is held by some authorities to be an equitable rule for plain work to measure all the walls and ceilings without deducting anything for an opening of less extent than 7 superficial yards (63 □ feet).

Find the cost of plastering

1. *The rooms,* they being of the uniform height of 11 ft., @ 40¢ a □ yard.

2. What would be the cost of the same work at the same price if allowance be made for doorways and windows, their dimensions being as follows:

Doorways—Front, 5 ft. by 9 ft.; the doors between parlor and library, and between reception-room and sitting-room, each 6 ft. by 9 ft.; all others, 3 ft. by 8 ft.

Windows—Front windows, 3 ft. 4 in. by 9 ft.; all others, 3 ft. 4 in. by 7 ft. 8 in.

Painting and Kalsomining.

Find the total cost of painting

1. *The base-boards* of the rooms. They are 9 in. wide, and require 3 coats of paint @ 10¢ a □ yd. per coat. (Deduct width of doorways.)

In practice, surfaces less than 6 inches wide are measured as 6, and if more than 6 and less than 12 inches wide, are measured as 12. The cost of painting is here computed accordingly.

2. *The doors and windows,* both sides, at 55¢ a □ yard, the number and dimensions of which are given above.

In finding the cost of painting doors it is customary to add one edge to each side, but here the dimensions may be used as given. The cost of painting the windows may be found as if they were plain surfaces of same dimensions. This is a very common rule when there are more than two lights in the window.

3. *The library and dining-room* floors @ 28¢ a □ yard.

4. *The outside,* measuring 18 ft. by 196 ft., 2 coats, each 9¢ a □ yd., deducting doors and windows.

5. Find the cost of kalsomining the *ceilings* (including hall) @ 5¢ a □ yd.

Paper Hanging.

270. Wall paper is sold only by the roll, any part of a roll being counted as a whole one.

American paper has 8 yd. in a roll, and is commonly 18 in. wide. (Foreign papers differ as to width and length of roll.) Borders are sold by the yard, and vary in width from 3 in. to 18 in. A very wide-boarder is called a friese.

The exact cost of papering a room can be ascertained only by taking account of the number of rolls actually used in doing the work, but it is useful to be able to make an approximate estimate, which may be done as follows: Find the distance around the room, omitting all openings. Divide the number of half yards thus found by the number of entire strips that can be cut from a roll, that is, by 2, if the height from baseboard to ceiling or cornice is more than 8 and less than 12 ft., or by 3, if the height is not more than 8 ft.

When the length of the strips is such as to leave much waste in cutting a roll, a double roll (16 yd.) can often be used to better advantage, thus: If the length of a strip be 9 ft. 6 in., a single roll will make two strips with 5 ft. waste, while a double roll will make 5 strips with only 6 in. waste.

When the paper cuts with little or no waste, an additional roll or two will be required for the spaces under and over the openings.

1. Find the number of rolls of paper, 8 yards long and 18 in. wide, required for each room, all being of the uniform height of 11 ft., allowing for doors and windows according to the widths given on page 265.

Carpeting.

271. The number of yards of carpeting needed for any given room can not always be ascertained by calculating the number of square feet or yards in the floor, for unless either the length or width of the room is a multiple of the width of the carpeting, and, furthermore, unless the carpet will match in the length required, more or less will have to be turned under or cut off at the end or side, and sometimes both.

A carpet with small figures will generally lose less in matching than one with large ones. Care should be taken to lay the carpet so as to lose as little as possible either in matching or in width.

Find the cost of carpeting

1. *The parlor* with Brussels carpet, 27 in. wide, @ $1.50, to be laid lengthwise, 3 in. being lost on each strip to match. If the floor is first covered with paper-lining, @ 9¢ per □ yard, what will be the additional cost?

2. *The sitting-room* with yard-wide three ply carpet, laid lengthwise, @ 95¢, 3 inches on each strip being lost to match.

3. *The library* with China matting, 36 in. wide, @ 60¢, there being no loss to match. (Allow $1\frac{1}{2}$ in. at each end of each strip for turning in.)

4. *The dining-room* with China matting, 36 in. wide, @ 50¢, to be laid with least loss, no loss to match. (Three in. on each strip being allowed for turning in, as above.)

5. *The reception-room* with Brussels carpet, 27 in. wide, @ $1.60, to be laid lengthwise, 6 in. being lost on each breadth to match.

6. *The hall* with Moquette carpet, 27 in. wide, @ $1.75.

7. What will be the cost of a rug for the dining-room, that is 3 yd. 1 ft. 6 in. by 4 yd., @ 85¢ per □ yard, with a border in addition, @ 65¢ per lineal yard? (Allow 1 yard of border for turning corners.)

8. Find the cost of carpeting a stairway having 20 steps, and each one having $11\frac{3}{4}$ in. tread and $6\frac{3}{4}$ in. rise, $\frac{1}{2}$ yard being allowed for the landing, $1\frac{1}{2}$ yard for the turning, and $\frac{1}{2}$ yard for moving up when the edges are worn. Carpet, $1.50 per yard.

Paving.

Find the cost of paving

1. *A sidewalk* 4 ft. wide and 63 ft. long, @ 21¢ per □ foot.

2. *A courtyard* 19 ft. × 19 ft. 6 in. with brick laid flat in sand, @ 75¢ a □ yard.

3. *A sidewalk* 37 ft. long 4 ft. 6 in. wide with brick, @ $1.36 a □ yard, the bricks to be laid on edge.

4. *A cellar* with cement, @ 39¢ a □ yard; dimensions, 27 ft. × 31 ft.

5. Find which would be the cheaper: to brick a sidewalk 4 ft. wide and 275 ft. long, @ 11¢ a □ foot, or to lay a stone walk 3 ft. 6 in. wide and of the same length, @ $1.89 per □ yard.

Bins, Tanks, and Cisterns.

It must be remembered that

2150.42 cubic inches = the contents of a bushel measure.
231 " " = " " " gallon liquid measure.

1. How many bushels of wheat may be contained in a box measuring 5 ft. long, 5 ft. wide, and 5 ft. deep?

Note.—If any number of cubic feet be diminished by $^1/_5$ of itself, the remainder will represent very nearly an equivalent in bushels, stricken measure.

Thus, the box above mentioned contains 125 cubic feet, $^1/_5$ being deducted the remainder is 100, which is the number of bushels to within less than $^1/_2$ bu. (.44).

2. First estimate and then compute exactly the number of bushels of grain in a box measuring 3 by 5 by 6 feet; also in one measuring $2^1/_2$ by 6 by 7 feet. What is the difference between the estimated and exact contents in each case?

3. What would be the difference between the estimated and exact number of bushels in a bin 8 by 7 by 5 feet?

4. I wish to build a tank 4 ft. square to contain 700 gallons; how deep must it be?

5. How many gallons will fill a circular reservoir 155 ft. in diameter and 15 ft. deep? (The contents of a circular reservoir or cistern is .7854 of a square one of equal depth and having sides equal to the diameter.)

6. How many gallons of water in a cistern 9 ft. in diameter and 10 ft. deep?

Estimating the Weight of Hay in a Mow.

272. The average weight of 450 cubic feet of meadow hay, or 550 feet of clover, dry and well settled in large mows or stacks, is about one ton.

1. The average height of the hay in a mow is $11^1/_2$ ft., the length of the mow is 30 ft., and the width 18 ft. What is the estimated weight of the hay? How much heavier than clover occupying the same space?

2. The length, width, and height of a stack of clover average 12 by 12 by 10 ft. What is its estimated weight?

Miscellaneous.

1. The pumping-engine at the Saratoga water-works in one week pumps 15,307,558 gallons of water. How long should a reservoir 150 wide and 35 ft. deep be to hold that quantity of water?

2. At \$2.80 per M, what must be paid for the shingles for a barn having a gable roof 37 ft. 6 in. long, and each slope being 16 ft. 6 in. wide? (1000 shingles of good quality, laid 4 in. to the weather, cover 120 □ feet.)

3. How many cords of wood can be piled under a shed 45 ft. long, 48 ft. wide, and 12 ft. high, and what would the wood be worth at \$4.75 a cord?

4. How many gallons will fill a water-tank $8\frac{1}{2} \times 6\frac{1}{4} \times 5$? How many bushels of wheat would the tank contain? How many bushels of potatoes or apples?

Note.—In measuring grains the measure is ***stricken***, or leveled, but in measuring potatoes, apples, etc., the measure is ***heaped***. The bulk of a bushel stricken measure is $\frac{1}{5}$ less than of the heaped measure, and the bulk of the heaped is $\frac{1}{4}$ greater than that of the stricken.

5. What will it cost to slate a gable roof, each slope being 35 ft. long by 18 ft. wide, @ \$14.75 a square?

6. In a pile of wood 175 ft. long, 16 ft. wide, 7 ft. 6 in. high, how many wagon loads of cord-wood, $\frac{3}{4}$ cord to a load, and what will it cost at \$5.65 a cord, 50¢ a load being paid for hauling?

7. A surveyor, in measuring a road, finds that it is 873 chains long. How many miles is that? (See Table, page 202, Art. 203.)

8. On each side of a lane 17 chains long a farmer puts a fence. How many rods of fence does he build?

9. If a street vender buys chestnuts a \$2 per bushel, and sells them at 10¢ per quart, liquid measure, how much does he gain in selling them?

10. What was the cost of excavating a cellar 47 ft. 6 in. × 39 ft. 6 in. and 8 ft. 6 in. deep, @ 65¢ a cubic yard?

Original Problems.

Suggestions to Pupils.

1. Take measurements of the school-room for finding the cost of joists, flooring, plastering, etc. A comparison of the measurements taken will suggest corrections of errors.

2. Take the actual measurement of some rectangular room, give the dimensions with such other information as is necessary to find the cost of carpeting or papering it.

3. Give the dimensions of a bin or box to find the quantity of grain, or potatoes, or apples it will contain.

4. Give the dimensions of a lot, and such details of information as may be needed to find the cost of fencing it. An inspection of a fence already constructed and inquiries made of workmen will suggest the points needed.

5. Tell where some sidewalk about your school-house is needed, and ask the members of the class to find what its cost would be. The pupils may determine what kind of walk they would have, and learn what it would cost per square yard or foot.

6. Take the measurements of a load or pile of cord wood; report the same to the class and ask how many cords and what it would cost at prevailing prices.

7. Find how many cubic feet of coal, such as is commonly used in your neighborhood, are estimated to weigh a ton; report the same to the class with the size of some coal bin, and ask how many tons of coal may be put into it.

8. Give the thickness of ice formed at some place near by, and ask how many tons can be taken from any given space. (The weight of 1 cubic foot of ice, at 32° Farenheit, is 57.5 pounds.)

Note.—It is not designed that all the pupils shall prepare questions on all the topics suggested, nor that they should be restricted to them alone. No pupil should present a question which he is not ready to answer if required.

CHAPTER XIV.

CALCULATIONS HAVING REFERENCE TO 100 AS A STANDARD.

Percentage.

Illustrations.—1. A farm hand, who works "on shares," receives from one farmer an offer of 7 bushels of corn out of every 16 bushels raised, another offers him 2 out of 5, another 5 out of 12. Which is the best offer?

Here it is difficult to determine which is the best offer, because the standards of comparison, 16, 5, and 12, are different.

Expressing the shares in the form of common fractions, we have $^7/_{16}$, $^2/_5$, and $^5/_{12}$, and reducing these to fractions having a common denominator, we obtain $^{105}/_{240}$, $^{96}/_{240}$, and $^{100}/_{240}$. Here 240 is the common standard of comparison, and the several offers are respectively equivalent to 105, 96, and 100 out of 240 bushels produced, whence we see that the first offer is the best.

But the offer of 7 bushels out of 16, or $^7/_{16}$ of a crop, is equivalent to the offer of $^7/_{16}$ of each 100 bu., or at the rate of $43\,^3/_4$ bu. out of 100. Comparing all the offers with this standard, we find that they are equivalent to $43\,^3/_4$, 40, and $41\,^2/_3$ bushels per hundred, respectively; whence we readily see how the several offers compare with each other.

2. In like manner compare the value of two iron ores, one of which produces 52 tons of metal from 65 tons of ore, and the other 42 tons of metal from 56 tons of ore.

Suggestion.—What common fractional part of each ore is metal? How many tons of metal can be produced from 100 tons of each ore?

Note.—Because of its simplicity and convenience, 100 has been adopted as a standard of comparison in almost every department of business and by all civilized nations; hence we hear of a boy's spelling a certain per cent. of the words dictated, that is, at the rate of so many in a hundred, and in like manner of the merchant gaining or losing a certain per cent. of the money he lays out for his goods, of an increasing per cent. of children who are near-sighted, etc., etc.

Definitions.

273. ***Per Cent.*** is an abbreviation of the phrase *per centum,* and signifies ***by the hundred.***

Caution.—The *abbreviation cent.* in the phrase *per cent.* has no reference to the cent of our decimal currency.

274. A ***Rate per cent.*** is a rate per hundred.

275. The sign % is annexed to the rate, and stands for the phrase ***per cent.***

Thus, 7% is read seven per cent., .07% is read seven hundredths of one per cent., $^1/_3$% is read $^1/_3$ of 1 per cent., .00$^1/_3$% is read $^1/_3$ of one hundredth of 1 per cent.

276. Any per cent. of a number is equivalent to so many hundredths of it.

Illustration.—If 100 marks be made, 4 in line and 25 in column, the following questions may be asked:

/ / / /
/ / / /
/ / / /
and 22 more lines

What *common fractional* part, what *decimal* part, what *per cent.* of the whole number, are in the top line? etc.

One mark is what common fractional part, what decimal part, and what per cent., of 25 marks?

Additional marks being made at the foot of a column, it may be asked: What per cent. is added to the marks of the column? that is, How many additional marks would be made in all if 1 were added to each 25 in the hundred?

What *fractional* part, and what *per cent.* of all the letters in the italicized lines below, are contained in the first word, in the first two? etc. In each part of *eighty-four?* In *watchful?* etc.

Manuscript, importance, regulation, house-plant, county maps, blind-mouse, eighty-four, be watchful, cash profit, spring-halt.

What per cent. of all the letters in either line are a's, b's? etc.

What per cent. of the letters in the word Oconomowoc are o's? What per cent. are c's? etc. What per cent. of the letters in *Ohio* are vowels? Are consonants? What % of the numbers from 1 to 100 are primes? From 101 to 200? etc.

SLATE EXERCISES.

The pupil who desires to become expert in computations of percentage should be able to convert per cents. into corresponding decimal and common fractions almost at sight.

Write the decimals and the common fractions that are equivalent to the following expressions (all common fractions should be given in lowest terms):

1.	10%	20%	30%	**4.**	80%	8%	16%
2.	25%	50%	75%	**5.**	14%	12%	5%
3.	4%	40%	60%	**6.**	15%	24%	65%

Example.—What common fractional part of a number is equivalent to $12\frac{1}{2}$ per cent. of it?

$$12\tfrac{1}{2}\% \text{ is equivalent to } .12\tfrac{1}{2} = \frac{12\frac{1}{2}}{100}$$

$$\frac{12\frac{1}{2}}{100} = \frac{25}{200} = \frac{1}{8}$$

Or, since the numerator, $12\frac{1}{2}$, is an aliquot part of the denominator 100, the reduction can be made directly by dividing both numerator and denominator by $12\frac{1}{2}$, thus:

$$\frac{12\frac{1}{2} \div 12\frac{1}{2}}{100 \div 12\frac{1}{2}} = \frac{1}{8}$$

In like manner the following rates per cent. are all reducible to simple common fractions.

What decimal and what common fractions are equivalent to

1.	$2\frac{1}{2}\%$	$18\frac{3}{4}\%$	$31\frac{1}{4}\%$	**5.**	$41\frac{2}{3}\%$	$58\frac{1}{3}\%$	$66\frac{2}{3}\%$
2.	$37\frac{1}{2}\%$	$43\frac{3}{4}\%$	$56\frac{1}{4}\%$	**6.**	$83\frac{1}{3}\%$	$91\frac{2}{3}\%$	$6\frac{1}{4}\%$
3.	$62\frac{1}{2}\%$	$68\frac{3}{4}\%$	$81\frac{1}{4}\%$	**7.**	$87\frac{1}{2}\%$	$93\frac{3}{4}\%$	$6\frac{2}{3}\%$
4.	$8\frac{1}{3}\%$	$16\frac{2}{3}\%$	$33\frac{1}{3}\%$	**8.**	$3\frac{1}{3}\%$	$13\frac{1}{3}\%$	$23\frac{1}{3}\%$

What per cents. are equivalent to the following fractions:

9.	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{3}{4}$	**14.**	.04	.40	$.41\frac{2}{3}$	.14
10.	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{1}{6}$	**15.**	.75	.075	$.91\frac{2}{3}$	$.12\frac{1}{2}$
11.	$\frac{5}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	**16.**	$.00\frac{1}{3}$	$.187\frac{1}{2}$	$.83\frac{1}{3}$	$.18\frac{3}{4}$
12.	$\frac{7}{8}$	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{7}{10}$	$\frac{1}{11}$	**17.**	.0075	.005	$.58\frac{1}{3}$	$.37\frac{1}{2}$
13.	$\frac{1}{25}$	$\frac{1}{2}$ of $\frac{1}{6}$		$\frac{1}{2}$ of $\frac{1}{8}$		**18.**	.625	.00625	$.56\frac{1}{4}$	$.00\frac{1}{3}$

Applications.—Example.—1. I bought a horse for $280 and sold it so as to gain 25%. *How much* did I gain?

Analysis.—At 1% (1 to a hundred) I would gain $2.80, and at 25% I wou'd gain 25 times as much. 25 × $2.80 = $70 *Ans.*

Or, since 25% = .25 or $^1/_4$ of a number, to obtain 25% of 280 we may take $^1/_4$ of it, $^1/_4$ of $280 = $70.

Written Work.

2.80 = 1%
25
1400
560
$70.00 = 25%

For exercises, see Case I, pages 276–279.

Example.—2. If the horse was bought for $280 and sold so as to gain $70, *what per cent.* was gained?

Analysis.—At 1% I would gain $2.80, and to gain $70 the rate of gain would have to be as many times 1% as $70 is times $2.80, which is 25. 25 × 1% = 25% *Ans.*

Or, the gain $70 is $^{70}/_{280}$ or $^1/_4$ of the price paid. $^1/_4$ of any number = 25% of it.

Written Work.

2.80)70.00(25
560
1400
1400

25 × 1% = 25%

For exercises, see Case II, pages 280, 281.

Example.—3. If the horse was sold so as to gain $70 at the rate of 25%, what was the *price paid?*

Analysis.—If $70 is 25% of the price, 1% is $^1/_{25}$ of $70 = $2.80, and 100% or the whole price = 100 times $2.8 = $280 *Ans.*

Or, if 70 is 25% or $^1/_4$ of the cost, the cost must be 4 times $70 = $280.

Written Work.

25)70.0(2.8 = 1%
50
200
200

100 × 2.8 = $280

For exercises, see Case III, page 281; and Case IV, page 282.

277. The three principal cases of percentage are presented in the foregoing examples. They are:

I. To find a required per cent. of a number.

II. To find what per cent. one number is of another.

III. To find a number from a given per cent. of it.

A fourth case is added, which differs from the third in nothing except that the rate per cent. to be operated with is derived from the given rate by adding it to or subtracting it from 100. (See Case IV.)

Definitions.

278. The result of taking any per cent. of a number is called a ***Percentage.***

279. The number of which a percentage is given or on which it is to be computed, is called the ***Base.***

280. The base plus the percentage is called the ***Amount.***

281. The base minus the percentage is called the ***Difference.***

Rules.

I. To find the Percentage, the Base and the Rate % being given.
***Rule.*—Multiply 1% of the base by the rate %.**

II. To find the Rate %, the Base and Percentage being given.
***Rule.*—Divide the percentage by 1% of the base.**

III. To find the Base, the Rate % and the Percentage being given.
***Rule.*—Divide the percentage by the rate %, and multiply the quotient by 100.**

Formulas.

282. The process of finding a given per cent. of a number may be indicated by signs, as follows :

I. Rate % × 1% of the Base = Percentage.

Since the rate per cent. is thus presented as the multiplier, 1% of the base as the multiplicand, and the percentage as the product, we derive from this formula two others, as follows :

II. Rate % = Percentage ÷ 1% of Base.
III. 1% of Base = Percentage ÷ Rate %.

Note.—It is best that the pupil should not become accustomed to depend on the formulas as he should not rely on rules to direct his solutions; but if he uses a formula at all, it is best that he should refer exclusively to the first. If he understands that, it will suggest the others readily enough.

283. Let the learner accustom himself to compute percentages in the shortest way possible. Thus, since 25% is equivalent to .25 or $^1/_4$ of a number, we may obtain 25% of it either by multiplying the number by .25 or by $^1/_4$, that is, taking $^1/_4$ of it. $12\,^1/_2$% is equivalent to $.12\,^1/_2$ or $^1/_8$ of a number, etc.

Case I.—To find the Percentage, the Base and the Rate % being given.

EXERCISES.

It is supposed that the majority of pupils will be able to note the results in the following exercises without the aid of written solutions. As far as practicable, the work should be purely mental.

Find

1. 20% of 5;	of 25;	of 35;	of 45;	of 75.
2. 25% of 4;	of 28;	of 36;	of 44;	of 144.
3. 4% of 25;	of 75;	of 125;	of 175;	of 350.

4. $12\frac{1}{2}$% of 1600;	$62\frac{1}{2}$% of 4000;	75% of 8000.
5. $16\frac{2}{3}$% of 1860;	$33\frac{1}{3}$% of 2424;	$18\frac{3}{4}$ of 1600.
6. 9% of 900;	7% of 800;	12% of 1200.

7. $8\frac{1}{3}$% of 24;	of 72;	of 1440;	of 84;	of 90.
8. $37\frac{1}{2}$% of 40;	of 84;	of 4000;	of 96;	of 14.
9. $66\frac{2}{3}$% of 36;	of 69;	of 36000;	of 53;	of 71.
10. $6\frac{1}{4}$% of 64;	of 32;	of 64000;	of 76;	of 80.
11. $31\frac{1}{4}$% of 80;	of 20;	of 80000;	of 60;	of 75.
12. $87\frac{1}{2}$% of 12;	of 94;	of 12000;	of 86;	of 63.
13. 8% of $37\frac{1}{2}$;	of $62\frac{1}{2}$;	of $87\frac{1}{2}$;	of $6\frac{1}{4}$;	of $4\frac{1}{6}$.
14. 9% of $22\frac{2}{9}$;	of $33\frac{1}{3}$;	of $66\frac{2}{3}$;	of $77\frac{7}{9}$;	of $44\frac{4}{9}$.
15. 50% of $\frac{1}{4}$;	of $\frac{1}{16}$;	of $\frac{8}{9}$;	of $\frac{10}{19}$;	of $\frac{11}{21}$.
16. 6% of 150;	of 375;	of 245;	of 180;	of 65.
17. $16\frac{2}{3}$% of 12;	of 42;	of 54;	of 66;	of 72.
18. $37\frac{1}{2}$% of 32;	of 48;	of 1.6;	of 2.4;	of 5.6.

Note.—By taking any common per cent. of two or more bases separately, and adding the percentages thus obtained, the same result is reached as by taking a like per cent. of the sum of the bases.

Thus, by taking 20% of 5, 25, 35, 45, and 75, we obtain 1, 5, 7, 9, and 15, the sum of which is *37*; and by taking 20% of 185, which is the sum of the bases 5, 25, 35, 45, and 75, we obtain the same result, *37*.

In this way each pupil may test the correctness of his results when a common per cent. of two or more bases is required, as in the several lines of the foregoing exercises. No answers are given to these exercises.

SLATE EXERCISES.

19. Find $41^2/_3\%$ of \$97.68.

Solution.

1% of \$97.68 = \$.9768.

$41^2/_3 = 41^2/_3 \times \$.9768 = \$40.70$.

Find

20. 31% of 6.25 ;	of 0.75 ;	of 4.55 ;	of $16^1/_2$;	of $19^2/_3$.
21. 29% of 752 ;	of 8.61 ;	of 58.4 ;	of .378 ;	of $^{21}/_{23}$.
22. 105% of 100 ;	of 20 ;	of 120 ;	of 140 ;	of 160.
23. 117% of 49 ;	of 51 ;	of 79 ;	of 117 ;	of $^1/_9$.

24.	**25.**	**26.**	**27.**	**28.**
Find 20% of	$16^2/_3\%$ of	$37^1/_2\%$ of	18% of	34% of
\$7.80	\$18.37½	\$24.85	\$13.12½	\$42.75
5.30	15.25	19.23	9.56	83.16
2.70	32.18¾	37.82	12.21	19.88
6.20	17.75	19.06	15.66⅔	71.21
1.18	12.62½	18.71	5.18¾	88.45
2.27	52.40	15.01	1.87½	.18
4.38	64.80	18.27	2.25	.24
1.07	51.13	1.65	12.68¾	1.31

Caution.—Let it be remembered that 20% of any number = $^1/_5$ of it, that $16^2/_3\% = ^1/_6$, etc. Shorten the work as much as possible.

No answers are given to Examples 24–28. The correctness of results may be tested by comparing the sum of the percentages with the like per cent. of the sum. (See Note, page 276.) Each column may be conveniently divided into two or more shorter ones.

Loss and Gain.

Example.—1. A lot is bought for \$1285 and sold at a gain of 15%. How much is gained, and what is the selling price?

Analysis.—1% of the cost is \$12.85, and 15% of the cost is 15 times \$12.85 = \$192.75, which is the sum gained. The gain \$192.75 being added to the cost of the lot, we obtain the selling price, \$1477.75 *Ans.*

$$
\begin{array}{rl}
\$12.85 & = 1\% \text{ of cost} \\
15 & \\
\hline
6425 & \\
1285 & \\
\hline
\$192.75 & = 15\% \\
1285 & \\
\hline
\$1477.75 & \text{Selling price}
\end{array}
$$

2. If the lot is bought for $1285 and sold at a loss of 15%, how much is lost, and what is the selling price?

Here the first step of the solution is the same as that of the preceding problem, but inasmuch as there is a loss of 15% in this case, $192.75 is deducted from the purchase price to find the selling price.

284. *When the purchase price and gain or loss per cent. are given to find the selling price, the following is the more convenient process:*

$$\begin{array}{rl} \$2.88 & = 1\% \\ \underline{145} & \\ \overline{1440} & \\ 1152 & \\ \underline{288} & \\ \$417.60 & = 145\% \end{array}$$

3. If I buy goods for $288 and sell them at a gain of 45%, what do I receive for them?

Analysis.—If the goods are sold at a gain of 45%, they are sold for 145% of what they cost me. 1% of the cost is $2.88, and 145% is 145 times $2.88, which is equal to $417.60 *Ans.*

If the goods were sold at a *loss* of 45%, the selling price would be 55% of what they cost me. 1% of $288 = $2.88, and 55% = 55 × $2.88 = $158.40 *Ans.*

4. To $75 add $33\frac{1}{3}$%
5. " 1.25 " $37\frac{1}{2}$%
6. " .48 " $62\frac{1}{2}$%
7. " 1.26 " $66\frac{2}{3}$%
8. " 1.54 " 18%
9. " $.81\frac{1}{4}$ " 25%
10. " .72 " $16\frac{2}{3}$%
11. " .84 " 45%
12. " .72 " $37\frac{1}{2}$%
13. " .96 " $12\frac{1}{2}$%
14. " 1.44 " $116\frac{2}{3}$%
15. From $0.64 take $18\frac{3}{4}$%
16. " .18 " 20%
17. " 2.65 " 15%
18. " 1.92 " 45%
19. " .94 " $83\frac{1}{3}$%
20. " 1.57 " 14%
21. " 9.85 " 23%
22. " 6.88 " $56\frac{1}{4}$%
23. " 3.04 " $43\frac{3}{4}$%
24. " 5.75 " $23\frac{1}{3}$%
25. " 9.58 " $87\frac{1}{2}$%

26. Find $12\frac{1}{2}$%, $18\frac{3}{4}$%, 25%, $6\frac{1}{4}$%, and $37\frac{1}{2}$% of $147.36, and add together the percentages. (Why is the sum equal to the base?)

27. If the sum of $18\frac{3}{4}$% and $31\frac{1}{4}$% of 1876 be subtracted from 1876, how many will remain? (Solve orally.)

28. How many will remain if $2\frac{1}{2}$% of 897,659 be subtracted from $12\frac{1}{2}$% of the same number? (Solve orally.)

Applications.—1. A little boy who has 8 apples gives 25% of them to his brother, $12\frac{1}{2}$% to his sister, and 50% to his mother. What per cent. and how many has he left?

2. Charles sold his sled, which had cost him \$1.75, at 20% below cost. How much did he get for it?

3. A lot of damaged calicoes are to be sold at 75% below the marked price. What prices must be asked for those that are marked 8¢, 10¢, $12\frac{1}{2}$¢, 16¢, 20¢, 30¢?

4. A grain dealer bought wheat for \$9384, and sold it at a gain of $4\frac{1}{2}$%. What did he receive for it?

5. If a man owes \$2500, and agrees to pay it in 4 instalments, the first to be 50% of the whole, the second 25%, the third 15%, the fourth 10%, what will each instalment be?

6. A man having 1000 bushels of apples, sold 5% of them at \$1.25 per bushel; 8% of the remainder at \$1 per bu.; 50% of what was then left at 75¢ per bu., and the rest at 60¢ per bu., thus receiving 10% more than he paid; how much did he pay for the whole quantity?

7. Mr. Brooks bought a farm, which was in very poor condition, for \$1586; and, after two years of careful cultivation, which paid for itself with some improvements, he sold it for 65% more than he paid for it. What did he sell it for?

8. The number of inmates in a workhouse 5 years ago was 110; this number has since increased 180%. How many inmates are there now?

9. A merchant bought goods for \$297.70, and paid an additional sum equal to 7% of the purchase price for cartage, freight, etc. What must he sell them for to gain 40% on the whole cost?

10. In a mixture of alcohol and water 85% is alcohol. How many gills of alcohol in 3 gallons of the mixture, and how many gills of water?

11. 560 bushels of wheat, bought at \$1.10 per bu., were sold at a profit of 10%. What did the wheat sell for?

Case II.—To find the Rate %, the Base and Percentage being given.

ORAL EXERCISES.

What per cent. of

1. 10 is 1? 5? 10? 20? 30? 40? 50? 60? 70? 80?
2. 50 is 9? 12? 15? 18? 30? 45? 50? 100? 125?
3. 200 is 25? 75? 125? 250? $12\frac{1}{2}$? $87\frac{1}{2}$? $16\frac{2}{3}$? $62\frac{1}{2}$?

SLATE EXERCISES.

4. $212.62\frac{1}{2}$ is what per cent. of \$486?

Solution.

$\$212.62\frac{1}{2} \div \$486 = 43.75.$

$43.75 \times 1\% = 43\frac{3}{4}\%$ *Ans.*

Analysis.—1% of \$486 = \$4.86, and $\$212.62\frac{1}{2}$ is as many per cent. as $\$212.62\frac{1}{2}$ are times \$4.86, which is $43\frac{3}{4}$. $43\frac{3}{4}$ times 1% = $43\frac{3}{4}\%$ *Ans.*

What per cent. of

5.* 225 is 9?	11.25?	29.25?	33.75?	38.25?
6. $\frac{3}{4}$ is .03?	.045?	.06?	.075?	.09?
7. $5\frac{1}{2}$ is 5.5?	1.1?	$5.22\frac{1}{2}$?	$2\frac{1}{5}$?	.825?
8. .25 is .0175?	.27?	.3?	.295?	$.337\frac{1}{2}$?
9. 6.45 is $.32\frac{1}{4}$?	$.25\frac{4}{5}$?	$.451\frac{1}{2}$?	$.580\frac{1}{2}$?	1.29?
10. 1 is $\frac{1}{50}$?	$\frac{2}{25}$?	$\frac{1}{10}$?	$\frac{2}{3}$?	$\frac{3}{4}$?
11. 45 is .3?	.25?	.36?	.15?	.05?
12. .1879 is 18.79?	$187\frac{9}{10}$?	281.85?	319.43?	394.59?
13. 55 is $167\frac{3}{4}$?	2000?	$660.27\frac{1}{2}$?	550.22?	$112\frac{3}{4}$?

---

14. What per cent. is $26\frac{1}{4}$, $29\frac{3}{4}$, $33\frac{1}{4}$, $36\frac{3}{4}$, of 175?
15. What per cent. is 49.5, 56.25, 58.50, 63, of 225?
16. What per cent. is $.024\frac{4}{9}$, $.4\frac{2}{3}$, $.06\frac{8}{9}$, $.09\frac{1}{9}$, of $\frac{2}{9}$?
17. What per cent. is $.4\frac{2}{5}$, $4.9\frac{1}{2}$, $4.67\frac{1}{2}$, $1.3\frac{3}{4}$, of $5\frac{1}{2}$?

---

* Answers: 4%, 5%, 13%, 15%, and 17%. The sum of these rates is 54, and 54% of 225 is 121.5, which is equal to the sum of the given percentages. In like manner the pupil may test for himself the correctness of his answers to the remaining questions.

Applications.—**1.** A boy buys an old pair of skates for 50¢ and sells them for 25¢. He then buys a pair for 25¢ which he sells for 50¢. What per cent. did he lose on the first pair? What per cent. did he gain on the second?

2. If a dealer buys a hat for $3, and sells it for $4, what % does he gain? If he buys it for $4 and sells it for $3, what per cent. does he lose?

3. One hundred pounds of beef were sold for $6, having been bought @ 4¢ a lb. What per cent. profit?

4. A retail dealer in boots and shoes sold 50 pairs of boots for $300. They cost him $5 a pair. What rate per cent. did he gain?

5. A merchant bought goods for $500. What per cent. would he gain by selling them for $530? For $525? For $550? For $540? For $560? For $575? For $600? For $1500?

6. The price of a single ticket from Glenwood to New York city is 30¢, but 20 coupon tickets can be bought for $5. What per cent. is saved by buying coupon tickets? What per cent. is lost by buying single tickets?

7. 10% of a flock of sheep were killed by dogs; $6^2/_3$% of the rest were lost; $33^1/_3$% of the remaining number were sold, and 28 then remained. What was the original number?

8. At harvest time a farmer sold 60 bushels of wheat, which was 25% of the quantity he sent to mill, and what he sent to mill was 40% of what he kept over till the next spring. How many bushels had he at first?

9. When a merchant sold his goods for $261, he gained twice as much as he would have lost had he sold them for $207. What was his gain per cent.? (How many times the loss is the difference between $261 and $207?)

10. A grocer sold butter at 12% profit. Had he sold it for 2¢ more per pound, he would have gained 20%. What did 50 pounds cost him?

Case III.—To find the Base, the Percentage and Rate % being given.

ORAL EXERCISES.

1. Ten apples are $^1/_2$ of how many apples? 50% of how many?

2. Eight bushels are $^4/_5$ of how many bushels? 16% of how many?

3. 25 tons are $^5/_6$ of how many tons? 25% of how many?

4. 9 is $^3/_5$ of what number? 20% of what number?

EXERCISES.

5. 234 is $56^1/_4$% of what number?

Solution.

$234 \div 56^1/_4 = 4.16 = 1\%.$

$416 = 100\%$ or the number.

Analysis.—If 234 is $56^1/_4$% of any number, 1% of the number is such part of 234 as is found by dividing 234 by $56^1/_4$, which is 4.16 and 100%, or the number itself is 416 *Ans.*

Or, since $56^1/_4$% of any number equals $^9/_{16}$ of it, 234 is $^9/_{16}$ of the number sought. If 234 is $^9/_{16}$ of the number, the number itself is 16 times $^1/_9$ of 234 = 416 *Ans.*

Find the number of which

6. 3 is 10%
7. 20 is 20%
8. 18 is $^2/_3$%
9. 56 is $2^1/_3$%
10. 75 is 1%
11. 125 is 95%
12. 40 is $62^1/_2$%
13. 7 is $12^1/_2$%
14. 11 is $87^1/_2$%
15. 20 is $33^1/_3$%
16. $14^2/_7$ is $14^2/_7$%
17. $19^1/_3$ is $62^1/_2$%
18. 5 is 20%
19. 45 is 5%
20. 22 is $^2/_9$%
21. 2 is 80%
22. 100 is $66^2/_3$%
23. 210 is 105%
24. 65 is $14^2/_7$%
25. 16 is $33^1/_3$%
26. 35 is $41^2/_3$%
27. 525 is 25%
28. $11^1/_9$ is $^5/_9$%
29. 232 is 29%
30. 38 is $3^4/_5$%
31. $12^1/_2$ is $12^1/_2$%
32. $\frac{^1/_2}{^2/_3}$ is $16^2/_3$%
33. 99.9 is 1.75%
34. .001 is $8^2/_3$%
35. 81 is 9%
36. 195 is 200%
37. $95^1/_4$ is $8^1/_3$%
38. $^3/_8$ is 0.9%
39. 2001 is $^1/_3$ %
40. 6.25 is $37^1/_2$%
41. 7 is $2^1/_3$
42. $999^9/_{10}$ is 100%
43. $87^1/_2$ is 50%

Case IV.—To find the Base, the Rate % and the amount or difference being given.

1. The retail price of a certain article is 68¢. How much can the retailer pay for it to realize a gain of $33\frac{1}{3}\%$?

$$
\begin{array}{l}
\underline{68¢ = 133\tfrac{1}{3}\% \text{ of cost}} \\
133\tfrac{1}{3})\ \ 68 \\
\underline{\ \ 3 \qquad\ 3} \\
4\cancel{0}\cancel{0}\ \ \underline{)2.04} \\
\qquad\ \ \underline{.51 = 1\%} \\
\qquad\ \ 51. = \text{cost}
\end{array}
$$

Explanation.—If when the article sells for 68¢ there is to be a gain of $33\frac{1}{3}$ per cent., 68¢ must be $133\frac{1}{3}\%$ of the cost; hence this problem is similar to that of Case III, which is to find the base, the percentage and rate % being given.

Or, if when the article sells for 68¢ there is to be a gain of $33\frac{1}{3}\% = \frac{1}{3}$ of the cost, 68¢ must be $\frac{4}{3}$ of the cost; hence the cost must be 3 times $\frac{1}{4}$ of 68. 3 times $\frac{1}{4}$ of 68 = 51¢ *Ans.*

What number increased by

2. 10 % of itself equals 110 ?

3. 75 % of itself equals $420 ?

4. $62\frac{1}{2}\%$ of itself equals $89.37\frac{1}{2}$?

5. 21.5% of itself equals 32.562 ?

6. $83\frac{1}{3}\%$ of itself equals $87.12 ?

7. $\frac{2}{3}\%$ of itself equals 9.06 ?

8. $\frac{8}{9}\%$ of itself equals $81.72 ?

9. $\frac{9\frac{1}{2}}{25}\%$ of itself equals $90.342?

10. $43\frac{3}{4}\%$ of itself equals $1.38?

11. I am charged $2.50 for a book, which the bookseller says is $33\frac{1}{3}\%$ less than it cost him. What was the cost ?

Explanation.—If when the book sells for $2.50 there is a loss of $33\frac{1}{2}\%$, the $2.50 must be $66\frac{2}{3}\%$ of the cost; hence this problem also is similar to that of Case III.

Or, since $2.50 is $66\frac{2}{3}\%$ or $\frac{2}{3}$ of the cost, the cost must have been 3 times $\frac{1}{2}$ of $2.50 = $3.75 *Ans.*

$$
\begin{array}{l}
66\tfrac{2}{3})2.50 \\
\underline{\ \ \ 3 \qquad\ 3} \\
2\cancel{0}\cancel{0}\ \ \ )7.50 \\
\qquad\ \underline{.0375 = 1\%} \\
\qquad\ \ 3.75 = 100\%
\end{array}
$$

What number diminished by

12. 5 % of itself equals $6.65 ?

13. 5 % of itself equals 19 ?

14. 20 % of itself equals 80 ?

15. 9 % of itself equals $9\frac{1}{10}$?

16. $87\frac{1}{2}\%$ of itself equals 10 ?

17. $16\frac{2}{3}\%$ of itself equals $95\frac{5}{18}$?

18. $5\frac{5}{8}\%$ of itself equals 67.95 ?

19. $\frac{2}{7}\%$ of itself equals 216.38 ?

Note.—No special rule is needed for Case IV, the process of solution being the same as that of Case III.

Applications.—1. William buys a penknife for 20¢ and sells it to James for 25¢, and James sells it to Fred for 20¢. What per cent. does William gain, and what per cent. does James lose?

2. If the 25 minutes of school time given to recesses are $8\frac{1}{3}\%$ of the daily session, how many hours in the session?

3. If a book is marked to be sold at 25% above cost, but it is sold at 20% below the marked price, what was the gain or loss per cent.?

4. If 80 pounds of coffee are exchanged for 120 pounds of sugar, what % is the coffee worth per pound more than the sugar?

5. What per cent. do I gain by selling an article for $3 for which I paid $2.25? What per cent. do I lose by buying an article for $3 and selling it for $2.25?

6. A drover sold a horse for $226, and thus gained 25%. What did he pay for him?

7. The assets of a business man are $135,700, which sum is 43% of his debts. What is his indebtedness?

8. A fruit dealer sold a lot of oranges for $337.50, which allowed him a profit of $12\frac{1}{2}\%$. What did he pay for them?

9. A city lot was sold for $25,500, the gain on the cost being 325%. What was the cost?

10. A grocer sold 300 bushels of potatoes for $285, which was $16\frac{2}{3}\%$ less than he had paid for them. How much did they cost him per bushel?

11. A. sold goods at a gain of 18%. His profit was $29.70. How much did he sell them for?

12. By selling a lot of goods for $380, I gain 3 times the per cent. that would be gained by selling them for $340. What per cent. is gained in the latter case? ($380 – $340 = 2 times the gain.)

13. In the schools of a village yesterday there were 1235 pupils present, which was 95% of the whole number belonging. How many belonged to the schools?

Trade Discount.

285. A ***discount*** is a deduction from a price, from the amount of a bill, or other account.

286. In some branches of business it is customary to have fixed price lists of certain kinds of goods, and, when a rise or fall of prices occurs, instead of changing every price on a long list, the rate of discount is changed.

287. The fixed price is called the ***List Price,*** and the discount is called ***Trade Discount.*** The ***Net Price*** is the list price minus the discount.

Example.—1. If penknives of a certain quality are sold at \$18 per doz., with a discount of $33\frac{1}{3}\%$, what is the net price?

2. How much must be paid on a bill of \$5560 for books if 20% discount is allowed on account of the great number of books sold, and a second discount of 5% is made for cash?

Each successive discount is made from the results of preceding discounts.

Find the net prices:

	List prices.	Discounts.		List prices.	Discounts.
3.	\$5.40	25% and 10%	8.	\$5.37	25% and 33%
4.	\$6.56	40% " 20%	9.	\$4.82	40% " 30%
5.	\$8.35	60% " 5%	10.	\$6.72	30%, 10%, and 5%
6.	\$7.80	50% " 30%	11.	\$3.98	40%, 20%, " 10%
7.	\$6.75	10% " 10%	12.	\$4.97	50%, 10%, " 10%

Note.—To find a single direct rate of discount equivalent to two successive discounts, deduct from the sum of the two rates either per cent. of the other.

Thus: 60 and 10 off $= 60 + 10 - 10\%$ of $60 = 70 - 6 = 64$.

---

13. A bill of hardware at list prices amounts to \$276.98, the discounts are 40%, $12\frac{1}{2}\%$, and 10%. What is due on the bill?

14. What is the difference on a bill of \$780, between a direct discount of 25% and successive discounts of 10%, 10%, and 5%?

15. If the list price of a certain size and quality of slates is \$12 per gross, shall I gain or lose by buying 15 gross of Mr. Brown, whose discounts are 25% and 10%, instead of from Mr. Green, whose discounts are 20% and 10% and 5%?

Insurance.

Insurance is security against loss by fire, water, accident, etc.

Life Insurance is a contract for the payment of a specified sum at the death of the insured or at the end of a specified time, though he may be still living.

The ***Premium*** is the sum paid for insurance. It is usually computed at a given rate per cent. on the sum insured.

The ***Policy*** is the written contract between the insurer and the insured.

The insurer is called an ***Underwriter***, because his name is written under the policy.

ORAL EXERCISES.

1. What will be the premium, if I insure my house for \$2000 at 1%? At $^1/_2$%? At 2%? At $^7/_8$%?

2. What is the premium on an insurance of \$600, \$400, \$800, \$1900, \$2400, \$100,000, at 1%? At 2%? At $^1/_2$%? At $1^1/_2$%? At $1^1/_4$%? At $2^1/_2$%? At $^3/_4$%?

3. A vessel is insured for \$45960 at $^1/_4$%. Find the premium.

WRITTEN EXERCISES.

4. A match factory is insured at $4^1/_2$%; the premium being \$217.50, for how much is it insured?

5. A barn was insured at the rate of $^3/_4$%; the premium was \$19.50. What did the owner receive when it was burned?

6. At a rate of 5%, a shipper pays \$213.95 for the insurance of $^3/_4$ of the value of his goods. What was their value?

7. Find the the sum of the premiums paid for the following insurances: \$4000 at $^5/_8$% for 1 year, \$3200 at $1^1/_4$% for 2 years, \$5000 at $1^1/_2$% for 3 years, \$2500 at $2^1/_2$% for 4 years, \$3500 at 2% for 3 years, \$2200 at $^7/_8$% for 1 year, \$5400 at $^3/_5$% for 1 year, \$3600 at $2^1/_4$% for 5 years, \$4700 at $1^1/_4$% for 2 years. (The rates here given are not annual, but for the times specified.)

Note.—Great risks are commonly distributed in small amounts to many different companies. (Why?)

8. A building is insured in 19 companies for \$2500 each, in 9 others for \$5000 each, and in 4 others for \$3500 each. What was the total annual premium at $^3/_5$%?

9. The goods in the building just mentioned were insured as follows: in 1 company for \$10000, in 1 for \$9000, in 16 for \$2500 each, in 7 for \$3500 each, in 4 for \$1500 each, and in 1 for \$1000. What was the total premium paid annually at 75¢ (per \$100)? (75¢ per \$100 = .75% or $^3/_4$%.)

10. What is the rate at which a factory is insured for \$5250, if the premium is \$65.62$^1/_2$?

11. The Ohio Mutual Insurance Company insured my house for \$5800 for a period of 3 years at 1$^1/_4$%. What was the premium?

12. The cargo of steamer Gallion, bound for Liverpool, is insured at $^1/_2$%. For what sum is it insured, the premium being \$1500?

13. My house cost me \$8400. I insured it for $^3/_4$ of its value, at $^2/_3$% per year. My books and furniture were insured for \$3000 at the same rate. What did I pay annually for insurance on both?

14. If you have your life insured for \$5000 at \$15.50 on \$1000 annually, what premium do you pay?

15. When 30 years of age a man insures his life for \$8500, at the annual rate of \$22.70 on \$1000. If he dies when 60 years old, how much more do his heirs receive than he had paid for insurance?

16. Suppose the man above mentioned had been insured from his 20th to his 60th birthday, how much would the sum of his annual premiums have fallen short of the sum insured, the rate being 1$^1/_2$%?

17. A manufacturing company paid \$214.80 premium for insurance on $^3/_4$ of the cost of its buildings and machinery, at 60¢ per \$100. What was their cost?

18. If in 1 year an insurance company takes the insurance of 1000 dwellings at $^2/_3$% on an average valuation of \$3000, and pays to its agents 15% of the amount received for premiums, what balance remains for profit and to meet the expenses of the company if 1 of the houses is totally destroyed by fire?

Commission and Brokerage.

An ***Agent*** is a person who is authorized to transact business for another. The person for whom he acts is his ***Principal.***

Commission is the allowance made to an agent for transacting the business of another. It is usually reckoned at a certain rate per cent. on the sum of money invested or realized, sometimes at a certain rate per bushel, barrel, bale, etc., bought or sold.

Agents are known by various names, as, commission merchants, brokers, collectors, correspondents, etc., according to the nature of their business.

A ***Broker*** effects bargains and contracts for and generally in the name of others. The broker does not take possession of the property bought or sold. The name of broker is erroneously applied to those who deal in stocks, bonds, etc., on their own account.

The commission allowed to a broker is called ***Brokerage.***

A ***Commission Merchant*** buys and sells on account of others, but in his own name. He has the merchandise in which he deals within his immediate control. His commission is usually greater than that of a broker.

A ***Consignment*** is a quantity of merchandise sent by one party to another. The one that sends it is called the ***Consignor;*** the one to whom it is sent is called the ***Consignee.***

The ***Gross proceeds*** of a consignment is the whole amount for which it is sold; the ***Net proceeds*** is the sum due the consignor after deducting commission and all other charges.

The calculations in commission and brokerage are simple applications of the rules of percentage.

ORAL EXERCISES.

1. If my commission for selling an article for \$450 is 4%, how much do I receive? How much at $4\frac{1}{2}$%? At 5%? At 15%? At 25%?

2. An agent sold a piano at \$350, and received \$35 commission; what rate per cent. is that? What rate if he had received \$14? If \$17.50?

3. What will be the fees of a collector of taxes on \$1,200,000 if allowed $1\frac{1}{2}$%? If $1\frac{1}{4}$%? If $1\frac{3}{4}$%?

4. Find the commission on \$200, \$220, \$250, \$300, \$580, at $\frac{1}{4}$%, $\frac{1}{8}$%.

5. A broker buys 5 tons of currants at \$8.50 per cwt. What is his brokerage at 2%?

WRITTEN EXERCISES.

6. An agent purchases 5 tons of raw sugar at $5\frac{1}{2}$¢ a pound, and charges $2\frac{1}{2}$% commission. How much money must be sent to him to cover the cost and commission?

7. I sell through my broker 7 tons of Brazil nuts at \$7.50 per cwt. How much do I receive if the broker charges 1% for selling?

8. A broker charged \$74.25 for effecting a loan of \$3300. What was the charge per cent.?

9. A fruit broker sold \$680 worth of apples, and after deducting 5% commission and 20% for freight and other charges, invested the balance in oranges. How much did he invest in oranges if he charged 2% for buying?

Explanation.—Charges and commission, together amounting to 25% of the whole sum received for the apples, being deducted from \$680, there is a remainder of \$510, with which the broker is to buy oranges and pay himself 2% on the *purchase price.*

If now the broker were to buy a dollar's worth of oranges at a time, and each time to pay himself 2¢, it is plain that he would expend for oranges only as many times \$1 as there are times \$1.02 in \$510.

10. John Wells & Co. sell \$150 worth of eggs for W. Smith, charging him $2\frac{1}{2}$% commission. They invest the proceeds in groceries, and charge 2% for buying. How much do they invest?

11. A shoe manufacturer forwarded 50 dozen pairs of shoes to his agent in New York, who sold them at \$42.60 per doz., charging 5% commission. He purchased leather with the proceeds, charging 2% for buying. What was his total commission? How much did he pay for the leather?

12. A cotton dealer in New Orleans ships \$10,000 worth of cotton to his broker in New York, with instructions to purchase dry goods and hardware with the proceeds. The broker charges $2\frac{1}{2}$% for selling the cotton and 2% for buying. How much does he invest, and what is his total commission?

13. A commission merchant having sold a consignment for \$3578, retains \$95.70 to pay charges amounting to \$6.25 and his own commission. What rate per cent. commission did he charge?

14. A commission merchant sold 500 lb. of butter at 18¢ per pound, and invested the proceeds in oats at 42¢ a bushel. He charged $4\frac{1}{2}$% for selling and $1\frac{1}{2}$% for buying. What was his total commission, and how many bushels of oats did he buy?

15. Sold a consignment of merchandise for $5000. What was the balance due the consignor after the deduction of $110.50 freight, $250 duty, cartage, and storage, $75.40 insurance, and 5% brokerage?

Stocks.

288. 1. Here the pupil needs to know what stocks are, and the meaning of some of the more common expressions used in relation to them. To this end the following illustration will serve better than mere definitions.

2. Suppose that the citizens of a town desire to have gas-light in their streets and houses, and that about $50,000 will be needed to construct the necessary works.

3. No one person or private company will be willing to risk so much money in the experiment, but if 500 persons will take a ***share*** in the enterprise and each put in $100, the plan can be carried out.

4. A subscription is started, and it is found that many are willing to put in $100, and that some are ready to take two or more shares, and possibly one or two who will take a hundred shares each. Thus it is found that the $50,000 can be raised.

5. The subscribers then obtain a charter, or legal authority to act as a company, and appoint a ***Board of Directors***, each subscriber having one vote in the election for each share of stock he has taken.

6. As soon as the company is ready for business the Board of Directors calls for the payment of the subscriptions, either at once or by instalments as the money is needed, until they are all paid.

289. A ***certificate of stock*** is now given to each ***stock*** or ***shareholder***, showing the number of shares he has taken and the *price paid per share.* The latter is called its ***face*** or ***par value.***

290. If the company is prosperous and pays in ***dividends*** (division of profits) more than the money could earn in other ways, the stock will be at a ***premium***, that is, worth more than its *par value;* but if the dividends are small, a share will be worth less than $100; then it will be said to be ***below par***, or at a ***discount.***

This is an illustration on a very small scale. The stocks of all the incorporated companies in the United States amount to more than a thousand millions of dollars, nd there are many men engaged in buying and selling them in all the great cities.

291. A ***stock broker*** is one who buys and sells stocks for others. For buying or selling stocks in the New York Stock Exchange the regular charge is $^1/_8$ of 1% on their *par value.*

292. A ***stock jobber*** buys and sells stocks on his own account.

The following report of the number of shares of certain stocks sold and the highest prices paid for them at the New York Stock Exchange, November 16, 1885, is taken from a long list to be found in the papers of the following day:

	Sales.	Highest.		Sales.	Highest.
Adams Express......	10	$142\,^1/_2$	Central Iowa.......	1700	$22\,^1/_4$
Atlantic & Pacific....	850	$10\,^3/_8$	Central Pacific.....	900	$47\,^1/_4$
Alb. & Susq........	15	140	C., C., C. & I.......	800	$64\,^1/_2$
Atch., T. & S. Fé....	100	88	Chic., B. & Q......	1500	$135\,^1/_2$
Canadian Pacific.....	1900	$55\,^1/_4$	Chicago & N. W....	15,580	$115\,^1/_4$

Note.—In the following problems it is supposed that all the transactions take place through brokers, in behalf of *outside* parties. Hence the person for whom stock is bought must pay the price of the stock plus the brokerage, and the seller will receive the price for which it is sold minus the brokerage.

Examples.—At the above quotations:

1. How much did the buyers pay for the several stocks sold above par, including brokerage?

Adams Express.

Solution.—10 shares at $142\,^1/_2$ per share	=	$1425
Brokerage at $^1/_8$%,		1.25
Cost of stock,		$1426.25 *Ans.*

2. How much did owners receive for the several stocks sold below par, brokerage being deducted?

3. What was the brokerage, at the usual rate, on the purchase of 1500 shares of Chic., B. & Q.?

4. How much did the buyer pay and how much did seller receive for 1900 shares of Canadian Pacific?

5. If I give my broker orders to sell 800 Central Iowa, and buy 50 Adams Express, what balance will he put to my credit after deducting brokerage on both sale and purchase?

6. How many shares of C., C., C. & I. could be bought for $10,381, including brokerage? What balance would remain?

Taxes.

293. A tax is a sum of money assessed on the income, person, or property of individuals for public purposes.

294. A tax on property is called a ***Property-Tax,*** that on the income is called an ***Income-Tax,*** that upon the person, a ***Poll-Tax*** or capitation-tax.

295. Fixed property, such as lands, houses, etc., is called ***Real Estate.*** Movable property, such as furniture, money, cattle, merchandise, etc., is ***Personal Property.***

296. The persons elected or appointed to estimate the value of property to be taxed are called ***Assessors*** or ***Appraisers.***

Example.—1. The sum of money to be raised by taxation in a certain city is $562,600, the total appraised value of the property is $44,800,000, and there are 25,000 persons subject to a poll-tax of $1 each. How much will Mr. Hunter have to pay, whose property is valued at $2560? *Ans.* $30.72.

Solution.—*First Step.*—Subtract the $25,000 to be received on the polls from the sum to be levied; the remainder will be the tax on property. $562,600 − $25,000 = ?

Second Step.—Divide the tax on property by the total appraised value of the property to find the tax on $1. $537,600 ÷ $44,800,000 = ?

The ***rate of taxation*** being thus found, the tax on Mr. Hunter's property is readily ascertained. To the property-tax must be added his poll-tax.

After the rate is determined, as above, the computation of the tax to be paid by each individual is greatly facilitated by a table like the following:

TABLE SHOWING TAXES AT THE RATE OF 12 MILLS ON $1.

$1	pays	$0.012	$10	pay	$0.12	$100	pay	$1.20
$2	pay	$0.024	$20	"	$0.24	$200	"	$2.40
$3	"	$0.036	$30	"	$0.36	$300	"	$3.60
$4	"	$0.048	$40	"	$0.48	$400	"	$4.80
$5	"	$0.060	$50	"	$0.60	$500	"	$6.00
$6	"	$0.072	$60	"	$0.72	$600	"	$7.20
$7	"	$0.084	$70	"	$0.84	$700	"	$8.40
$8	"	$0.096	$80	"	$0.96	$800	"	$9.60
$9	"	$0.108	$90	"	$1.08	$900	"	$10.80

The tax on $2, $3, etc., being found by multiplying the rate by 2, by 3, etc., the rates for $10, $20, etc., are found by removing the decimal points of the first column one place to the right, and for $100, $200, etc., two places, etc.

2. Find the amount of taxes Mr. A. has to pay on property assessed at \$2475.

\$2000 =	\$24.00
400 =	4.80
70 =	0.84
5 =	0.06
\$2475 =	\$29.70

Explanation.—From such a table as the above we would take \$24.00, the tax on \$2000, \$4.80, the tax on \$400, \$.84, the tax on \$70, and .06, the tax on \$5, and adding these together we would find Mr. A.'s tax on his property to be \$29.70.

3. My real estate is estimated at \$4500, my personal property at \$1345, and I have to pay \$2 poll-tax. How much tax will I have to pay, the rate being $12\frac{1}{2}$ mills on the dollar?

4. Find the amount of taxes my neighbor will have to pay on \$9876, and \$1 poll-tax. Same rate.

5. Find the amount of taxes my three neighbors across the street pay, at the same rate, on \$2732, \$3695, \$8351; each paying \$1 poll-tax.

6. Find how much a non-resident must pay on his real estate, which is listed at \$6129. (No poll-tax.)

7. A person has to pay \$100.20 taxes at 12 mills on the dollar, there being no poll-tax? What is the assessed value of his property?

8. Suppose my property, real and personal, to be listed at \$1500, and that I have to pay 3 mills on the dollar for state purposes, 2 mills for county purposes, 2 mills for township purposes, 5 mills extra for school purposes, and $2\frac{1}{2}$ mills for corporation (village) expenses; how much in all would I have to pay?

9. In a state of Europe 1% is required to be paid on incomes from \$100 to \$300, $1\frac{1}{2}$% on incomes from \$300 to \$500, and $2\frac{1}{2}$% on incomes from \$500 to \$800. Mr. A.'s income was \$450, Mr. B.'s \$175, Mr. C.'s \$760. What income-tax did each have to pay?

10. If my property is valued at \$2500, and the rate of taxation for school purposes is 5 mills on the dollar, what does the tuition of each one of my three children cost me if all of them attend the public schools?

11. Allowing 5% for taxes uncollectible, and 2% for collection, what sum must be levied that $50,000 may be realized for the building of a school-house?

$1 must be *collected* for every 98¢ needed for the school-house, because 2¢ out of every hundred go to the collector, and $1 must be *levied* for every 95¢ supposed to be collectible, since those who do not pay will keep back 5¢ of every hundred levied.

12. The people of Abdera wish to levy a tax which will net them $18,979, after paying the expense of collection, which will be 3%. The assessed value of the real and personal property is $1,260,000, and there are 323 polls, each taxed $2. How much will $1 be assessed?

13. Make out a table similar to that on page 292.

From the table (Ex. 13) find how much

14. Mr. W. M. Hart	pays on $6000	17. Mr. H. Kidd	pays on $10000
15. Mr. John Hanly	" " $5583	18. Mr. L. B. Pease	" " $7534
16. Mr. E. G. Eliot	" " $5354	19. Mr. R. J. Luck	" " $5821

20. For the purpose of building a town-house, a tax of $15,961.60 is to be levied on property valued at $1,856,000. What will be the tax on Mr. Burns' property, which is valued at $8650?

21. A bridge costing $18,135 was built by the proceeds of a tax levied upon the property of a town, the rate of taxation being 50¢ on $100 (5 mills on $1), the cost of collection being $2\frac{1}{2}$%. What was the assessed valuation of the property?

22. If the assessed value of the real and personal property of a city is $80,000,000, and a special tax is desired for the construction of sewers, what must be the rate of levy to realize $188,160 for the purpose, if 2% be allowed for collection and 4% of the levy be uncollectible?

Note.—The answers given to problems such as the preceding ones are based on the method of analysis given under Example 11, but, since the amount of tax uncollectible can never be known beforehand, the sum to be assessed for any given use can be determined with sufficient exactness by adding to the sum needed the estimated percentage of taxes uncollectible and the percentage charged for collection. In States where the collector is paid a fixed salary, the cost of collection would not be taken into account.

Miscellaneous Problems in Percentage.

1. Of 480 persons in a village, 30 moved away within one year. What per cent. of the whole number remained?

2. If two hundred pounds of wheat make 150 lb. of flour, what per cent. of the weight of the wheat is the weight of the flour?

3. Twenty pounds of coffee lose $4\frac{4}{5}$ lb. in weight by roasting; what %?

4. A village of 1250 inhabitants has 200 children attending school. What per cent. of the whole population in school?

5. A person paid $\$22\frac{1}{2}$ tax on his income at the rate of $1\frac{1}{2}\%$. What was his income?

6. A house was sold by an agent for $5600. The agent's commission was $1\frac{1}{2}\%$. How much did the owner receive?

7. A real estate agent collects rents as follows:

For Mr. Williams,	$2384.20	John Jones,	$936.18	Mr. Cook,	$786.15
" Mr. Johnson,	856.75	Henry Jones,	1852.00	Mr. Doan,	885.

What is the amount of his commissions at 3 per cent.?

8. One and a quarter per cent. of the inhabitants of the kingdom of Prussia are annually called into military service. How many men do the city of Breslau, with 240,000 inhabitants, and the city of Hildesheim, with 23,000 inhabitants, have to furnish?

9. A farmer bought a team of horses, but could pay only $155 in cash, $37\frac{1}{2}\%$ remaining unpaid. What was the price of the horses?

10. An inspector of coal mines, having a salary of $2400 a year, pays $560 rent for house and barn, $1\frac{1}{2}\%$ taxes on an assessment of $480, and $\frac{1}{2}\%$ for insurance of books and furniture valued at $1250. What % of his salary does he pay for rent, taxes, and insurance, respectively?

11. On one occasion the price of a barrel of petroleum fell from 90¢ to 78¢. What per cent. was the decline? Shortly after the price rose again to 90¢. What per cent. was the advance?

12. Twenty pounds flax, when spun, make $17^3/_5$ lb. of yarn. What per cent.?

13. Eight pounds of beef are reduced 1 lb. in weight by boiling and $1^2/_5$ lb. by roasting. What per cent. of weight is lost by each process?

14. If a single railway fare to the city is 30¢, what per cent. would I save by the purchase of 100 tickets for $20?

15. The Union Steel Screw Co. declared a dividend of $17^3/_4$% upon its stock. What did stockholders receive who had respectively $900, $2000, $4700, $2300, and $1100 worth of stock?

16. A merchant sends out bills for collection as follows:

$184.75	$57.61	$384.21	$728.13
136.54	98.13	17.86	564.21
19.81	156.22	918.54	1986.54
5.78	7.61	12.32	.95
846.00	887.60	50.65	18.70

The collector receives 6% on all sums less than $100, 4% on amounts from $100 to $500, and 2% on all sums greater than $500. What will be his commission if all are collected?

17. Church bells commonly contain 80% of copper, 5.6% of zinc, 10.1% of tin, and the rest is lead. At that rate, how much of each is contained in the great bell at Moscow, which weighs 443,772 pounds? What per cent. is lead?

18. A carpet dealer reduced the price of certain goods $12^1/_2$%, which amounted to 12¢ on the yard. What did the goods sell at per yard before and after the reduction?

19. Twelve quarts of good milk will give $1^1/_5$ qt. of cream. What per cent. of the milk is cream?

20. The liabilities of a bankrupt merchant are $7200, his assets only $3200. How much will his creditors get, to whom he owes respectively $2572, $856, $782, $1025, $1912, and $53?

21. The daily wages of a workman were increased 25¢, or $6^2/_3$%. What did he get before and after the increase?

22. Charles has a salary of $750, his brother $850. What % does his brother receive more than he? By what per cent. is Charles's salary less than his brother's?

23. If eight pounds of imperial tea may be had for $9, and a single lb. of the same kind costs $1.20, what is the per cent. saved by buying 8 lb. at a time?

24. A gentleman is insured for $5000. His premium is $96.25. How much does he pay on a thousand? What per cent.?

25. The butcher estimates a beef to weigh 980 pounds, of which 57% is salable as meat and $6\frac{1}{4}$% tallow. What is the weight of the meat and tallow together?

26. Colonel A. has to pay $1200 rent, Major B. $1000. The owner of the two houses raises the rent $100 on each. What per cent. is the major's rent raised more than that of the colonel?

27. The weight of a cubic foot of

American pine,	when	green,	is 44.75,	when	seasoned,	30.7.
Ash	"	"	" 58.18	"	"	50.
Beech	"	"	" 60	"	"	53.37.
Cedar	"	"	" 32	"	"	28.25.
English oak	"	"	" 71.6	"	"	43.5.

What per cent. does each lose in weight by being seasoned?

28. What per cent. income does Mr. Abel have more than Mr. Bain, if A. has $2500 and B. $2000? If A. has $3000 and B. $2500? If A. has $2000 and B. $1500? If A. has $1750 and B. $1250? If A. has $600 and B. $100?

29. Find what per cent. the lower income is of the higher in each of the above-mentioned cases.

30. Anna bought 8 yd. of tape for 5¢; Emma, 25 yd. for 15¢. What per cent. did Anna pay per yard more than Emma? What per cent. did Emma pay per yard less than Anna?

31. Five men in a factory accomplish as much work as 8 boys. What per cent. of a man's work does a boy do? What per cent. of a boy's work does a man accomplish?

32. If beech timber is worth $16^2/_3\%$ more than pine, what is the value of 5 cords of the former when 3 cords of the latter is worth $12 ?

33. In 1885 there were 90,920,707 shares of stock bought and sold in the New York Stock Exchange. What did the brokerage amount to at $^1/_8$¢ per share for both sale and purchase ?

34. In 1885 the highest price paid per share for Manhattan Consolidated stock was $123^1/_2$, the lowest, 65 ; for Louisville and Nashville, highest, $51^3/_4$, lowest, 22 ; for Pacific Mail, highest, 70, lowest, $46^3/_4$. What per cent. would have been lost by buying at the highest and selling at the lowest rate in each case ?

35. If a boy buys 5 tops and sells 4 for as much as the 5 cost him, what per cent. does he gain on the tops sold ? What per cent. would he gain on the tops sold if he sold 3 for what 4 cost ? If he sold 2 for what 3 cost ? If he sold 1 for what 2 cost ?

36. What per cent. would the boy lose if he sold the 5 for what 4 cost ? What per cent. would he lose if he sold 5 for what 3 cost ? 5 for what 2 cost ? 5 for what 1 cost ?

37. In 1885 there were 12,480,423 shares of C., M. and St. P. reported to have been bought and sold in the New York Stock Exchange. What did the commissions of the brokers amount to at $^1/_8\%$ commission for buying and selling ?

38. If 18 horses draw as much as 30 oxen, what per cent. less than a horse does an ox draw ? What per cent. does a horse draw more than an ox ?

39. In the schools of a city there are in the first year's course 8666 pupils ; in the second, 3205 ; in the third, 3960 ; in the fourth, 2456 ; in the fifth, 2012 ; in the sixth, 1125 ; in the seventh, 654 ; in the eighth, 640. What per cent. of the whole number in each ? (What per cent. in all ?)

40. A merchant had marked some calicoes 36¢ per yard, which was 20% above cost, but finally sells them at $33^1/_3\%$ below the marked price. What per cent. on first cost does he lose ?

41. Mr. Williams has an insurance on his life for \$6000. His annual premium is \$185, but each year he has the benefit of a dividend of 40% on the premium of the preceding year. What is the net cost of his insurance per year? (40% of \$185 = ? \$185 minus dividend = ?)

42. In consequence of a rise in the market, a merchant marks up calicoes 10%, which were already marked to sell at 25% advance on cost. What per cent. advance on first cost is the latter price?

43. 28,000 bricks are needed for a building. How many have to be ordered, if $6^2/_3$% be allowed for waste?

	Selling price.	Gain per cent.		Selling price.	Loss per cent.
44.	\$155.00	$3^1/_3$%	**50.**	\$6.80	25%
45.	\$110.00	25%	**51.**	\$117.00	$16^2/_3$%
46.	\$234.00	4%	**52.**	\$25.50	$3^1/_3$%
47.	\$95.00	$66^2/_3$%	**53.**	\$30.00	$66^2/_3$%
48.	\$187.00	10%	**54.**	\$3.20	$8^1/_3$%
49.	\$5.60	12%	**55.**	\$72.00	$8^1/_3$%

What was the cost? What was the cost?

56. A merchant sold $^1/_4$ of a certain lot of goods at 10% profit, $^1/_3$ at 20% profit, and $^1/_6$ at 15% profit. The remainder, on which he lost 5%, he sold for $\$142^1/_2$. How much did he get for the whole?

What per cent. is gained or lost when I buy

57.	For \$5	and sell for	\$7?	**61.**	For 20¢	and sell for	23¢?
58.	" \$30	" "	\$45?	**62.**	" 4¢	" "	3¢?
59.	" \$25	" "	\$21?	**63.**	" \$35	" "	\$42?
60.	" \$40	" "	\$36?	**64.**	" \$15	" "	$\$13^1/_2$?

65. Mr. James sent his check for \$2500 to a broker, with instructions to buy good stocks. The broker bought bank stock, then selling at $105^1/_2$ per share (par value, 100). How many shares did he buy? What sum remained to Mr. James's credit after deducting the broker's commission?

Original Problems.

1. Construct five problems, each pertaining to business transactions:

   1. Giving a base and rate per cent., to find the percentage.
   2. Giving base and percentage, to find the rate.
   3. Giving rate and percentage, to find the base.
   4. Giving a rate and amount, to find the base.
   5. Giving rate and difference, to find the base.

2. Get from some friend engaged in any line of business a statement of some transaction requiring a calculation in percentage, and form a proper question for the class. This may be in the line of trade discount, insurance, taxes, etc., etc.

3. Find in the daily papers statements of stock sales. They will furnish a great variety of problems.

4. On occasion of a fire in your city or neighborhood, ascertain the facts concerning insurance, and inquire what advantage was gained by insurance, if any, or what loss resulted from failure to insure.

5. Construct a problem in taxation on the model of any one or more of those found on page 294.

6. Suppose yourself a commission merchant buying or making sales, or both, for some party in a distant city. Your supposition may range from a transaction in a few bushels of hickory nuts to millions of bushels of grain.

7. Suppose the case of a broker buying and selling real estate for yourself, and ask his commission on various imaginary transactions.

8. Find, if you can, a list price of books, magazines, or other articles of merchandise, with the discounts given on the same, and ask the rates per cent. gained by dealers.

9. Tell the class the number of different boys absent from school during any week or month, and ask what per cent. of the ole number of scholars were absentees.

CHAPTER XV.

INTEREST.

297. ***Interest*** is compensation for the use of money.

We pay *rent* for the use of a house, *hire* for the use of a horse, *interest* for the use of money.

298. The ***Principal*** is the sum of money, for the use of which interest is paid.

299. The ***Rate of Interest*** is the rate per cent. allowed for the use of money for one year or other specified time.

Note.—Any given rate is understood to be the rate for one year, unless the time be specified; as, per month, per day, etc.

In this book, when no rate is mentioned, 6% is understood. Thus, in the question "What is the interest of $50 for 6 months?" *6% per annum* is understood.

300. ***Legal Interest*** is a rate fixed by law for cases in which no rate is specified in the agreement between the parties interested.

In most of the States a limit is fixed to the rate of interest which may be received. Interest at a rate exceeding the limit allowed by law is termed ***usury,*** to which some penalty is usually attached.

301. The ***Amount*** is the sum of the principal and interest.

If we hire money we return it and pay interest for the use of it, as we return a hired horse and pay for his use.

302. ***Simple Interest*** is interest on the principal alone, and is payable with the principal.

303. In the common method of computing interest, 12 months of 30 days each or 360 days are reckoned as 1 year.

Note.—Though this method of reckoning time is not exact, it is the most common because it is the most convenient, and in most States it is legalized by statute.

ORAL EXERCISES.

1. Find the interest of \$600 at 5% for 1 year.

Analysis.—The interest of \$600 for 1 year at 1% is \$6, at 5% it is 5 times \$6, or \$30.

2. What must be paid for the use of \$800 for $1^1/_2$ years at 6% ?

Analysis.—The interest of \$800 for 1 year at 1% is \$8. For $1^1/_2$ years it is $1^1/_2$ times 8 = \$12, and at 6% it is 6 times \$12 = \$72.

3. Find the interest of \$200 for 1 year at 4%; of \$500 at $3^1/_3$%; of \$800 at 6%; of \$700 at 8%; of \$1000 at 10%; of \$400 at 5%.

4. Find the interest of \$400 for 2 years at $3^1/_2$%; of \$500 at $4^1/_2$%; of \$600 at $7^1/_2$%.

5. Find the interest for 6 months of \$1000 at 5% per annum; of \$150 at $4^1/_2$%; of \$275 at 4%; of \$1000 at $4^1/_2$%.

SLATE EXERCISES.

1. What is the interest of \$1230 for 1 year? **(What rate is here understood?)**

2. Find the interest of \$120 at $4^1/_2$%; of \$140 at $3^1/_2$%; of \$260 at 5%; for 3 years.

3. What is the interest of \$160 at $6^1/_4$% for 3 years?

Analysis.—The interest of \$160 at 1% for 1 year is \$1.60, at $6^1/_4$% it is $6^1/_4$ times \$1.60, or \$10, and for 3 years it is 3 times \$10, or \$30.

Find the interest for one year of

4. \$450 at $4^1/_2$%	9. \$2630 at $4^1/_2$%	14. \$7428 at $5^1/_2$%
5. \$680 at $3^1/_2$%	10. \$4920 at 5%	15. \$9654 at 6%
6. \$960 at $7^1/_2$%	11. \$5000 at $3^3/_4$%	16. \$7851 at $6^1/_2$%
7. \$840 at $5^1/_2$%	12. \$3720 at $3^1/_2$%	17. \$9643 at 7%
8. \$1720 at $6^1/_2$%	13. \$4680 at $4^1/_2$%	18. \$5430 at 5%

19. How much must be paid for the use of \$80 for 9 months at 5%?

Analysis.—The interest of \$80 for 1 year at 1% is 80¢, at 5% it is 5 times 80¢ = \$4, and for 9 months it is $^3/_4$ of \$4 = \$3.

Find the interest of

20. \$72 at 5% for 4 years.
21. \$70 at $4\frac{1}{2}$% for 5 years.
22. \$84 at $4\frac{1}{6}$% for $2\frac{1}{2}$ years.
23. \$97 at $6\frac{1}{4}$% for $3\frac{1}{3}$ years.
24. \$56 at 7% for $5\frac{1}{4}$ years.
25. \$675 at $3\frac{1}{3}$% for 6 months.
26. \$780 at 5% for 5 months.
27. \$825 at 4% for 7 months.

28. Find the interest of \$228.50 for 5 mo. 18 d.

Solution.

\$2.285	Int. for 1 yr. at 1%.
6	
\$13.710	" " " " 6%.
168	
360)2303.280	
\$6.398	" " 5 mo. 18 d. = 168 d.

Since 360 d. are reckoned as 1 year, the interest for 168 d. is $^{168}/_{360}$ of the interest of 1 year. Hence, we multiply \$13.71, the interest for one year, by 168, and divide the product by 360, as above.

Cancellation.—If we desired to shorten the work by cancellation, we would arrange all the factors of the dividend on the right side of a vertical line, and place the divisor on the left, and cancel as follows:

$$\begin{array}{r|rr} & \cancel{\$2.285} & \$.457 \\ \cancel{5}\quad \cancel{60}\quad \cancel{360} & \cancel{6} & \\ & \cancel{168} & 14 \end{array}$$

$$14 \times \$.457 = \$6.398\ \textit{Ans.}$$

304. The computation of interest as already presented does not involve any process or principle with which the pupil is not entirely familiar. They are such as would be adopted by any one acquainted with the elements of arithmetic, though he might not have had any instruction in "Interest" as taught in the books. It is therefore sometimes called the ***general method.*** But any one who wishes to become expert in computing interest without the aid of Tables should be able to use shorter methods.

The 60 Day Method.

305. At the rate of 6% per year, the rate for 2 months or 60 days is 1%, which is equal to .01 of the principal; and for 6 days it is $1/10$ of 1%, which is equal to .001 of the principal.

Example.—1. What is the interest of $562 for 2 mo. 6 d.?

Solution.

Int. of $562 for 2 mo. =	$5.62
" " " 6 d. =	.562
2 mo. 6 d.	$6.18

2. Find the interest of $328, $532, $690, $1085, $52, $780, and $630 each, for 2 mo. 6 d. at 6% per annum.

306. Thus we see that when the rate of interest is 6% per year we may find the interest of any principal for two months by removing the decimal point *two* places to the left, and for six days by removing it *three* places to the left, prefixing ciphers when needed.

307. By taking such multiples and parts of these results as the given time requires, and adding them together, the interest may be found for any given time.

3. What is the interest of $280 for 9 mo. 15 d.?

Solution.

2 mo.'s int., $2.80; 6 d.'s int., $.280.

$8.40	Int. for 6 mo. (3 × 2 mo.)
4.20	" " 3 mo. (1/2 of 2 mo.)
.56	" " 12 d. (2 × 6 d.)
.14	" " 3 d. (1/2 of 6 d.)
$13.30	" " 9 mo. 15 d. *Ans.*

4. What is the interest of $3275 for 63 d.?

Solution.

$32.75	Int. for 60 d.
1.6375	" " 3 d.
$34.3875	" " 63 d. *Ans.*

5. Find the interest of $48,225 for 93 d.

6. Find the interest of $72.85 for 3 years 5 mo. 27 d.

Complete Solution.

	.7285	Int. for 2 mo.
14	570	Int. for 40 mo. (20 × 2 mo.)
	36425	" " 1 mo.
	29140	" " 24 d. (4 × 6 d.)
	036425	" " 3 d. ($^1/_2$ of 6 d.)
	$15.262075	" " 3 yr. 5 mo. 27 d.

For business purpose it is sufficiently exact to carry the work to mills or to tenths of mills at furthest, as in the shorter process.

In this process, when the decimal in the fourth place is less than 5 it is rejected. When 5 or greater than 5 the number of mills represented in the third order is increased by 1.

Note. — The result of the shorter process is 1 cent greater than that of the complete solution, but in a business transaction this difference would be disregarded. Answers are given as found by the shorter process.

Shorter Process.

	.729	Int. 2 mo.
14	58	" 40 mo.
	365	" 1 mo.
	291	" 24 d.
	036	" 3 d.
	$15.272	" 3 yr. 5 mo. 27 d.

7. How much must be paid for the use of $125.25 for 117 days at 6% per annum? What will be the amount?

Complete Solution.

Principal, $125.25; Int. 60 d., 1.2525; Int. 6 d., .12525.

			Shorter Process.
	1.2525	Int. 60 d.	1.253
	62625	" 30 d.	627
	4175	" 20 d.	418
	12525	" 6 d.	125
	020875	" 1 d.	021
	$2.442375	Int. 117 d.	2.444
Principal,	$125.25		$125.25
Amount,	$127.692375		$127.694

8. Find the interest of $9280 for 1 yr. 7 mo. 7 days.

9. What is the interest of $13985 for 2 years 23 days?

10. What is the interest of $18.56 for 1 year 8 mo. 16 days?

11. What is the interest of $198 for 2 years 11 mo.?

If it be required to find only the amount of a given sum at interest for a given time, the process may be made somewhat shorter than in the preceding example by adding the principal with the several items of interest, as follows:

12. Find the amount of $328 for 2 yr. 11 mo. 26 d.

Principal, $328; Int for 2 mo., $3.28; Int. 6 d., .328.

$3.28 17 2296 328 $55.76	$328.	Principal.
	55.76	Int. for 34 mo.
	1.64	" " 1 mo.
	1.312	" " 24 d.
	.109	" " 2 d.
	$386.821	Amount.

Interest at other Rates than 6%.

From the interest of any given sum at 6% the interest at any other rate can be readily found.

Example.—What is the interest of $329.75 for 1 yr. 5 mo. 23 d. at 6%? At 3%? 4%? 5%? 7%? 8%? 9%?

Interest at 6%.

Solution.

Prin., $329.75; Int. 2 mo. = $3.2975; Int. 6 d. = .32975.

			Shorter Process.
	$26.3800	Int. 1 yr. 4 mo. (8 × 2 mo.)	$26.38
	1.64875	" 1 mo. ($^1/_2$ of 2 mo.)	1.649
	1.09917	" 20 d. ($^1/_3$ of 60 d.)	1.099
	.16488	" 3 d. ($^1/_2$ of 6 d.)	.165
Ans.	$29.29280	Int. for 1 yr. 5 mo. 23 d.	*Ans.* $29.293

Interest at other per cents.

Having thus obtained the interest of $329.75 for 1 yr. 5 mo. 23 d. at 6%, the interest of the same sum for the same time—

At 3% = $^1/_2$ of 29.293	= $14.647	Or,	**3% divide by 2**
" 4% = $^2/_3$ " "	= $19.528	from	**4% subtract $^1/_3$**
" 5% = $^5/_6$ " "	= $24.411	int. at	**5% " $^1/_6$**
" 7% = $^7/_6$ " "	= $34.175	6% to	**7% add $^1/_6$**
" 8% = $^4/_3$ " "	= $39.056	find int.	**8% " $^1/_3$**
" 9% = $^3/_2$ " "	= $43.941	at	**9% " $^1/_2$**

SLATE EXERCISES.

Find the interest:

	Principal.	Rate.	Time.
1.	$100,	5%,	2 yr.
2.	$100,	6%,	2 yr. 6 mo.
3.	$100,	7%,	8 yr.
4.	$100,	8%,	7 yr.
5.	$100,	8%,	7 yr. 6 mo.
6.	$100,	9%,	6 yr. 8 mo.
7.	$274,	10%,	4 yr. 10 mo.
8.	$1200,	7%,	8 yr. 5 mo.
9.	$796,	12%,	6 yr. 9 mo.
10.	$1126.84,	9%,	4 yr. 4 mo.
11.	$964.50,	8%,	12 yr. 6 mo.
12.	$360,	11%,	11 yr. 5 mo.
13.	$10,	6%,	1 mo. 12 d.
14.	$10,	6%,	6 mo. 24 d.
15.	$100,	8%,	12 mo. 9 d.
16.	$600,	9%,	20 mo. 10 d.
17.	$240,	5%,	19 mo. 27 d.
18.	$396,	10%,	25 mo. 8 d.
19.	$840,	11%,	15 mo. 20 d.
20.	$900,	12%,	22 mo. 13 d.
21.	$100,	6%,	1 yr. 6 mo. 15 d.
22.	$240,	5%,	2 yr. 8 mo. 10 d.
23.	$360,	7%,	3 yr. 8 mo. 20 d.
24.	$1200,	8%,	2 yr. 4 mo. 10 d.
25.	$810,	9%,	4 yr. 2 mo. 10 d.
26.	$720,	10%,	5 yr. 4 mo. 24 d.
27.	$132.48	11%,	3 yr. 1 mo. 6 d.
28.	$120.48	12%,	6 yr. 6 mo. 6 d.
29.	$600,	5%,	4 yr. 4 mo. 5 d.
30.	$720.84,	8%,	12 yr. 3 mo. 10 d.
31.	$900,	6%,	7 yr. 9 mo. 25 d.
32.	$840.80,	10%,	2 yr. 2 mo. 2 d.
33.	$270.36,	9%,	4 yr. 8 mo. 12 d.
34.	$143.33,	11%,	6 yr. 4 mo. 5 d.
35.	$360.96,	12%,	5 yr. 6 mo. 10 d.
36.	$770.21,	7%,	2 yr. 8 mo. 15 d.

Find the interest and amount:

	Principal.	Rate.	Time.
1.	$1080.50,	7%,	1 yr. 9 mo.
2.	$420.25,	8%,	2 yr. 9 mo.
3.	$960,	9%,	3 yr. 4 mo.
4.	$576.48,	10%,	3 yr. 6 mo.
5.	$645,	12%,	5 yr. 10 mo.
6.	$1200,	5%,	6 yr. 3 mo.
7.	$1200,	10%,	12 yr. 6 mo.
8.	$828,	6%,	8 mo. 16 d.
9.	$972.36,	8%,	17 mo. 18 d.
10.	$600.60,	10%,	23 mo. 14 d.
11.	$1165.17,	12%,	40 mo. 6 d.
12.	$894,	7%,	14 mo. 17 d.
13.	$1248,	9%,	9 mo. 25 d.
14.	$840,	6%,	1 yr. 8 mo. 15 d.
15.	$960,	7%,	1 yr. 9 mo. 24 d.
16.	$1296,	8%,	2 yr. 3 mo. 9 d.
17.	$1080,	9%,	2 yr. 9 mo. 21 d.
18.	$1800,	10%,	3 yr. 6 mo. 15 d.
19.	$600,	11%,	4 yr. 7 mo. 18 d.
20.	$796,	12%,	5 yr. 10 mo. 6 d.
21.	$976.28,	7%,	7 yr. 9 mo. 27 d.
22.	$869.44,	9%,	8 yr. 4 mo. 17 d.
23.	$1126.56,	11%,	10 yr. 5 mo. 1 d.
24.	$1295.28,	8%,	13 yr. 4 mo. 29 d.

To find the time between the dates here given, follow the method of Ex. 1, page 231.

Find the amount:

	Principal.	Rate.	Time.
1.	$542,	7%,	From 1872, Oct. 27, to 1880, May 12.
2.	$684,	8%,	" 1868, Sept. 19, to 1870, June 1.
3.	$960,	6%,	" 1872, Dec. 31, to 1879, Oct. 1.
4.	$1100,	10%,	" 1839, Jan. 1, to 1850, Dec. 20.
5.	$1186.20,	11%,	" 1827, April 1, to 1847, July 28.
6.	$1260.48,	12%,	" 1868, Aug. 31, to 1879, Nov. 1.
7.	$1040.25,	8%,	" 1829, Feb. 20, to 1841, May 10.
8.	$1097.76,	6%,	" 1865, Mar. 15, to 1877, Jan. 15.
9.	$976.80,	7%,	" 1849, June 19, to 1869, April 7.
10.	$896.84,	9%,	" 1877, Nov. 24, to 1880, Nov. 30.
11.	$1272.24,	10%,	" 1876, Sept. 27, to 1879, Dec. 9.
12.	$1284.96,	12%,	" 1866, Dec. 8, to 1871, May 1.
13.	$1200,	11%,	" 1875, Dec. 25, to 1878, May 28.
14.	$989,	12%,	" 1876, Mar. 21, to 1879, June 30.

6% Method.

To find the interest of $1 for any given time at 6% affords an excellent mental exercise.

1. At 6% per annum, what is the interest of $1 for 3 yr. 5 mo. 7 d.?

Solution.—The interest of $1 at 6% for 41 mo. is 41 × 5 mills = 205 mills, and for 7 d. it is $1\,^1/_6$ mills, hence for 3 yr. 5 mo. and 7 d. it is $206\,^1/_6$ mills.

In like manner find the interest of $1 for—

2. 1 yr. 2 mo.	**8.** 3 yr. 1 mo. 3 d.	**14.** 8 mo. 2 d.
3. 5 yr. 8 mo.	**9.** 4 yr. 1 mo. 10 d.	**15.** 3 yr. 7 d.
4. 3 yr. 1 mo.	**10.** 9 yr. 7 mo. 8 d.	**16.** 11 yr. 11 mo.
5. 1 yr. 9 mo.	**11.** 5 yr. 4 mo. 1 d.	**17.** 11 mo. 5 d.
6. 10 yr. 4 mo.	**12.** 1 yr. 11 mo. 20 d.	**18.** 1 yr. 4 d.
7. 4 yr. 6 mo.	**13.** 6 yr. 6 mo. 14 d.	**19.** 7 mo. 3 d.

From the interest of $1 for any given time the interest of any other sum may be readily obtained for the same time and at the same rate per cent. For some uses this method is preferable to any other. See Partial Payments.

308. To find the Rate; Principal, Interest, and Time being given.

ORAL EXERCISES.

1. If 8¢ are paid for the use of $2 for 1 year, what is the rate per cent.?

Analysis.—At 1% per annum $2 (200¢) would earn 2¢, but since the given interest (8¢) is 4 times 2¢, the rate must be 4 times 1% or 4%.

2. Thirty-six cents are paid for the use of $3 for 2 years. What is the rate per cent. charged?

Analysis.—At 1% per annum the interest of $3 (300¢) for 1 year would be 3¢, and for 2 years it would be 6¢, but since the interest paid (36¢) is 6 times 6¢, the rate must be 6 times 1% or 6%.

3. Find the rate per cent. if the annual interest on $840 is $42; if on $850 the interest is $34; if on $725 the interest is $43 1/2; if on $75 the interest is $3.

Find the rate:

	Principal.	Interest.	Time.		Principal.	Interest.	Time.
4.	$200,	$18,	1 yr.	7.	$120,	$18,	2 1/2 yr.
5.	$150,	$90,	10 yr.	8.	$2000,	$90,	1 yr.
6.	$180,	$76,	5 yr.	9.	$4000,	$340,	2 yr.

SLATE EXERCISES.

Find the rate per cent.:

	Principal.	Interest.	Time.	Time.*
1.	$960,	$88.80,	1 yr. 6 mo. 15 d.	4 yr. 7 mo. 15 d.
2.	$796.20,	$171.98,	2 yr. 8 mo. 12 d.	8 yr. 1 mo. 6 d.
3.	$897.50,	$251.30,	3 yr. 6 mo.	10 yr. 6 mo.
4.	$1224.72,	$481.04,	5 yr. 7 mo. 10 d.	16 yr. 10 mo.
5.	$1152,	$403.20,	3 yr. 10 mo. 20 d.	7 yr. 9 mo. 10 d.
6.	$867.40,	$320.94,	7 yr. 4 mo. 24 d.	2 yr. 5 mo. 18 d.
7.	$1231.36,	$923.52,	8 yr. 4 mo.	2 yr. 9 mo. 10 d.

* **Note.**—The "time" given in this column being a multiple or aliquot of the corresponding "time" in the first column, the rate for the second column may be readily found from that of the first. But it must not be forgotten that, if the time is twice as long, the rate must be only 1/2 as great to produce the same interest, etc.

SLATE EXERCISES.

Applications.—1. What is the rate per cent. when $650 yields $32½ interest per annum? When $1250 pays $43¾? When $320 pays $14.40?

2. What is the rate per cent. per annum when I pay $45 for the use of $450 for two years?

3. What rate, if I pay $94½ on $900 for 2¼ years?

4. Mr. Pierce had $8000 at 4½%, and $1000 at 5%. At what rate must he loan both sums to one person to obtain the same annual interest?

5. A house, bought for $12,500, paid $1000 rent. If $200 were paid for taxes and repairs, what rate of interest did the purchase money yield?

6. At what rate of interest will $1268 double itself in 5 years? In 16⅔ years? In 12½ years? (At what rate will any sum double itself in the times specified?)

7. Mr. Williams borrowed $8590 on the 1st of June; on the 25th of the following January he paid the amount, $9036.68. What % per annum did he pay?

8. Mr. Knell wishes to obtain a loan of $480, but is not willing to pay more than $1½ interest per month. What rate % per annum would that be?

9. A residence costing $7500 is rented for $56¼ per month. What rate per cent. per annum does the money yield?

10. Mr. Dill had money at 3 different banks: in one he had $300 at 3%, in another $400 at 4%, and in the third $500 at 5%. Find at what % he should loan the three sums together to get the same interest.

11. Mr. Ball put out $1200 at 5% for 6 years. What per cent. should he charge to get the same amount of interest in 5 years?

12. Seven months after date a note for $1800 amounted to $1878.75. What was the rate?

309. To find the Time; Principal, Interest, and Rate per cent. being given.

ORAL EXERCISES.

1. In what time will \$2 yield 12¢ interest, at the rate of 3% per annum.

Analysis.—In 1 year, at 3%, \$2 (200¢) yield 6¢ interest, and it will require as many times 1 year to produce 12¢ interest as there are times 6¢ in 12¢, which is 2. Hence it will require 2 years, etc.

2. How long will it take \$3 to produce an interest of 54¢ at 6% per annum?

Analysis.—In 1 year, at 6%, \$3 will produce an interest of 18¢, and it will require as many years to produce 54¢ interest as there are times 18¢ in 54¢, which is 3. Hence it will require 3 years, etc.

3. In how many years will \$100 at 4% pay \$28 interest?

Find the time:

	Principal.	Rate.	Interest.		Principal.	Rate.	Interest.
4.	\$200,	$4\frac{1}{2}$%,	\$45.	7.	\$31,	5%,	\$31.
5.	300 fr.,	4%,	15 fr.	8.	\$1200,	5%,	\$75.
6.	£15,	$3\frac{1}{3}$%,	£$\frac{1}{4}$.	9.	\$3400,	7%,	\$119.

10. \$$50\frac{5}{8}$ interest is due on \$450 at $4\frac{1}{2}$%. How long has the interest remained unpaid?

11. Mr. Long paid \$48 interest. For what period did he pay it, the principal being \$640 and the rate 5%?

12. In how many years will the interest equal the principal at 3%? At $4\frac{1}{2}$%? At $6\frac{1}{4}$%?

(In what time will \$1 principal produce \$1 interest at the rate of 3% a year?)

SLATE EXERCISES.

Find the time:

	Principal.	Rate.	Interest.		Principal.	Rate.	Interest.
1.	\$840,	$3\frac{1}{3}$%,	\$70.	5.	\$4000,	5%,	\$50.
2.	\$1050,	4%,	\$$136\frac{1}{2}$.	6.	\$650,	4%,	\$78.
3.	320 mark,	$4\frac{1}{2}$%,	72 m.	7.	\$820,	5%,	\$$215\frac{1}{4}$.
4.	180 lire,	5%,	$4\frac{1}{2}$ l.	8.	\$450,	4%,	5¢.

Applications.—1. Mr. Hill invested $3000 in government bonds, bearing $5\frac{1}{2}\%$ interest. In how many years will the interest have increased this sum to $4320?

2. $600 were put at interest at $3\frac{1}{3}\%$, and $750 at 4%. For what time were they loaned, if both sums together with interest amount to $1525?

(How much does the first pay per year? The second? Both?)

3. In what time will $8000, invested at $4\frac{1}{2}\%$, produce an interest of $2400? How long will it be until $\frac{2}{3}$ of the interest will amount to as much as the principal?

Find the time:

	Principal.	Rate.	Interest.
4.	$896,	6%,	$80.64.
5.	$768,	7%,	$144.853.
6.	$984,	8%,	$288.64.
7.	$645.75,	9%,	$206.64.
8.	$727.35,	12%,	$418.954.
9.	$366.40,	11%,	$347.065.
10.	$978.60,	10%,	$518.658.

Find the time:

	Principal.	Rate.	Interest.
11.	$998.52,	5%,	$185.145.
12.	$1092.24,	7%,	$338.958.
13.	$1129.32,	9%,	$582.729.
14.	$1192.80,	8%,	$751.464.
15.	$1200,	6%,	$1200.
16.	$1268.40,	12%,	$1268.40.
17.	$1288.88,	10%,	$1261.142.

18. What would be the effect on the time if the rates in problems 4–17 were doubled, etc., etc.? If they were $\frac{1}{3}$ as great as those given?

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310. To find the Principal; the Interest, Rate per cent., and Time being given.

ORAL EXERCISES.

1. What principal at 5% will pay a yearly interest of 45¢?

Solution.—$1 at 5% will yield an annual interest of 5¢, and it will require as many dollars to produce an interest of 45¢ as there are times 5¢ in 45¢, which is 9. Hence it will require $9, etc., etc.

---

2. What principal at 6% will produce 36¢ interest in 2 years?

Solution.—$1 in 2 years at 6% will produce an interest of 12¢, and it will require as many dollars to produce an interest of 36¢ as there are times 12¢ in 36¢, which is 3. Hence it will require $3, etc.

3. Find the principal which pays per annum \$40 at 5%; \$53$^3/_4$ at 5%; \$100 at 3$^1/_3$%.

4. What principal at 5% will in one year yield \$30 interest? \$50? \$60? \$80? \$90?

5. What sum will pay \$368 interest in 20 years at 5% per annum?

6. What is the principal, if at 4% it pays \$387 interest in 25 years?

7. Find the principal that will yield \$441 at 3$^1/_3$% in 30 years, and one which will yield \$111 at 2$^1/_2$% in 40 years.

SLATE EXERCISES.

Find the principal:

	Rate.	Time.	Interest.
8.	5%,	6 months,	\$6.25.
9.	4$^1/_2$%,	6 "	\$4.50.
10.	6%,	6 "	\$27.00.
11.	4%,	1 month,	\$1.50.
12.	5%,	1 "	\$1.80.

Find the principal:

	Rate.	Time.	Interest.
13.	5%,	3 months,	\$61.00.
14.	4$^1/_2$%,	3 "	\$20.25.
15.	6%,	3 "	\$25.50.
16.	4$^1/_2$%,	1 month,	\$3.00.
17.	6%,	1 "	\$7.50.

18. How many times greater is the principal than the annual interest, when the rate is 2%? 2$^1/_2$%? 2$^2/_3$%? 3$^1/_8$%? 3$^1/_3$%? 4%? 4$^1/_6$%? 5%? 5$^5/_9$%? 6$^1/_4$%? 6$^2/_3$%? 7$^1/_2$%?

19. What is the principal which yields \$41 interest per year at 2$^1/_2$%? At 3$^1/_3$%? At 6$^1/_4$%? At 2$^2/_9$%?

20. What sum yields \$47.25 interest per year at 4$^1/_2$%?

Find the principal:

	Rate.	Time.	Interest.
21.	3$^1/_2$%,	1 year,	\$45$^1/_2$.
22.	5$^1/_2$%,	1 "	\$41$^1/_4$.
23.	4$^1/_2$%,	$^1/_2$ "	\$25$^1/_2$.
24.	3$^3/_4$%,	$^1/_2$ "	\$3$^3/_5$.
25.	8%,	$^3/_4$ "	\$18.
26.	2$^1/_2$%,	6 "	\$52$^1/_2$.

Find the principal:

	Rate.	Time.	Interest.
27.	5%,	7 years,	\$29.75.
28.	3$^1/_3$%,	4$^1/_2$ "	\$94.50.
29.	4%,	1$^3/_4$ "	\$68.25.
30.	4$^1/_2$%,	1$^1/_4$ "	\$47.25.
31.	6%,	5$^2/_3$ "	\$170.00.
32.	3$^1/_5$%,	4$^1/_4$ "	\$136.00.

SLATE EXERCISES.

Find the principal:

	Interest.	Rate.		Time.
1.	$42.70,	7%,	From	Jan. 1, 1880, to Sept. 1, 1881.
2.	$197.80,	8%,	"	Jan. 1, 1880, to July 12, 1882.
3.	$26.08,	6%,	"	Jan. 1, 1880, to Sept. 9, 1882.
4.	$60.75,	5%,	"	Jan. 1, 1880, to Oct. 10, 1881.
5.	$97.875,	9%,	"	Jan. 1, 1880, to July 1, 1881.
6.	$366.32,	10%,	"	Jan. 1, 1880, to Oct. 18, 1883.
7.	$90.06 +	11%,	"	Jan. 1, 1880, to July 1, 1882.
8.	$561.56,	12%,	"	Jan. 1, 1880, to Oct. 1, 1884.
9.	$445.19,	7%,	"	Jan. 1, 1880, to July 24, 1885.
10.	$277.76,	8%,	"	Jan. 1, 1880, to Nov. 15, 1883.
11.	$315.64 +	5%,	"	Jan. 1, 1880, to Aug. 6, 1885.
12.	$95.97,	6%,	"	Jan. 1, 1880, to Nov. 1, 1882.
13.	$700.70,	9%,	"	Jan. 1, 1880, to Oct. 10, 1889.
14.	$1150.86,	12%,	"	Jan. 1, 1880, to July 20, 1887.

Applications.—1. Mr. Day wishes to invest such a sum that at 5% he may draw $10,000 interest in 20 years. What sum must he invest?

2. The interest for $1\frac{1}{2}$ years of a principal drawing $3\frac{1}{3}\%$ interest per annum is paid with 7 bu. of wheat at $\$1\frac{1}{14}$ a bu., and 30 bu. of potatoes at $1 a bu. What is the principal?

3. Mr. A. has invested $5500 at 5% interest. Mr. B. receives for his money $25 more interest, though invested at $1\frac{1}{4}\%$ less than A.'s. What is B.'s principal?

4. Mr. Jones proposes to buy the house in which he resides. The rent he pays is $540 per year. He does not wish to pay more for interest, insurance, taxes, and repairs, than he now pays for rent. What sum can he offer for the house, if he allows $4\frac{1}{2}\%$ for interest and $1\frac{3}{4}\%$ for taxes and repairs?

5. What may I offer for a house which pays $1350 rent per year, so that the money I invest may bear $7\frac{1}{2}\%$ interest?

311. To find the Principal; the Amount, Time, and Rate per cent. being given.

1. What principal at 5% will produce an amount of \$2.10 in 1 year?

Solution.—\$1 at 5% will amount to \$1.05 in 1 year, and to produce an amount of \$2.10 in the same time will reqnire as many dollars, principal, as there are times \$1.05 in \$2.10, or \$2.

2. What principal at 6% will amount to \$3.54 in 3 years?

Solution.—\$1 at 6% will amount to \$1.18 in 3 years, and to produce an amount of \$3.54 in the same time will require as many dollars, principal, as there are times \$1.18 in \$3.54, or \$3.

3. What sum of money must be put to interest at 6% to amount to \$5.30 in 1 year? To \$6.72 in 2 years? To \$3.72 in 3 years?

4. Find the principal which will in two years amount to \$8.72 at $4\frac{1}{2}$% per annum.

ORAL AND SLATE EXERCISES.

What sum must be put at interest for

5.	2 yr. at	4% to amt.	\$5.40?	11.	$2\frac{1}{2}$ yr. at	2%	to amt.	\$5.30?
6.	4 "	6% "	\$12.40?	12.	$3\frac{1}{3}$ "	6%	"	\$12.00?
7.	6 "	$2\frac{1}{2}$% "	\$9.20?	13.	$7\frac{1}{2}$ "	8%	"	\$8.00?
8.	3 "	3% "	\$8.72?	14.	$4\frac{1}{4}$ "	3%	"	\$4.51?
9.	10 "	7% "	\$3.40?	15.	$9\frac{3}{4}$ "	1%	"	\$8.78?
10.	8 "	5% "	\$15.40?	16.	$6\frac{2}{3}$ "	5%	"	\$12.00?

17. What sum must Mr. Day invest, that it may amount to \$10,000 in 20 years at 5%?

18. An investment at 5% made by Mr. Palmer 15 years ago, now amounts to \$1984.40. What was the sum invested?

19. Ten years after buying a city lot, Mr. D. sold it for \$18,678, thereby obtaining interest at the rate of 12% per annum on the sum paid for it. What was the purchase price?

20. What principal at interest for 4 years at 8% will amount to the same as \$3500 at 4% in the same time?

Present Worth.

312. The present worth of a debt, payable at a future time without interest, is such a sum of money, that, if put at interest, it would amount to the debt when it becomes due.

The problem in present worth is similar to the preceding one, that is, it is required to find the principal, the amount, time, and rate per cent. being given.

313. The difference between the face of a debt and its present worth is ***True Discount.***

True discount is so called to distinguish it from *bank discount.* True discount is the interest on the present worth of a debt. Bank discount is interest on the debt. The difference is the interest on the true discount.

Examples.—1. Find the present worth and the true discount of a debt of $48.36 payable in 6 months without interest.

Analysis.—Since one dollar, at interest for six months, will at the end of the time amount to $1.03, every dollar and three cents in $48.36, due at the end of six months, is worth one dollar now; hence the solution $48.36 ÷ $1.03 = 46.951.

2. Find the present worth of $59.50 payable in 3 mo. without interest.

3. Find the present worth of $118.20 payable in 8 mo. without interest.

4. What is a debt of $100 worth now, which in 1 month is to be paid without interest? Suppose the debt to be $48.35.

5. Find the true discount of a debt of $763.30 due in 9 months and bearing no interest.

6. Find the true discount of a debt of $364.43 due without interest in $6\frac{1}{2}$ months; also of $37.44 due in 8 months without interest.

7. Mr. Hall owes me $968, due in two months without interest. If he desires to pay me now, what sum should I accept if the use of the money is worth 8% per annum?

8. What is the present worth of the amount which will be due on a note for $3500 having 1 yr. 2 mo. 18 d. to run, at 6%, if the current rate of interest is 12%?

Exact Interest.

314. In all the foregoing calculations 30 days have been taken for one month, and 360 days for one year. According to this method the interest on $6000 for 30 days at 6% would be $30; but if the period of 30 days were reckoned as only $^{30}/_{365}$ of a year, the interest would be $29.52. The latter is called Accurate or Exact Interest.

315. ***Accurate*** or ***Exact Interest*** is interest for the exact time in days, the days being reckoned as 365ths of a year.

The United States Government pays *exact interest.*

316. To obtain accurate or exact interest for any number of days, we find the exact time in days, and take as many 365ths of a year's interest as there are days in the given time.

Example.—1. $350 is at interest from January 16 to March 29, 1880. What is the accurate interest?

Exact Time.		Solution.
From Jan. 16 to 31 =	15 d.	$3.50 = int. at 1% for 1 yr.
In February	29 d.	6
To March 29	29 d.	21.00 = int. at 6% for 1 yr.
	73 d.	$\frac{73 \times \$21}{365} = \$4.20$ = int. at 6% for 73 d.

	Principal.	Time.	Rate.
2.	$5000,	From Jan. 1 to Aug. 16, 1875,	6%.
3.	$16500,	" Feb. 10 to Sept. 15, 1879,	6%.
4.	$24800,	" Mar. 8 to Oct. 3, 1879,	6%.
5.	$7500,	" July 2 to Nov. 9, 1880,	5%.
6.	$9800,	" May 5 to Dec. 1, 1879,	7%.
7.	$187.44,	" Jan. 20 to April 14, 1881,	$7^{3}/_{10}$%.
8.	$768,	" Jan. 24 to July 30, 1877,	6%.
9.	$840,	" June 15 to Oct. 25, 1878,	7%.
10.	$480,	" Mar. 31 to Oct. 4, 1866,	8%.
11.	$1080,	" Jan. 1 to July 16, 1867,	5%.
12.	$975,	" Sept. 15 to Dec. 30, 1852,	9%.
13.	$1104,	" Feb. 28 to Aug. 31, 1870,	10%.
14.	$1020,	" April 5 to Oct. 26, 1874,	7%.
15.	$1121,	" Sept. 24 to Dec. 21, 1872,	6%.

Common Business Method.

317. Bankers and most other business men, in computing interest for short periods of time, usually take the exact number of days as above, but reckon each day as $^1/_{360}$ of a year.

Days of Grace.—Formerly granted as a favor, the custom of allowing three days beyond the time agreed upon for the payment of a debt has grown into a very general law. They are of no advantage to the debtor, since interest is in all cases charged up to the time of payment. In California and some other States they are not required (see p. 416). In almost all other States they are required, and suit can not be brought for the payment of a debt till the expiration of the three days of grace.

In the following problems, days of grace must be added to the given time.

Example.—1. Find the interest of $150 from April 27 to June 26, 1885, at 9%.

In April,	3 d.	1.50	Int. for 60 d. at 6%
In May,	31 d.	75	3 " "
In June,	26 d.	2)1.575	63 " "
Grace,	3 d.	.787 $^1/_2$	
	63 d.	2.362 $^1/_2$	Int. for 63 d. at 9%

Find the interest of

	Principal.	Time.	Rate.
2.	$450,	From Aug. 10 to Nov. 8, 1885,	6%.
3.	$720,	" Jan. 25 to April 7, 1885,	7%.
4.	$960,	" Feb. 3 to Mar. 19, 1884,	8%.
5.	$540,	" April 8 to May 18, 1870,	9%.
6.	$100,	" Jan. 30 to Mar. 6, 1872,	4%.
7.	$900,	" Feb. 12 to Mar. 4, 1873,	7 $^1/_2$%.
8.	$240,	" May 31 to Nov. 27, 1875,	10%.
9.	$333,	" Aug. 1 to Nov. 29, 1876,	5%.
10.	$672,	" Feb. 28 to Oct. 25, 1880,	4 $^1/_2$%.
11.	$60,	" June 19 to Nov. 10, 1881,	12%.
12.	$600,	" July 4 to Oct. 20, 1882,	3%.
13.	$630,	" Feb. 1 to Aug. 20, 1883,	5 $^1/_7$%.
14.	$480,	" Jan. 21 to Dec. 2, 1871,	5%.
15.	$270,	" May 10 to July 29, 1874,	6%.
16.	$386,	" Oct. 13 to Dec. 12, 1885,	9%.

Bank Discount.

Illustration.—If a merchant desires to obtain a loan for any short period of time, say about $1000, for 60 days, he may take his own note, or the note of another party, for $1000, to a bank, and if the note is properly indorsed or its payment is otherwise secured, the bank will take it, pay him $1000 less the interest for 63 days, and collect the $1000 when it becomes due.

The interest on $1000 for 60 days + 3 days of grace, computed by any of the preceding methods, is $10.50. $1000 − $10.50 = $989.50 = the sum which the merchant will receive on the note.

318. The ***Avails*** or ***Proceeds*** of a note is the sum that the bank pays upon it after deducting the interest on the face or amount of the note.

319. The sum deducted for the payment of proceeds before a note becomes due is called ***Bank Discount.***

320. A note is said to be ***payable*** at the promised time of payment. Its date of ***maturity*** is three days later, the last day of grace. It then becomes ***legally due.***

If a note is not paid at maturity, a written notice of the failure should be sent on the last day of grace to the indorser or indorsers, otherwise they can not be held for its payment. Such notice is called a ***protest***, and should be made out by a ***Notary Public.***

321. The dates showing when a note is payable and the date of maturity are written thus: Aug. $^5/_8$.

322. If the third day of grace falls on a Sunday or on a legal holiday, the note is due the day before.

SLATE EXERCISES.

Example.—1. What was the bank discount and what were the proceeds of a note of $645, dated July 22, payable Sept. 20, and discounted at 6% on the day the note was drawn?

Due Sept. $^{20}/_{23}$.			*Discount.*	
In July = 9 d.	$6.45	Int. 60 d.	$645	Face.
In Aug. = 31 d.	215	" 3 d.	6.67	
In Sept. = 23 d. (3 d grace)	$6.665	" 63 d.	$638.33	Proceeds.
63 d.				

Find the bank discount and proceeds of a note for

2. $440,	payable in	90 d.,	discounted at	6%	on the day	drawn.
3. $500,	"	60 d.,	"	9%	"	"
4. $1000,	"	45 d.,	"	5%	"	"
5. $140.25,	"	30 d.,	"	$4^1/_2$%	"	"

6. $970, dated Feb. 9, 1884, payable in 60 days, and discounted at 6% on March 13, 1884. (Due April $^9/_{12}$. Reckon discount from March 13th to April 12th = 30 d.)

7. $638, dated Jan. 21, 1880, payable in 3 mo., discounted at 8% on Feb. 4, 1880. (Due April $^{21}/_{24}$. Discounted for 80 d.)

8. $1235, dated May 10, 1881, payable in 4 mo. with interest at 6%, discounted at 6% on June 5, 1881. (Due Sept. $^{10}/_{13}$, with interest for 4 mo. 3 d. The amount is the sum to be discounted.)

9. $6040, dated July 12, 1885, payable in 90 d. with interest at $4^1/_2$%, discounted at 10% on Aug. 4, 1885. (Due Oct. $^{10}/_{13}$. Reckon interest for 93 d. at $4\,^1/_2$% and discount from Aug. 4th to Oct. 13th.)

10. $12008, dated May 12, 1870, payable in 9 mo. with interest at 8%; indorsed $5000, Aug. 12, 1870, discounted Nov. 12, 1870. (Due Feb. $^{12}/_{15}$.)

Find interest on $12008 for 3 mo. at 8%; apply the partial payment, $5000, and find new principal; reckon amount of this new principal for 6 mo. 3 d.; on this amount reckon discount from Nov. 12, 1870, to Feb. 15, 1871 = 95 d.

11. Find the face of a note, payable in 57 d., that will yield $792 proceeds when discounted at 6%.

Analysis.—Since a face of $1 would yield $.99 proceeds, to yield $792 proceeds will require as many dollars face as $.99 are contained times in $792 = 800 times $1 = $800.

12. Find the face of a note, payable in 30 days, that will yield $477.36 proceeds when discounted at 6%.

13. I bought a bill of goods for $864 on 4 mo. credit, but, being offered 5% off for cash, I borrowed the money at a bank by having my note, due in 117 days, discounted at 6%, and paid the bill. What was the face of the note, and how much did I gain?

Promissory Notes.

323. A ***Promissory Note,*** or simply a ***Note,*** is a written promise to pay a specified sum of money on demand or at a certain time to some person named in the note.

324. The sum promised to be paid is the ***Face*** of the note.

325. The signer is called the ***Maker*** or ***Drawer.***

326. The person to whom it is payable is the ***Payee.***

327. The owner of the note is called the ***Holder.***

The following is a simple form of a promissory note :

$50.75. *New York, June 15, 1880.*

Three months[1] *after date I promise to pay* *JOHN JONES*[2] *fifty 75/100 dollars,*[3] *for value received.*

AMOS WILSON.

(Who is the maker of the above note? Who the payee? What is its face? What is its date?)

Notes.—1. **Time.** *a.* The time of a note should be written in words, and may be expressed in days, months, or years, or the date on which it is to be paid may be specifically stated. Thus, "***on the first day of May, 1887, I promise,***" etc. Notes in which a certain time is given for payment are called ***Time Notes.***

b. If, in place of "*sixty days after date,*" the words "*on demand*" were used, the note would be payable on demand. It would then be called a "***demand note.***"

2. **Payee.** *a.* As it now stands, the note is payable only to Mr. John Jones in person. A note thus drawn can not be transferred, and is said to be "***not negotiable.***"

b. But if the note read, to "John Jones ***or order,***" Mr. Jones could make it payable to whomsoever he might order. Thus, if he wrote

Pay to William Jackson,
John Jones,

then William Jackson would be the only person to whom it would be payable. But if Mr. Jones wrote

Pay to William Jackson, or order,
John Jones,

then Mr. Jackson could in his turn make it payable to whomsoever he pleased, and in this way it might pass through dozens of hands.

c. If the words "*or bearer*" followed Mr. Jones's name, it would be payable to any one who might present it. Or, if Mr. Jones should write only his name on the back it would then be payable to any one who might hold it, as if made payable *to bearer*. Such an indorsement is called an ***indorsement in blank.***

d. A note that may be transferred from one person to another, either by delivery or indorsement, is said to be ***negotiable.***

3. **The face.** The number of dollars for which the note is drawn should be specified in words, the cents may be expressed as hundredths of a dollar.

4. **Interest.** The note as printed would not bear interest till due. If not then paid it would thereafter be subject to legal interest. If the words "with interest" were used, and no rate specified, the note would draw interest at the legal rate from date to time of payment. If the rate were any other than the legal rate it would have to be specified, as, "with interest at 8%."

5. **The place of payment.** If the words, *at the First National Bank*, or other place, were written after the "fifty $^{75}/_{100}$ dollars," then the note should be presented at the bank or other place named, on the last day of grace. If no bank or other place of payment is specified, the note is payable at the drawer's place of business or residence.

6. If the words "value received" are omitted, the holder may have to prove that the drawer received its value.

EXERCISES IN WRITING NOTES.

1. Draw a note payable by John Doe to Richard Roe; another to Richard Roe *or order;* another to Richard Roe *or bearer.*

2. Draw a *demand note;* a *time note.* Draw a *time note* with interest at legal rate. (The words *with interest* mean as much as *with interest at legal rate.*)

3. Draw a note in which the date of payment is specifically stated.

4. Draw a time note in which the rate of interest is other than the legal rate.

5. Draw a time note and indorse it in blank.

6. Draw a demand note, payable to Henry Hudson, or order, and properly indorse it to Miles Standish, or bearer.

7. Draw a note to mature 3 months from the present date.

8. Draw a note payable to order for any given sum, specifying the place and time of payment.

Partial Payments.

328. ***Partial payments*** are payments in part of any indebtedness. It is customary ***to indorse*** partial payments on the back of a note. The ***indorsement*** consists of the date and amount of the payment.

Example.—1.

$100. *Cincinnati, O., May 10, 1876.*

For value received, on demand, I promise to pay to WARREN HASTINGS, *or order, one hundred dollars with interest at 6%.*

ROBERT MOULTON.

On this note partial payments were indorsed as follows: Nov. 10, 1876, $23; May 10, 1879, $17; May 10, 1881, $7; Sept. 10, 1882, $33. What was the amount due Jan. 10, 1883?

Solution.

Principal from May 10, 1876	$100
Interest to Nov. 10, 1876 (6 months)	+3
Amount	103
First payment, Nov. 10, 1876	−23
New principal	80
Interest on $80 to May 10, 1879 ($2\,^1/_2$ years)	+12
Amount	92
Second payment, May 10, 1879	−17
New principal	75
Interest on $75 to May 10, 1881 (2 years)($9)	
(Here the payment ($7) is less than the interest, and if we were to form a new principal, as in the cases preceding this, it would be equivalent to adding the unpaid interest ($2) to the principal. But, this being illegal, we continue the interest on $75 till the sum of the payments equals or exceeds the interest; hence we find the)	
Interest on $75 from May 10, 1879, to Sept. 10, 1882	15
Amount	$90
Third and fourth payments ($7 + $33)	−40
New principal	50
Interest on $50 to Jan. 10, 1883 (4 months)	+1
Amount due on settlement, Jan. 10	**$51**

This calculation is in accordance with the

329. United States Rule for Partial Payments.

Find the amount of the principal to the time when a payment or the sum of two or more payments equals or exceeds the interest. From this amount deduct the payment or the sum of the payments.

With the remainder as a new principal, proceed as before.

Note.—This rule is founded on the principle that neither interest nor payment shall draw interest.

Examples.—2. A note of $250 is dated June 1, 1878. Indorsement: June 1, 1879, $85. What was due at the time of settlement, June 1, 1880, interest at 5%?

3. A note of $1000, dated May 1, 1875, was indorsed as follows: Nov. 25, 1875, $134; March 7, 1876, $315.30; Aug. 13, 1876, $15.60; June 1, 1877, $25; April 25, 1878, $236.20. What was the amount due on Sept. 10, 1878, interest 6%?

When there are many dates to deal with, as in the preceding problem, it will aid the accountant to avoid confusion and consequent danger of error to write out the dates as in the first column below, and the differences as in the second. This arrangement affords a ready means of testing the accuracy of the work, inasmuch as the sum of the differences in the second column must be equal to the difference between the first and last dates in the first.

Then, if we adopt the 6% method (see page 308), that is, multiply the interest of $1 for each period by the number of dollars at interest during that period, we may test the correctness of each item, since the sum of the several items must equal the interest of $1 for the whole time.

Dates.	Time elapsed.	Int. of $1.	Principal.	Int. of principal.	Amount.	Payment.
Yr. mo. d.	Mo. d.					
1875, 5, 1						
1875, 11, 25	6, 24	$.034	$1000	$34	1034	134
1876, 3, 7	3, 12	.017	900	15.30	915.30	315.30
1876, 8, 13	5, 6	.026	600	15.60	615.60	15.60
1877, 6, 1	9, 18	.048	600	28.80	661.20	25.
1878, 4, 25	10, 24	.054	600	32.40		236.20
1878, 9, 10	4, 15	.0225	400	9	409	——
3, 4, 9	40, 9	.2015	$409 balance remaining unpaid.			

4. A note of \$600, dated March 10, 1877, is indorsed as follows: Sept. 10, 1877, \$100; June 10, 1878, \$100; Dec. 10, 1878, \$100; Dec. 10, 1879, \$200. What amount was due on Oct. 10, 1880, at the time of settlement, interest 6%?

5. I held a note for \$600, bearing interest at 6% from March 8, 1869. I received as partial payments (1) \$140 on Sept. 10, 1870; (2) \$50 on July 20, 1872. What amount was due on settlement, Oct. 15, 1873?

6. A note of \$300, dated Jan. 1, 1878, had the following indorsements: Aug. 1, 1878, \$50; Dec. 1, 1878, \$50; July 1, 1879, \$100. What amount was due on Jan. 1, 1880, interest at 7%?

7. On the 1st of June, 1879, H. R. Fox borrowed of Charles Lever \$800, and gave his note for that sum with interest at 7%. Sept. 1, 1879, Fox made a payment of \$240, and a new note was made out for the balance. What was the face of this note?

(Write this new note out in proper form, dating it at Richmond, Va.)

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8. *\$575.* *Cleveland, O., Feb. 1, 1879.*

Eight months after date I promise to pay C. F. CUTTER & Co., or order, five hundred seventy-five dollars, with interest at 7%, for value received.

R. W. CANE.

Indorsements: Oct. 1, 1879, \$300; July 1, 1880, \$150.

What balance was due on settlement, Dec. 1, 1880?

9. Aug. 1, 1873, F. Critland gave to Robert Ingham a note for \$143.50, with interest at 7%. He made payments as follows: Dec. 17, 1873, \$37.40; July 1, 1874, \$7.09; Dec. 22, 1875, \$13.13; Sept. 9, 1876, \$50.50. What amount was due Dec. 28, 1876?

10. A note for \$2800, dated June 30, 1884, has the following indorsements: Dec. 23, 1884, \$50; May 16, 1885, \$90; June 3, 1885, \$10; April 23, 1886, \$150; May 30, 1886, \$22. What balance remained due at the last payment? (Why the original principal?)

The Mercantile Rule.

330. The following method of finding the balance due on a mercantile account or other debt running for a year or less is very commonly adopted when partial payments have been made.

The principle on which it is based will be readily understood from the statement of the rule.

331. ***Rule.*—1. Compute the amount of the debt from its date to the time of settlement.**

2. Compute the amount of each payment from its date to the date of settlement.

3. Subtract the sum of the amounts of the several payments from the amount of the debt.

The difference will be the balance due.

The answers given to the following problems are based on the common method of finding differences of time (compound subtraction—360 days to the year) though other methods may be used.

SLATE EXERCISES.

1. On a debt of $420, contracted March 15, 1885, payments were made as follows: June 1, 1885, $150; Sept. 6, 1885, $130; Oct. 14, 1885, $75. What was the balance due Dec. 24, 1885, at 7% interest?

2. On a note for $645, at 6% interest, dated Jan. 1, 1886, and maturing 9 months after date, the following indorsements were made: Mar. 4, 1886, $50; Apr. 2, 1886, $75; Aug. 10, 1886, $200. What was the balance due at time of payment?

3. Mr. Thomas gave his note, dated Feb. 15, 1885, to Amos King, for $1940, payable Jan. 1, 1886, with interest at 8%. When due, the note had the following indorsements: Aug. 1, 1885, $440; Sept. 6, 1885, $500; Oct. 1, 1885, $300; Nov. 15, 1885, $400. What was the balance due Jan. 1, 1886?

4. On May 1st, goods were purchased to the value of $1250, on which the following payments were made: Aug. 1, 1884, $250; Sept. 4, 1884, $300; Oct. 15, 1884, $450; Dec. 8, 1884, $120. What was the balance due Dec. 20, 1884?

Annual Interest.

332. ***Annual Interest*** is simple interest payable annually.

In some States annual interest, if not paid when due, draws interest at the same rate as the principal, in others at the legal rate, whatever may be the rate paid on the principal; but in some it is illegal to collect interest on unpaid annual interest.

Example.—1. Mr. Hart gives his note for $2000, payable in 4 years with interest annually at 5%. What will be the amount due when the note matures, provided Mr. Hart pays interest as agreed upon?

Solution.—The interest on $2000 at 5% is $100 per year. Hence Mr. Hart should pay $100 each year till the close of the 4th, when he should pay the last year's interest ($100) with the principal, making together $2100; but

2. If Mr. Hart does not pay the interest annually, as agreed, his account at the close of the 4 years would stand as follows:

The principal,			$2000
Total annual interest ($100 per year for 4 yr.)	=	$400	
Interest on 1st ann. int. ($3 \times 5 \times \$1$)	=	15	
" " 2d " ($2 \times 5 \times \$1$)	=	10	
" " 3d " ($1 \times 5 \times \$1$)	=	5	430
Amount due at the close of the 4th year,			= $2430

This amount differs from the amount of the same principal at simple interest only by the $30 interest on the deferred payments.

Applications.—1. The annual interest on $2000 invested at 6% for 6 years remaining unpaid, what is the amount due?

2. I invested $550 for 3 years at 5% interest, payable annually. What was due at the end of the three years, interest for the first only having been paid?

3. At the end of 3 years, what is due on a debt of $500, with interest payable annually?

4. Find the amount due in 8 years on $320, invested at 7% interest, payable annually, the interest for the first 3 years having been paid when due.

5. The interest on a note of $400, payable after 6 years, with annual interest at $5\frac{1}{2}$%, has not been paid. What is the note worth at the time of its becoming due?

6. A note for \$1000, dated July 1, 1865, at 7% interest, payable annually, was paid January 1, 1868. What was the amount due at maturity?

7. No interest having been paid on a note for \$500, dated June 1, 1878, with interest payable annually, find the amount due September 1, 1880.

Miscellaneous Problems.

1. A gentleman has at interest \$10,640 at 5%, \$37,500 at 6%, and \$26,000 at $6\frac{1}{2}$%. What income does he derive from these sums per annum?

2. Mrs. Stone has 24 government bonds of \$1000 each, bearing 4% interest. What is her income per quarter?

3. \$1850 yields $\$55\frac{1}{2}$ int. in 6 months. What rate is paid?

4. A principal of \$500 was increased by \$35 interest per year. What was the rate per cent.? In how many years will the principal be doubled by simple interest?

5. A sum of money was invested at $3\frac{1}{2}$% interest. After 3 years 4 months the amount was \$10,887.50. Find the principal.

6. What is the principal, if after $\frac{1}{2}$ year the amount is \$413, the rate being $6\frac{1}{2}$% per annum?

7. Mr. A. had a mortgage on his house, and paid $6\frac{1}{4}$% interest. In 4 years the amount was \$5262.50. What was the debt?

8. A lady inherits \$6480, and desires to derive \$32.40 per month from it. At what rate must it be invested?

9. In what time will \$462.50 produce \$37 interest at 4%?

10. In what time will \$723 produce $\$60\frac{1}{4}$ interest at 5%?

11. What sum will yield \$35 interest at 7% in 1 yr. 4 mo.?

12. What principal will pay \$21.59 interest at 5% in $8\frac{1}{2}$ months?

13. Mr. A. borrowed a sum of money at $5\frac{1}{2}$%, and after $1\frac{1}{2}$ yr. paid the amount \$4330. What was the principal?

14. What principal gives $83\frac{1}{3}$ interest per month at 5% ?

15. A gentleman draws $2940 per year; $\frac{4}{5}$ of his money bears 4%, $\frac{1}{5}$ 5%. Find the principal.

16. A principal of $6000 has by simple interest grown to $8820 in a number of years; $\frac{1}{3}$ of the time it brought 3%, $\frac{1}{4}$ of the time 5%, and the remainder of the time 4% per annum. How long did the principal stand?

17. The discount at 6% on a note due Nov. 1, and sold on May 1, was $13\frac{1}{2}$. What was the face of the note?

18. I had a note for $250, due in $2\frac{1}{2}$ months, and sold it at a discount of 1% per month. How much did I get for it?

19. If a person wishes to get the same interest for $1200 in 4 yr. which he receives for $1000 at 4% in 6 years, what rate must he charge?

20. In how many years will $820 at $6\frac{1}{2}$% produce $278.82\frac{1}{2}$ interest?

21. In how many years will a principal of $5000 grow to be $8000, if put out at the rate of 6% ?

22. The sum of $3360 is at $4\frac{1}{2}$% interest, and has thus far yielded $1058.40 interest. How long has the principal been standing?

23. What is the rate of discount if a note of $300, due June 20th, is sold April 20th for $297\frac{1}{2}$?

24. What is the face of a note, sold 1 month before it fell due at 9% discount, the discount amounting to $15\frac{3}{4}$?

25. Mr. M. buys a house by paying $\frac{13}{20}$ of the purchase money, and securing the remainder, $3500, by a mortgage at $4\frac{1}{2}$%. At the end of 1 year he pays the remainder. How much does the house cost him?

25. Three principals, of which the first, $600, has been at 6% interest for $2\frac{1}{2}$ years; the second, $425, at 4% for 3 years; and the third, $550, for 2 years, together yield $190.50 interest. At what rate was the third principal invested?

27. What principal will bear as much interest in 6 years at 5% as \$840 in 8 years at 4%?

28. What interest will you get on \$960 in 7 years, if \$280 in 5 years yields \$63 interest at the same rate?

29. If I borrow \$480 at 5% per annum, and the interest for the first year is deducted at once, what per cent. do I really pay?

30. \$400 were at 5% interest for $3\frac{1}{2}$ years, and \$350 at 4% for a longer time. The principals with interest, when collected, amounted to \$890. How long was the second principal standing?

31. A gentleman borrows \$60, and promises to pay \$70 in 3 months. What rate per cent. will he pay?

32. A guardian put his ward's money, \$17,500, at interest: $\frac{1}{5}$ of the money at 4%, and the remainder at $6\frac{1}{2}$%. How much could he lay by annually for the future benefit of his ward, after deducting \$650 per year for expenses?

33. For what time does Mr. B. pay interest, if he pays \$84 on \$5600 at the rate of $4\frac{1}{2}$%?

34. Mr. Frank buys a house and pays $\frac{2}{5}$ of the price in cash. The remainder, \$5400, is secured by a mortgage, and paid in 2 years together with $7\frac{1}{2}$% interest. How much was paid, including the interest?

35. A man was asked what money he had at interest. He said: One half of it yields $4\frac{1}{2}$%, the other half 5%, and I receive on the whole \$114 a month. How much had he at interest?

36. Mr. Conklin having bought a piano for \$350, he rented it at once for fifteen months at \$$4\frac{1}{2}$, payable monthly. Then he sold it for \$325. How much did he gain by the transaction, taking into account the interest on the cost and interest on payments received for rent?

37. I paid \$600 per year rent for a factory which I afterward bought for \$12,000. I gave \$5000 cash (which was worth 6%) and a 4% mortgage for the balance. How much per year did I save?

Compound Interest.

333. Compound interest is interest on interest.

Payment of compound interest, or interest on interest, can not be enforced by law.

334. Interest is usually compounded at specified intervals, as annually, semi-annually, etc., by adding interest to principal, and computing interest on the amount.

Example.—1. Find the compound interest of $500, at 6%, for 3 yr. 5 mo.

$500.00	Principal.
30.00	Interest 1st year.
$530.00	Amount.
31.80	Interest 2d year.
$561.80	Amount.
33.708	Interest 3d year.
$595.508	Amount.
14.8877	Interest 5 mo.
$610.3957	Amount 3 yr. 5 mo.
$500	Original principal.
$110.3957	Compound int. 3 yr. 5 mo.

$530	
6	
$31.80	Int. 2d year.
$561.80	
6	
$33.7080	Int. 3d year.
$5.95508	Int. 2 mo.
$11.91016	Int. 4 mo.
2.97754	" 1 "
$14.8877	Int. 5 mo.

2. What is the compound interest on $7325 for 2 yr. 2 mo. at 7%? (Carry the work to four decimal places.)

3. Find the compound interest on $3333, at $3\frac{1}{3}$% semi-annually, for 1 yr. 7 mo.

4. What amount was due March 25, 1886, on $1512, borrowed Jan. 25, 1885, with compound interest at $1\frac{1}{2}$% quarterly?

5. What is the amount of $4615, at compound interest, for 2 yr. 5 mo. at 8%?

6. Find the amount of $3500, at compound interest, from Oct. 29, 1884, to Nov. 15, 1885, at 2% quarterly.

7. How much greater, at compound than at simple interest, would be the amount of $1568 in 3 yr. 8 mo. at 6%?

8. Find the amount due Sept. 18, 1876, on $450, loaned Sept. 18, 1873. Interest compounded annually at $4\frac{1}{2}$%.

The use of the following table will greatly shorten calculations in compound interest:

TABLE SHOWING THE AMOUNT OF $1 AT DIFFERENT RATES, COMPOUND INTEREST, FROM 1 TO 15 YEARS.

Yrs.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.	6 per cent.	Yrs.
1	1.030000	1.035000	1.040000	1.045000	1.050000	1.060000	1
2	1.060900	1.071225	1.081600	1.092025	1.102500	1.123600	2
3	1.092727	1.108718	1.124864	1.141166	1.157625	1.191016	3
4	1.125509	1.147523	1.169859	1.192519	1.215506	1.262477	4
5	1.159274	1.187686	1.216653	1.246182	1.276282	1.338226	5
6	1.194052	1.229255	1.265319	1.302260	1.340096	1.418519	6
7	1.229874	1.272279	1.315932	1.360862	1.407100	1.503630	7
8	1.266770	1.316809	1.368569	1.422101	1.477455	1.593848	8
9	1.304773	1.362897	1.423312	1.486095	1.551328	1.689479	9
10	1.343916	1.410599	1.480244	1.552969	1.628895	1.790848	10
11	1.384234	1.459970	1.539454	1.622853	1.710339	1.898299	11
12	1.425761	1.511069	1.601032	1.695881	1.795856	2.012196	12
13	1.468534	1.563956	1.665074	1.772196	1.885649	2.132928	13
14	1.512590	1.618695	1.731676	1.851945	1.979932	2.260904	14
15	1.557967	1.675349	1.800944	1.935282	2.078928	2.396558	15

9. Find the compound interest of $1250, at 6%, for 3 years.

Analysis.—According to the foregoing table, each dollar at 6% compound interest for 3 years would amount to $1.191016, and the amount of $1250 for the same time and at the same rate would be 1250 times as much = $1488.77. The principal, $1250, being subtracted from $1488.77, the remainder is the compound interest = $238.77 *Ans.*

Note.—Except for large sums of money, it is not necessary to use more than four decimal places, as in the following problems.

10. Find the amount of $750, at 5%, compound interest, from March 10, 1883, to Sept. 10, 1886.

11. $325 was placed at compound interest at 4% semi-annually, on Jan. 1, 1882. What was due Jan. 1, 1886?

12. What is the compound interest of $650, at 3½% annually, for 1½ years?

13. Find the amount paid Oct. 31, 1886, for $1225, borrowed Dec. 10, 1881. Interest compounded at 3% semi-annually.

CHAPTER XVI.

EQUATION OF PAYMENTS.

ORAL EXERCISES.

In what time will the use of $1 balance the use of

1. $3 for 1 month? **4.** $3 for 2 mo.? **7.** $8 for 5 mo.?

2. $7 for 1 month? **5.** $7 for 2 mo.? **8.** $12 for 8 mo.?

3. $9 for 1 month? **6.** $9 for 2 mo.? **9.** $15 for 4 mo.?

In what time will the use of $1 balance the use of

10. $8 for 4 mo. + $5 for 5 mo. + $9 for 2 mo.?

11. $7 for 2 mo. + $4 for 6 mo. + $5 for 10 mo.?

12. $8 for 2 mo. + $9 for 3 mo. + $8 for 5 mo.?

13. How long should $2 be kept in use to balance the use of $1 for 4 months?

How long:

14. $3 to bal. $2 for 2 mo.? **17.** $300 to bal. $200 for 6 yr.?

15. $5 to bal. $3 for 10 mo.? **18.** $400 to bal. $300 for 12 yr.?

16. $7 to bal. $9 for 14 mo.? **19.** $5000 to bal. $250 for 20 yr.?

20. How long should $8 be kept at interest to balance the interest of $6 for 2 months + the interest of $2 for 12 mo.?

How long:

21. $7 to bal. the interest of $3 for 14 mo. + $4 for $3\frac{1}{2}$ mo.?

22. $15 to bal. the interest of $9 for 5 mo. + $6 for $7\frac{1}{2}$ mo.?

23. $38 to bal. the interest of $20 for 9 mo. + $18 for $11\frac{1}{9}$ mo.?

24. How long a time may $600 be kept in use to balance the use of $400 for 3 years + $200 for 9 years?

Definitions.

335. ***Equation of Payments*** is the process of determining the date at which two or more debts due at different times may be paid in one sum, without loss of interest to either party.

336. The ***Term of Credit*** is the time allowed for the payment of a note or account.

337. A debt is said ***to mature*** at the expiration of the term of credit.

338. The ***Equated*** or ***Average Time*** of payment is the date at which several items of debt due at different times may be equitably paid in one sum.

WRITTEN EXERCISES.

Example.—Find the average term of credit of \$200 due in 3 mo., \$450 due in 4 mo., \$500 due in $4\frac{1}{2}$ mo., \$350 due in 5 mo.

$$
\begin{array}{lll}
\$200 \times 3 & \text{mo.} = & 600 \text{ mo.} \\
\$450 \times 4 & \text{mo.} = & 1800 \text{ mo.} \\
\$500 \times 4\frac{1}{2} & \text{mo.} = & 2250 \text{ mo.} \\
\$350 \times 5 & \text{mo.} = & 1750 \text{ mo.} \\
\hline
\$1500 & & 6400 \text{ mo.}
\end{array}
$$

Ans. $4\frac{4}{15}$ mo. = 4 mo. 8 d.

The use of \$200 for 3 mo. is equivalent to the use of \$1 for 200 times 3 mo. = 600 mo.

The pupil may make a similar explanation for each item.

The use of \$1500 in parts, as specified in the example, that is, \$200 for 3 mo., \$450 for 4 mo., etc., is thus found to be equivalent to the use of one dollar for 6400 mo. But the use of \$1 for 6400 mo. is equivalent to the use of \$1500 for $\frac{1}{1500}$ of 6400 mo. = $4\frac{4}{15}$ mo., or 4 mo. 8 d. *Ans.*

Hence, to find the average term of credit for several sums of money, due at different times, by the

Method of Products.

339. ***Rule.***—**Multiply each item of the debt by its term of credit, and divide the sum of the products by the sum of the items; the quotient will be the average term of credit.**

Notes.—1. In computing terms of credit, it is customary to reject the cents in any item if less than 50; and, if 50 or greater, to reckon them as one dollar.

2. Less than $\frac{1}{2}$ day in a result is rejected; a $\frac{1}{2}$ day or greater fraction is counted as 1 day.

The Interest Method.

340. The time in which the *sum* of several items of indebtedness would become justly due is the time in which the use of the sum would balance the use of the items for the several terms allowed for their payment. Thus, in the foregoing problem,

The use of	$200	for 3	mo. at	6%	would be worth	$3.00
" "	$450	" 4	"	"	" "	$9.00
" "	$500	" $4\frac{1}{2}$	"	"	" "	$11.25
" "	$350	" 5	"	"	" "	$8.75
	$1500					$32.00

Thus we find that the use of the several items as allowed by agreement is worth $32. The question then is, How many months' use of $1500 would be worth as much? Hence the

Analysis.—The use of $1500 for 1 mo. at 6% per annum is worth $7.50, and, that its use may be worth $32, it must remain at interest as many times 1 mo. as there are times $7.50 in $32 = $4\,^4/_{15}$ times; $4\,^4/_{15}$ times 1 mo. = 4 mo. 8 d. *Ans.*

341. Any per cent. may be used in finding equated time by the method of interest. In solving the foregoing example, for instance, we may use 12% per ann. = 1% per month, as follows:

The interest of	$200	for 3	mo. at	1% per mo.	= $6.00
" "	$450	" 4	"	" "	= $18.00
" "	$500	" $4\frac{1}{2}$	"	" "	= $22.50
" "	$350	" 5	"	" "	= $17.50
	$1500				$64.00

$\$64 \div \$15 = 4\,^4/_{15}$. $4\,^4/_{15}$ mo. = 4 mo. 8 d.

The use of the several items for the terms of credit allowed having been found, as above, to be worth $64, we divide $64 by the interest of $1500 for 1 month at 1%, to find the term of credit for $1500. The interest of $1500 for 1 month at 1% per month = $15. $\$64 \div \$15 = 4\,^4/_{15}$. *Ans.* $4\,^4/_{15}$ mo. = 4 mo. 8 d. Hence the following:

342. ***Rule.***—**1. Find the interest of each item of debt for its term of credit at any assumed rate per cent.**

2. Divide the sum of the interests by the interest of the whole debt for any unit of time (day, month, or year), and the quotient will be the average term of credit in the denomination of the unit selected.

SLATE EXERCISES.

1. May 1, 1880, I purchased property for $8500, paid cash $1500, and gave notes, one for $3000, payable in 2 years, and another for $4000, payable in 4 years. Find the average term of credit on the notes.

2. Sept. 1, 1881, I bought goods, as follows: $200 on 2 mo. time, $400 on 3 mo., and $450 on 4 mo. What was the average term of credit, and the average date of maturity?

Ans. The average term of credit was 3 mo. 7 d., which, being added to Sept. 1, brought the average date of maturity on Dec. 8, 1881.

3. Jan. 15, I bought a bill of goods amounting to $900, $275 of which was on 30 days' credit, $300 on 60 days, and $325 on 90 days. What was the equated time of payment?

4. James Hudson, June 12, owes $317.75 due in 4 mo., $216.38 due in 5 mo., and $170 due in 6 mo. Find the equated time of payment and date of maturity.

5. William Owens bought a farm of 320 acres at $68 per acre, $1/4$ payable in cash, $1/4$ in 1 year, $1/3$ in 3 years, and the remainder in 5 years. What was the average term of credit?

6. Find the average term of credit:

$189.50 on 90 d.	$560.00 on 90 d.	$120.00 on 90 d.
$150.00 " 60 d.	$33.50 " 45 d.	$80.00 " 30 d.
$70.00 " 90 d.	$15.00 " 60 d.	$480.00 " 90 d.

7. Mrs. Handy bought a city lot, May 1, 1879, paying $60 cash, $120 in 10 mo., $150 in 15 mo., and $200 in 20 mo. What was the average term of credit on deferred payments?

8. On Feb. 1, 1880, Mrs. Handy paid the whole amount due; what discount was allowed her at 6 % (bank discount)?

9. Bought 5000 bu. of coal at 13¢ a bu., payable in 30 days. But I paid $300 after 10 days; what term of credit was I entitled to on the balance?

10. A debt of $2400 was contracted March 6, 1876, payable in 8 mo., but $400 was paid in 2 mo., $600 in 5 mo., $800 in 7 mo. What was the equitable time for paying the balance?

11. A stock of groceries was purchased Jan. 1, 1886, the purchase price payable as follows: $^1/_4$ in 1 mo., $^1/_4$ in 3 mo., $^1/_6$ in 4 mo., $^1/_3$ in 5 mo. When may the whole amount be equitably paid in one sum?

Inasmuch as the relation of each item of indebtedness to the whole amount, and its term of credit, are the only conditions necessary to be known, \$1 may be assumed as the purchase price, and the operation in many cases be performed orally, as here indicated.

$$
\begin{array}{rl}
\$\,^1/_4 \times 1 \text{ mo.} = & ^1/_4 \text{ mo.} \\
^1/_4 \times 3 \text{ mo.} = & ^3/_4 \text{ mo.} \\
^1/_6 \times 4 \text{ mo.} = & ^2/_3 \text{ mo.} \\
^1/_3 \times 5 \text{ mo.} = & ^5/_3 \text{ mo.} \\
\hline
\$\,1 \qquad\qquad = & 3^1/_3 \text{ mo.}
\end{array}
$$

12. John Doe sells to Richard Roe goods to the amount of \$3600; $^1/_4$ on 2 mo. credit, $^1/_3$ on 3 mo., and the balance on 4 mo. What is the average term of credit?

13. Nov. 1, 1881, I sold a horse and carriage for \$650, $^1/_4$ payable in 3 mo., $^1/_4$ in 4 mo., and $^1/_2$ in 6 mo. Find the equitable date for the payment of the whole sum.

14. Jan. 12, 1880, Thomas Kline sold a farm for \$2890, payable $^1/_4$ in cash, $^1/_3$ in 1 year, and the balance in 2 years (no interest). When should interest begin on the deferred payments?

15. A book dealer bought a stock of books and stationery for \$2400 on 4 mo. time, but in one month he paid \$600, and in 2 mo. \$800. In what time might he equitably pay the balance?

$$
\begin{array}{rcl}
600 \times 3 & = & 1800 \\
800 \times 2 & = & 1600 \\
\hline
1400 & & 3400
\end{array}
$$

$$2400 - 1400 = 1000$$

$$^1/_{1000} \text{ of } 3400 = 3\,^2/_5$$

$$4 \text{ mo.} + 3\,^2/_5 \text{ mo.} = 7\,^2/_5 \text{ mo. } \textit{Ans.}$$

In making the payments before due he lost the use of \$600 for 3 mo., and of \$800 for 2 mo., which, by the process just given, we find to be equivalent to the use of \$1 for 3400 mo. He was therefore entitled to keep the remainder of the debt (\$1000) *beyond the stipulated time*, long enough to balance the loss. The use of \$1 for 3400 mo. = the use of \$1000 for the $^1/_{1000}$ of 3400 mo. = $3\,^2/_5$ months. 4 mo. + $3\,^2/_5$ mo. = $7\,^2/_5$ mo. = 7 mo. 12 d. *Ans.*

16. Austin & Co., Oct. 12, 1880, bought a bill of goods to the amount of \$2480, on a credit of 4 mo., but paid \$700 on Nov. 9, and \$850 on Nov. 30. Find the equitable date for paying the balance.

When the terms of credit begin and mature at different dates.

Example.—Find the equitable date of a note which may be given in the settlement of the following account:

Levi Little

to *Nelson New* DR.

1881.	March	10,	to mdse. on	2	mo.	credit	$800
"	"	19,	"	3	"		$620
"	April	8,	"	4	"		$420
"	May	18,	"	3	"		$560

Taking the dates of maturity instead of the dates of purchase as given in the account, and finding the number of days from the earliest date to each of the succeeding ones, we obtain the average date of maturity as follows:

Due dates.	Items.		Days.		Products.
May 10,	$800	×	0	=	0
June 19,	$620	×	40	=	24,800
Aug. 8,	$420	×	90	=	37,800
" 18,	$560	×	100	=	56,000
	$2400				118,600

118,600 ÷ 2400 = $49\,{}^5/_{12}$. May 10 + 49 d. = June 28 *Ans.*

Or, taking the difference of time between the latest and each preceding date of maturity, and computing the average by a process similar to the foregoing one, we may determine the equated time of payment by counting backward instead of forward:

Dates.	Items.		Days.		Products.
May 10,	$800	×	100	=	80,000
June 19,	$620	×	60	=	37,200
Aug. 8,	$420	×	10	=	4,200
" 18,	$560	×	0	=	0
	$2400				121,400

1214 ÷ 24 = $50\,{}^7/_{12}$ d. Aug. 18 − 51 d. = June 28.

Thus we see that either the earliest or latest date of maturity may be assumed as the focal date, and that either process may be used to test the accuracy of the other.

EXAMPLES.

Find the average term of credit, and equated time of payment:

1. Purchased:				2. Purchased:			
Mar. 4,	$450,	2 mo. cr.		Aug. 20,	$500,	3 mo. cr.	
May 10,	$316,	1	"	Sept. 3,	$380,	2	"
July 9,	$420,	2	"	Oct. 19,	$295,	4	"
Aug. 1,	$500,	3	"	" 30,	$400,	1	"

3. A young man, having money advanced to help him pay his way through college, received:

Sept. 1, 1878,	$75.	Feb. 15, 1880,	$86.
Feb. 15, 1879,	$80.	Sept. 20, 1880,	$128.
Aug. 31, 1879,	$95.	Aug. 30, 1881,	$175.

What was the equated time at which he should date a single interest-bearing note for the whole sum?

4. Five years from the date of the first loan, the above-mentioned note was paid, with interest at 4%; what was the amount?

5. What is the average time at which the following bills become due? Feb. 10, 1882, $400 on 2 mo. credit; May 10, 1882, $300 on 4 mo. credit; June 16, 1882, $350; Aug. 6, 1882, $150.

6. Find the equitable date for a single note given on the following bills for merchandise: June 1, 1885, $20, July 1, $30, Aug. 1, $30, Sept. 1, $20, each on 2 mo. credit.

7. Bought goods of Messrs. Holt & Co., as follows: Mar. 11, $35, on 30 d. credit; July 20, $95, on 2 mo. credit; Sept. 8, $215, on 3 mo. credit. What was the average term of credit?

8. A credit of 5 mo. on $400, one of 3 mo. on $900, and one of 7 mo. on $600, are equivalent to a credit on how many dollars for one year?

9. Sold Mr. Long the following goods: May 2, two dozen ulsters, @ $18 each, on 3 mo. credit; June 21, six dozen vests, @ $2.50 each, on 2 mo. credit; Aug. 1, three dozen pique pants, @ $4 each, on 40 days' credit. Find the average term of credit and the equated time of payment.

Debit and Credit Accounts.

1. Find the equitable balance of the following account:

DR. *John Lock in account with Geo. Putnam.* CR.

1880.			1880.		
Oct. 2,	To mdse.	$180	Nov. 18,	By mdse. (2 mo.)	$150
Nov. 8,	" (3 mo.)	$120	Dec. 24,	" cash,	$200
Dec. 16,	" (4 mo.)	$240			

Explanation.—It is to be noticed that April 16, 1881 (4 mo. after Dec. 16), was the latest date for the complete maturity of any transaction recorded in the foregoing account. We therefore look into it to see how it stood on that day.

If the value of the goods bought, and the money paid, were the only matters for consideration, the sum due to Putnam, April 16, would have been $190. But each had also had from the other the use of money from the dates when it became due, or the days of payment, to the time of settlement. Hence, we find how many days' use of $1 is equivalent to the advantage which each one thus received from the other from and after each date to April 16.

Solution.

Due.	Am't.		Days.		Product.	Due.	Am't.		Days.		Product.
Oct. 2,	$180	×	196	=	35,280	Jan. 18,	$150	×	88	=	13,200
Feb. 8,	$120	×	67	=	8,040	———	$200	×	113	=	22,600
April 16,	$240	×	0				$350				35,800

$540 × ? = 43,320 no. days' int. on $1, due Putnam by Lock.
$350 35,800 " " $1, which Lock had paid.
Leaving $190 cash bal. and 7,520 bal. of days' interest on $1 due to Putnam.

$7520 \div 190 = 39\,^{11}/_{19}$ d. Practically, 40 days. *Ans.*

From this solution we find that, on the 16th day of April, Lock owed Putnam not only $190, but also an *interest balance* equivalent to the interest of $1 for 7520 days, which was equal to $1.25.

Evidently this difference could have been adjusted by Lock paying Putnam $190 + $1.25 = $191.25. This was the *cash balance* due April 16. But if not prepared to pay the cash, Lock would have given his note for $190, with interest, dated backward 40 days from April 16, so that it might be worth $191.25 on the day it was given.

If the *balance of items* and the *interest balance* had been on the other side of the account, Putnam would have been the debtor, and would have had to give his note dated back to the equated time of maturity, just as Lock was supposed to do.

But if the *balance of items* had been on one side, and the *interest balance* on the other, the average date of maturity would have been thrown forward instead of backward, and the note would have been dated accordingly.

SLATE EXERCISES.

2. When did a note given in settlement of the following account begin to bear interest:

DR. *L. R. Clem.* CR.

1880.			1880.		
July 2,	To mdse. (3 mo.)	$580	Aug. 14,	By cash,	$450

3. When did interest begin on the following account, and what was due on settlement, Jan. 1, 1882:

DR. *C. L. Hoosack.* CR.

1881.			1881.		
June 17,	To mdse. (2 mo.)	$270	June 30,	By mdse.	$250
Sept. 20,	" (3 mo.)	$650	Oct. 1,	By cash,	$500
Oct. 1,	" (1 mo.)	$100	Nov. 30,	By mdse.	$150

4. Find the cash balance due on the following account on the latest day of maturity:

DR. *W. M. Davis.* CR.

1882.			1882.		
Mar. 30,	To mdse. (60 d.)	$300	Mar. 10,	By mdse.	$180
April 2,	" (90 d.)	$700	June 20,	"	$980
July 16,	" (60 d.)	$150	July 27,	By draft,	$290

5. Find the equated time for the payment of the balance due on the following account:

DR. *W. T. Dawes.* CR.

1882.			1882.		
Mar. 1,	To mdse. (60 d.)	$200	Mar. 6,	By mdse.	$200
May 10,	" (60 d.)	$900	May 16,	By cash,	$150
June 20,	" (90 d.)	$400	June 26,	"	$360
July 30,	" (30 d.)	$700	July 1,	"	$990
Aug. 14,	" (60 d.)	$100	Aug. 28,	By mdse.	$240

Original Problems.

1. Any actual business transactions that may with propriety be reported to the class can be made a subject for one or more original problems.

2. Obtain from parents or friends copies of notes, the names thereon being changed; and ask the class to compute interest, amount, and proceeds at bank at current rates.

3. Ascertain about what it costs per year to board and clothe a school boy or girl, and how much money must be invested at current rates to produce that much interest.

4. Suppose that you buy a vacant lot for a given sum, and in a number of years sell it for less than you gave for it. Ask the class how much you would lose by the transaction, supposing the use of your money to be worth current rates.

5. Construct for the class a problem in annual, compound, exact interest, etc.

6. Ask what sum you should put at interest that in a given number of years it may amount to enough to erect a public library building at any cost you think desirable.

7. Find the rent paid for some particular house, and what the taxes on it are, and ask the class whether it would be profitable to buy it at the price asked, not forgetting insurance at current rates and the use of money.

8. Ask which is to be preferred by the creditor, compound or annual interest, and how much one would be worth more than the other on any given sum for any given time and rate per cent.

9. Ask the yearly interest on some of the national, State, or city loans that may be noticed in the newspapers from time to time. On the debt of your own city.

10. Write a promissory note, indorse three or four partial payments, and ask the amount due, etc.

11. Suppose some business transactions with a school-mate, which require an equation of payments.

CHAPTER XVII.

PROPORTION.

ORAL EXERCISES.

1. A boy buys 3 lemons for 7¢. How much, at the same rate, would he have to pay for 6 lemons? 9, 12 lemons?

Suggestion.—6 lemons are how many times 3 lemons? How many times as much will they cost?

2. If 3 dozen of eggs are worth 25¢, what is the worth of 9 doz.? 15 doz.? 12 doz.?

3. Mr. Jones walks 17 miles in 5 hours. At the same rate, how many miles will he walk in 15 h.? 30 h.? 40 h.?

4. If 6 oranges can be had for 21¢, what will 2 cost?

Suggestion.—What part of 6 oranges is 2 oranges? What part of the cost of 6 should be the cost of 2?

5. If 14 bl. flour cost $35, how many can be had for $105?

6. If a number of hands can plow 5 acres in 6 hours, how many acres can they plow in 48 hours?

7. Sixteen lb. cost 36¢. At the same rate, what will 12 lb. cost?

8. A lot, 32 ft. wide, costs $500. What will a lot, measuring 96 ft. in width, cost at the same rate?

9. A courier travels on an average 156 miles in 3 days? How far will he travel in 12 days? 18 d.? 24 d.?

10. What is the height of a steeple, that casts a shadow of 300 feet, if at the same time a staff, 2 feet high, casts a shadow of 3 feet?

SLATE EXERCISES.

1. If a man can earn \$23 in 13 days, how much can he earn, at the same rate, in 221 days?

Solution.—We know that, at the same rate of wages per day,

221 days' wages must be as many times 13 days' wages as 221 days are times 13 days.

But, since the wages of 13 days are \$23, we may as well have written

221 days' wages are as many times \$23 as 221 days are times 13 days.

And since 221 days are 17 times 13 days, the wages of 221 days must be 17 times \$23 = \$391 *Ans.*

Inasmuch as the words printed in small type are invariably the same, whatever the nature of the question, signs may be substituted for them. Thus,

$$\text{221 days' wages} \div \$23 = \text{221 days} \div \text{13 days.}$$

This form may be read exactly as the preceding one, though it is more common in this case to use the colon (:) for the sign of division (÷) and a double colon (::) for the sign of equality. Thus,

$$\text{221 days' wages} : \$23 :: \text{221 days} : \text{13 days.}$$

Since 221 days are 17 times 13 days, hence the wages for 221 days must be 17 times the wages of 13 days. 17 times \$23 = \$391 *Ans.*

2. A man who travels at the rate of 258 miles in 13 days will travel how far in $32\frac{1}{2}$ days?

$$\text{Miles traveled in } 32\tfrac{1}{2} \text{ days } (?) : \text{miles } 258. :: \text{days } 13\tfrac{1}{2} : \text{days } 13.$$

Explanation.—The man will travel as many times 258 miles in $32\frac{1}{2}$ days as $32\frac{1}{2}$ days are times 13 days. Having found, from the second pair of terms, that $32\frac{1}{2}$ days are $2\frac{1}{2}$ times 13 days, and hence knowing also that the first term of the first pair must be $2\frac{1}{2}$ times the second term, we *make it so* by multiplying 258 by $2\frac{1}{2}$, and thus obtain the required answer.

3. A lady bought 15 yards of calico for \$1.40. At the same rate, how much would she have paid for 42 yards?

Statement.

Cost of 42 yds.		Cost of 15 yds.		Yards.		Yards.
?	:	\$1.40	::	42	:	15

Solution.—$\$1.40 \times \frac{42}{15} = \$3.92.$

4. James purchased 15 acres of farm-land for \$72. How much did William have to pay for $37\frac{1}{2}$ acres at that rate?

5. A locomotive runs 18 miles in 30 min. How many miles does it run in 50, 65, 72, 81 min.?

6. A ship sailed $47\frac{1}{2}$ miles in 5 hours. How long will she be in sailing 180 miles?

7. If 7 men eat $10\frac{1}{2}$ loaves of bread a week, how many will 25, 67, 39 men eat at the same rate in the same time?

8. What is the height of a tower which casts a shadow of 210 feet, when a pole, 15 feet high, casts a shadow of 18 feet?

Inverse Proportion.

Example.—1. A house was painted by 8 men in 6 days. How many men would have been required to do the same work in 12 days?

Note.—The following erroneous statement is likely to be made:

The number of men required to do the work in 12 days	is as many times	the number of men required to do the same work in 6 days	as 12 days is times 6 days.

When solved, it would lead us to the conclusion that it would take 16 men 12 days to do a work which requires only 8 men 6 days, which is absurd. If we allow double the time for any work, we do not need twice as many men to do it, but only half as many.

The statement then should be:

The number of men required to do the work in 12 days	is such part of	the number of men required to do the work in 6 days	as 6 d. is of 12 d.

	Men.		Men.		Days.		Days.
Or, using the shorter form:	?	:	8	::	6	:	12.

We reason that, since 6 days is one half of 12 days, the number of men required to do the work in 12 days is only one half as many as would be required to do it in 6 days. *Ans.*, 4.

In all the problems preceding this, *more required more*; that is, *more goods* required *more money*, *more work* required *more men*, etc., etc. But, as we see in this example, there are problems in which *more requires less* and *less requires more.*

ORAL EXAMPLES.

2. If a man can perform a journey in 6 days, traveling 12 hours a day, how many days will be required if he travels only 6 hours a day? 4 hours? 3 hours?

3. The owner of a livery stable sends 16 horses to pasture for 8 days. How many could he send for the same money for 16 days? 4 days?

4. Six masons can perform a certain work in 24 days. How long will it take 12 men? 18 men?

5. Six horses consume a certain quantity of oats in 12 days. How long will it feed 36 horses?

6. Fifteen farm hands will mow the harvest of a farm in 8 days. How many will do it in 2 days? In 24 days?

SLATE EXERCISES.

7. Fred has to walk 250 steps from his house to school, each of his steps measuring 18 in. His brother's steps measure 27 in. each. How many steps does his brother take?

8. A messenger who traveled 12 miles an hour reached his point of destination in 3 hours. How long would it have taken him had he traveled only 8 miles per hour? 6 miles? 4 miles?

9. If he had traveled $4^1/_2$ miles an hour, how many hours would it have taken him.

10. Thirty sailors can subsist on their provisions 4 months. At the same rate, how long will the same provisions last 20 sailors?

11. Suppose the same provisions would last 30 men $4^1/_2$ months. Find how many months and days they would last 20 men.

12. A carter agrees to transport $7^1/_2$ cwt. 6 miles for a certain sum. How far will he carry 9 cwt. for the same money?

13. A bridge was built by 15 workmen in 4 weeks and 4 days. How many would have built it 8 days sooner?

14. A certain number of trees was felled by 28 men in 4 weeks and 3 days. How many could have done it in 12 days?

Ratio and Proportion.

343. A comparison of two numbers is made by showing how many times, or what part, one number is of another.

344. In any comparison of numbers there must be at least *two numbers*, or *quantities*, compared.

345. Two numbers or quantities thus compared are together called a ***couplet.*** The *first* is called the ***antecedent*** (the one going before); the *second*, the ***consequent*** (the one coming after).

346. The antecedent and consequent are called the ***terms*** of the couplet.

347. The consequent is the ***standard of comparison.***

348. The relation of two numbers, that is, the *quotient* obtained by dividing the antecedent by the consequent, is called the ***ratio*** of those numbers.

349. ***Proportion*** is an ***equality of ratios.***

For instance, the following is a proportion:

$$\frac{\text{The cost of 9 yd.}}{37\tfrac{1}{2}¢} \ \text{is as many times}\ \frac{\text{the cost of 3 yd.}}{12\tfrac{1}{2}¢} \ \text{as}\ \frac{\text{yd.}}{9} \ \text{are times}\ \frac{\text{yd.}}{3}$$

The arithmetical form for the statement of which is:

$$\overset{\text{Cost.}}{37\tfrac{1}{2}¢} : \overset{\text{Cost.}}{12\tfrac{1}{2}¢} :: \overset{\text{Quantity.}}{9\ \text{yd.}} : \overset{\text{Quantity.}}{3\ \text{yd.}}$$

350. The double colon :: is the special sign of a proportion.

Note.—The colon (:) is a sign of division, the line between the dots being omitted. The sign of equality (=) is often used instead of the double colon (::). Thus the statement of a proportion frequently appears in this form:

$$\overset{\text{Cost.}}{37\tfrac{1}{2}¢} \div \overset{\text{Cost.}}{12\tfrac{1}{2}¢} = \overset{\text{Quantity.}}{9\ \text{yd.}} \div \overset{\text{Quantity.}}{3\ \text{yd.}}$$

351. A proportion must contain at least two couplets. The first and second terms make the *first couplet;* the third and fourth terms, the *second couplet.*

352. The first and fourth terms of a proportion are called the ***extremes;*** the second and third are called the ***means.***

EXERCISES IN FINDING RATIOS

Find the ratios of the following couplets :

The consequent being the standard of comparison, the question to be answered, in each case, is, How does the antecedent compare with the consequent? *How many times*, or, *What part of*, the consequent is the antecedent?

1. 18 : 9 =	**4.** 95 : 19 =	**7.** 13 : 4 =	**10.** 42 : 8 =
2. 36 : 18 =	**5.** 18 : 12 =	**8.** 16 : 15 =	**11.** 48 : 5 =
3. 72 : 12 =	**6.** 85 : 17 =	**9.** 51 : 3 =	**12.** 88 : 9 =

Note.—The quotient, arising from dividing one number by another, may be expressed in the form of a fraction, thus, $15 \div 3 = {}^{15}/_{3}$, which, being narrowed down to lowest terms, is equal to 5. In writing out the foregoing exercises, the pupil may therefore adopt the following form: $15 \div 3 = {}^{15}/_{3} = 5$. He should recollect that $15 \div 3$, 15 : 3, and ${}^{15}/_{3}$, indicate the same thing, viz., that 15 is to be divided by 3.

Questions.—When we compare a greater number with a less, is the ratio greater or less than a unit?—Can a ratio be expressed by a mixed number? By a fraction?—Why is the ratio of 32 : 8 (read, 32 to 8) not greater than that of 8 : 2?—Give other couplets having the same ratio as 32 : 8. As 15 : 3, etc.—If you double both numbers does the ratio increase? Why not?—Is the ratio of the halves of two numbers the same as the ratio of the numbers themselves? Why?

13–28. Find the ratios of the following numbers :

Note.—Express the ratio in the form of a fraction, and reduce to lowest terms.

5 : 25 = ($^5/_{25} = {}^1/_5$)	18 : 54 =	27 : 108 =	15 : 25 =
9 : 72 =	28 : 42 =	17 : 93 =	33 : 132 =
3 : 33 =	16 : 96 =	23 : 69 =	31 : 124 =
6 : 72 =	19 : 104 =	14 : 98 =	13 : 117 =

29–52. Find the ratios of the following fractions :

$^1/_2 : {}^1/_8$ =	$^1/_8 : {}^1/_2$ =	0.6 : 0.12 =	0.16 : 0.4 =
$^1/_3 : {}^1/_5$ =	$^3/_4 : 1^1/_2$ =	0.5 : 0.05 =	0.33 : 0.3 =
$^3/_4 : {}^1/_3$ =	$2^1/_3 : 5^1/_6$ =	0.35 : 0.07 =	3.5 : 6.5 =
$^1/_6 : {}^1/_{12}$ =	$23^3/_8 : 8^1/_2$ =	27.2 : 3.6 =	2.6 : 10.4 =
$^1/_2 : {}^1/_2$ =	$4^3/_5 : 8^2/_5$ =	3.35 : 0.07 =	8.45 : 10.25 =
$^1/_2 : {}^1/_3$ =	$2^1/_4 : 3^1/_2$ =	0.26 : 0.07 =	0.01 : 0.5 =

Fill the blanks in the following statements:

1. $42 : — = 7$	**5.** $4 : — = 1/2$	**9.** $10.4 : — = 4$
2. $— : 19 = 3$	**6.** $4/8 : — = 3/4$	**10.** $23\,5/8 : — = 2\,3/4$
3. $18 : 1/3 = —$	**7.** $3 : 15 = —$	**11.** $10 : — = 2$
4. $27 : — = 1/3$	**8.** $36 : — = 7\,1/2$	**12.** $— : 3 = 9$

Suggestion.—To fill the blank in the first problem the pupil has to answer the question, "42 is 7 times what number?" The second, "What number is 3 times 19?" The third, "18 is how many times $1/3$?" The fourth, "27 is $1/3$ of what number?"

From the foregoing definitions and exercises we may derive the following

Rules.

353. ***Rule.*—1. Multiply the consequent by the ratio; the product will be the antecedent.**

2. Divide the antecedent by the ratio; the quotient will be the consequent.

13–18. Prove the following proportions to be correct:

$3 : 4 :: 6 : 8$	$3 : 8 :: 6 : 16$	$3/5 : 5/6 :: 9/10 : 1\,1/4$
$2 : 8 :: 6 : 24$	$1.05 : 8.4 :: 1 : 8$	$0.05 : 7 :: 0.3 : 42$

19–33. Fill the blanks in the following statements, determining the ratios from the completed couplets:

$7 : 56 :: — : 16$	$— : 3 :: 16 : 9$	$— : 5 :: 3\,1/3 : 6\,2/3$
$20 : 5 :: — : 2$	$5 : — :: 8 : 64$	$18 : 6 :: 21 : —$
$15 : — :: 6 : 18$	$3/4 : 1 :: — : 12$	$1/1 : 1/8 :: — : 24$
$— : 21 :: 56 : 8$	$5/7 : 1 :: 60 : —$	$0.2 : — :: 6 : 108$
$2/3 : — :: 6 : 9$	$17 : 51 :: — : 48$	$10\,1/2 : — :: 19 : 95$

From the preceding principles and exercises we derive the following

354. Rules for finding the Missing Term of a Proportion.

***Rule.*—1. Find the ratio of the complete couplet; then,**

2. If the antecedent of the incomplete couplet be wanting, multiply the consequent by the ratio; or,

3. If the consequent be wanting, divide the antecedent by the ratio.

4. The result will be the term required.

EXERCISES IN PROPORTION.

1. If 24 hats cost \$44, what will 150 hats cost?

Note.—To avoid error in the statement of a proportion, an arrangement of the terms of the question such as the following is recommended:

Complete couplet.	Incomplete couplet.
?	is the cost of 150 hats
if \$44	" " 24 "

It matters not whether the complete or the incomplete couplet is placed first, nor whether the sign for the wanting term be the first or last of the proportion; but it is essential, in all questions in which more requires more and less requires less, that, if the upper term be taken as the antecedent in one couplet, the same should be done with the other.

In a problem in which more requires less, and less requires more, it is necessary to invert the terms of the complete couplet in the statement. For example

2. If it requires 7 men to build a wall in 27 days, in how many days would 9 men perform the same work?

Preliminary Arrangement.

Days.	Men.
?	↓ 9
27	7

Note.—An arrow pointing downward may be used to indicate terms to be inverted.

Since 9 men require less time to do the work than 7 men, the terms of the complete couplet are inverted, and the statement is:

$$\underset{\text{Days.}}{—} : \underset{\text{Days.}}{27} :: \underset{\text{Men.}}{7} : \underset{\text{Men.}}{9} \qquad 27 \times \frac{7}{9} = 21 \text{ days } Ans.$$

3. How many tons of hay will 325 acres produce if, at the same rate, 13 acres produce 40 tons?

4. What time would it require for 7 men to mow a field, if 3 men can mow it in $3\frac{1}{3}$ days?

5. At Christmas 8 eggs were sold for 25¢. What was the cost of 6 dozen?

6. Farmer Black pays $\$52\frac{1}{2}$ rent for 24 acres of land. At the same rate, what will he have to pay for 51 acres?

7. Mr. H. agrees to do certain work in 15 days, thereby earning \$3.20 a day. How much will he earn a day if he does the work in 10 days? In $13\frac{1}{2}$ days?

8. In canning 5 lb. of raspberries 3 lb. sugar are needed. How many pounds sugar for 38 lb. of berries?

9. If with the money I have, I can buy 84 lb. of coffee at 25¢ a lb., how many lb. could I buy for the same money at 30¢ a lb.?

10. If 3 yd. of calico cost 20¢, what will $^4/_5$ yd. cost?

Arrangement.	Statement.	Solution.
? is the cost of $^4/_5$ of 20 " " 3	? : 20 :: $^4/_5$: 3	$20 \times \frac{^4/_5}{3} = 5\,^1/_3$ *Ans.*

11. If wall paper be 20 inches wide, I shall need 7 rolls to paper a room. How many rolls will suffice if the paper be 24 inches wide? If 30 inches wide?

12. If $750 will yield $120 interest in a certain time, what interest will $600 yield in the same time?

13. A man, whose step measures $^5/_8$ yard, counts 1200 steps from his house to his office. How many steps will his son have to take, whose step measures $^1/_2$ yd.?

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14. If each man on board ship consumes daily $1\,^1/_4$ lb. bread, their bread will last $5\,^1/_5$ months. How much will each man get per day if it is to last $6\,^1/_2$ months?

15. The rate of two pedestrians is as 5 : 4. How many miles will the first travel in the same time in which the second travels $84\,^1/_2$ miles?

16. At the rate of $180 for $^3/_{10}$ acre, what will 5 acres cost?

17. The heat produced by a cubic yard of beech-wood is to that produced by a cu. yd. of pine as 9 : 7. How many cu. yd. of beech-wood are needed to produce the heat of 50 cu. yd. of pine?

18. If $1\,^3/_4$ yards of velvet cost $\$5\,^1/_2$, what will 9 yd. cost?

19. A farmer sowed 3 bu. of buckwheat on $2\,^2/_5$ acres. How much would he need for a field containing $4\,^1/_2$ acres?

20. $^2/_7$ of a sum of money is $800. How much is $^5/_8$ of it?

21. If bread is 7¢ a loaf when flour is sold at $6 a barrel, what should flour be worth when bread is sold at 8¢ a loaf?

Compound Proportion.

In the foregoing exercises the *ratio* for the *incomplete couplet* is found from one complete couplet; for example:

1. If 8 boys can pile up 7 piles of cord wood in a day, how many boys would be required to lay up 21 piles of the same size in the same time?

Here we know the ratio of the required number of boys to the given number, from the fact that it must be the same as that of 21 piles to 7 piles. Hence the proportion is:

Statement.

Boys.		Boys.		Piles.		Piles.
?	:	8	::	21	:	7.

Solution.—$8 \times \frac{21}{7} = 24$ boys.

But sometimes the ratio of the incomplete couplet depends on the ratios of two or more complete ones; for instance, let problem 1 be changed by the addition of the words printed in italics, as follows:

2. If 8 boys can pile up 7 piles of cord wood, *each 12 feet long,* in 1 day, how many boys could pile up 21 piles, *each 6 feet long,* in the same time, the height and width being the same?

In this problem the number of boys required evidently depends—first, on the number of piles to be made; and, second, on the length of the piles.

The way to indicate this dependence of a term on two or more ratios is to write one of the completed couplets under the other, as follows:

$$? : 8 :: \begin{matrix} 21 : 7 \\ 6 : 12 \end{matrix}$$

Solution.—$8 \times \frac{21}{7} \times \frac{6}{12} = 12$ boys.

Taking it for granted that wood is as easily piled 6 feet high as 4 feet, the question may be again extended, as follows:

3. If 8 boys can pile up 7 piles of cord wood, each pile being 12 feet long and *4 feet high,* how many boys can pile up 21 piles, each 6 feet long and *6 feet high?*

Here the number of boys required depends—first, on the number of piles; second, on the length of each; and, third, on the height.

This dependence is indicated thus:

$$? : 8 :: \left\{ \begin{matrix} 21 : 7 \\ 6 : 12 \\ 6 : 4 \end{matrix} \right.$$

Solution.—$8 \times \frac{21}{7} \times \frac{6}{12} \times \frac{6}{4} = 18$ boys.

The question may be still further extended by introducing the element of time; thus:

4. If 8 boys can pile up 7 piles of cord wood, each pile 12 feet long and 4 feet high, *in 2 days*, how many boys can pile up 21 piles, 6 feet long and 6 feet high, *in 3 days?*

In this example the number of boys required depends—first, on the number of piles; second, on the length; third, on the height; and, fourth, on the number of days.

When there are so many conditions in a problem, it is convenient to make a preliminary arrangement of all the terms, as follows:

$$\begin{array}{llllll} ?\text{ boys,} & 21\text{ piles,} & 6\text{ feet long,} & 6\text{ feet high,} & \big\downarrow & 3\text{ days.} \\ 8\ \text{"} & 7\ \text{"} & 12\ \text{" "} & 4\ \text{" "} & & 2\ \text{"} \end{array}$$

It is to be observed that the greater the number of days the less will be the number of boys required. Hence the last couplet will have to be inverted in the statement.

To avoid making a misstatement in such cases, the couplet which is to be inverted should be distinguished from the rest, and since in each preceding couplet the upper number is taken for the antecedent, an arrow may be placed before this one, the head pointing downward to indicate that here the lower term is to be taken as the antecedent.

The statement will then be:

$$? : 8 :: \left\{\begin{array}{l} 21 : 7 \\ 6 : 12 \\ 6 : 4 \\ 2 : 3 \end{array}\right. \qquad \textbf{Solution.}—8 \times \frac{21}{7} \times \frac{6}{12} \times \frac{6}{4} \times \frac{2}{3} = 12 \text{ boys.}$$

Note.—The pupil will observe that the preliminary statement is all-sufficient for the solution, care being taken to invert the terms whose ratios are inverse.

Definitions.

355. A ***Simple Proportion*** is an equality of two simple ratios. Thus, 8 : 32 :: 9 : 36 is a simple proportion.

356. A ***Compound Proportion*** is an equality between a simple and a compound ratio or between two compound ratios. Thus,

$$12 : 8 :: \begin{array}{l} 21 : 7 \\ 6 : 12 \end{array} \text{ and } \begin{array}{l} 6 : 4 \\ 3 : 9 \end{array} :: \begin{array}{l} 2 : 6 \\ 3 : 2 \end{array} \text{ are compound proportions.}$$

EXERCISES IN COMPOUND PROPORTION.

1. Five clerks use 25 quires of paper in 8 days. At the same rate, how much paper will 6 clerks use in 10 days?

2. Six lamps consume 2 gallons of petroleum in 8 days. How many lamps will consume 3 gal. in 14 days?

3. Two workmen dig a ditch of 24 yd. in length and 3 ft. in width in 5 days. How long will it take 3 workmen to dig a ditch 30 yd. long and 4 ft. wide?

4. Eight persons spend \$296 on a journey of 7 days. How long will \$300 last 7 persons at that rate?

5. If a block of marble 5 ft. long, 3 ft. wide, 2 ft. thick, weighs 4850 lb., what will a block weigh measuring 7 ft. in length, 4 ft. in width, and 3 ft. in thickness?

6. Ten cwt. of merchandise cost $\$2^1/_2$ freight for 245 miles. What will 5 cwt. cost for 210 miles?

7. If \$700 at interest amounts to \$770 in 15 months, what sum must be put at the same rate to amount to \$845 in the same time?

8. From the milk of 20 cows, each giving 18 qt. daily, 16 cheeses of 50 lb. each are made in 38 days. How many cows, giving but 16 qt. daily, will be needed to make 33 cheeses of 60 lb. each in 28 days?

9. Being asked to find the number of bricks in a wall 10 ft. high, 922 ft. long, and 16 in. thick, I found that a part of the wall, 4 ft. high, 4 ft. long, and 16 in. thick, contained 448 bricks. How many in the whole wall?

10. Being asked to find the probable cost of a lot on Seventh Street, 52 ft. front and 98 ft. deep, I thought of my own lot close by, which is 24 ft. front and 75 ft. deep, and which cost me \$1680. What should the answer be?

11. If 450 copies of a book containing 300 pages require 12 reams of paper, how much paper will be needed to print 1500 copies of a book of 170 pages?

Miscellaneous Problems in Proportion.

1. Seven men need 16 days' time to repair a dam. How many men will be required if the work must be completed in 14 days?

2. The prices of rye and wheat are to each other as 5 : 6. What is the price of wheat if rye sells at 80¢? At 75¢?

3. For $^7/_{10}$ yd. lace a lady pays \$5.60. At the same rate, what does the merchant ask for the whole piece of 16 yd.?

4. The spire of Nicolai Church at Hamburg throws a shadow of 49 yd. in length when a vertical staff $2\,^1/_2$ yards high throws a shadow of $^{245}/_{308}$ yd. What is the height of the spire?

5. John takes 1200 paces in a mile. How many paces must Harry make, who makes 9 paces to every 8 of John's? What would be the ratio of John's paces to Harry's?

6. A pavement was supposed to be 329 ft. long, but the measuring-line being found 50 ft. $3\,^1/_2$ in. instead of 50 ft., it is required to find the true length of the pavement without another measurement.

7. A bag of coffee was supposed to weigh 250 lb., but the 50 lb. weight used in weighing was really 50 lb. 5 oz. Find the true weight of the coffee.

8. The liter of the metric system = 1.0567 qt. What will represent metrically the contents of a gallon?

9. A gram is equal to 0.03527 oz. Avoirdupois. What will represent metrically the weight of a pound?

10. If a meter is 39.37 inches, what is the length of a yard by the metric system?

11. If a man owes \$15,850, and has but \$9750 to pay it with, what will a creditor receive to whom \$1200 are due?

12. If a wheel, $6\,^1/_4$ ft. in circumference, turns 884.8 times in going a given distance, how many times will a wheel, $9\,^1/_2$ ft. in circumference, turn in going the same distance?

13. If $8\frac{1}{4}$ lb. butter will pay for $4\frac{2}{5}$ lb. tea, how much butter will pay for 100 lb. tea?

14. If \$360 gain \$40.80 in 1 yr. 5 mo., what sum will \$480 gain in 2 yr. 10 mo. at the same rate?

15. A., B., and C. are partners. A. invests \$5050 in the business of the firm, B. \$7070, and C. \$3030. They gain \$4545. What is the share of each in the gain?

Suggestion.—Each man's share of the gain is to the whole gain as each man's stock is to the whole stock.

16. Three men freight a steamer: the first puts \$24,000 worth of merchandise aboard; the second, \$18,000 worth; the third, \$15,000 worth. During a storm \$1900 worth of the freight is thrown overboard. What is each man's share of the loss?

Suggestion.—Each man's share of loss is to the whole loss as each man's freight is to the whole freight.

17. If it takes 21.78 paving blocks to pave 5 rods square, how many □ rods will 1,197,900 blocks cover?

18. I sent to my agent \$2575 to invest after deducting his commission of 3%. What was his commission? What sum did he invest?

19. I send to my broker \$1224 for him to invest after deducting his commission of 2%. What does he invest? What is his commission?

20. When a bushel of wheat was sold for \$1 the price of a loaf of bread was 5¢. What will be the price of a loaf of equal weight when wheat is sold at $\frac{4}{5}$ dollar a bushel?

21. If it costs \$380 to construct a wall 30 yards long, 10 feet high, and 16 inches wide, when labor is worth \$2 per day of 10 hours each, and the price of bricks is \$7 per M; what will it cost to build a wall 40 yards long, 8 feet high, and 12 inches thick, when the price of labor is \$1.80 per day of 8 hours each, and bricks are worth \$9 per M?

CHAPTER XVIII.

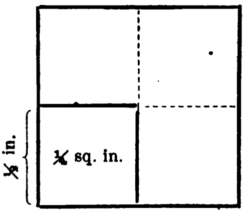
SQUARES AND CUBES.

357. A square measuring 3 in. on each side contains 9 sq. in.; hence 9 is said to be the square of 3. For a like reason, 16 is said to be the square of 4, 25 of 5, $\frac{1}{4}$ of $\frac{1}{2}$, etc.

Find the squares of

2	3	7	9	$\frac{1}{2}$	.05
0.8	$\frac{1}{5}$	$\frac{2}{3}$	$\frac{4}{7}$	$\frac{8}{9}$	$\frac{6}{11}$

It may seem strange to the pupil that the square of $\frac{1}{2}$ is $\frac{1}{4}$, but, to convince himself that it is so, he has only to draw a square having a side one inch in length, and to divide it into four equal parts, as the figure in the margin. He will readily see that a square, measuring $\frac{1}{2}$ inch on each side, will contain but $\frac{1}{4}$ of a square inch.



In the same way illustrate the squares of .3, $\frac{1}{5}$, $\frac{1}{3}$, $\frac{2}{3}$.

358. A cubic block, measuring 3 in. on the edge, contains 27 cubic inches, hence 27 is said to be the cube of 3. For a like reason, 64 is said to be the cube of 4, $\frac{1}{8}$ of $\frac{1}{2}$, etc.

What are the cubes of

1.	.1	$\frac{1}{4}$	$\frac{4}{9}$	5.	.3
$\frac{2}{5}$	$\frac{4}{5}$	6.	.7	$\frac{1}{6}$	$\frac{3}{8}$

Note.—The cubes of integers and fractions should be frequently illustrated by the use of modeling clay, cubic blocks, etc.

359. To *square a number,* we multiply the number by itself; that is, we use it twice as a factor to produce the square. Hence, the square of a number is also called its ***second power.***

360. To *cube a number*, we multiply the square of the number by the number itself; that is, we use the number three times as a factor to form the cube. Hence, it is said that the cube of a number is its ***third power***.

Definitions.

361. A ***power of a number*** is the number itself, or the product obtained by the use of the number two or more times as a factor. The number itself is called the ***root*** of the power.

362. The number of times a root is employed as a factor is indicated by an ***exponent***, which is commonly a small figure written to the right and a little higher than the root.

7^2 indicates the second power of 7, hence $7^2 = 49$.

7^3 indicates the third power of 7, hence $7^3 = 343$.

363. The process of raising a number to any required power is called ***Involution***.

The process of involution is a process of simple multiplication; therefore no rule is necessary, except that the root is to be used as a factor as many times as there are units in the exponent.

EXAMPLES FOR PRACTICE.

Raise the following numbers to the powers indicated:

1. 17^3	4. 38^2	7. 12^3	10. 33^3	13. 2.15^2
2. 3.2^2	5. 1.3^3	8. 0.1^3	11. 128^2	14. $(5\frac{1}{3})^2$
3. $(\frac{1}{7})^3$	6. $(\frac{1}{6})^2$	9. $(\frac{2}{3})^3$	12. 325^2	15. $(7.1\frac{1}{4})^3$

Find the values of

16. $12^2 \times 2$	18. $(7^2 \times 3^3) + 10$	20. $(9^2 \div 3^3) \times 7^2$
17. $9^2 \times 2^2$	19. $3^3 \times (4^2 + 2)$	21. $\dfrac{7^2 - 3^2}{(5^2 \times 2) + 10}$

Raise to the second power:

22. 97	24. 35	26. 128	28. 826	30. 5287
23. 98	25. 47	27. 371	29. 981	31. 6520

Write answers to the following questions:

1. How many places are there in the second power of a number expressed by one figure? By two figures? By three figures? By four figures?

2. Does the power always contain twice as many figures as there are in the root? Does it ever contain more than twice as many? May it contain less?

Suggestion.—To answer the foregoing questions correctly, and with confidence, the pupil should square the greatest and the smallest numbers that may be expressed by one figure (1 and 9), by two figures (10 and 99), etc.

3. How many decimal places are there in the second power of a decimal expressed by one figure? By two figures? By three figures? By four figures? In the square of a decimal, should we ever have more than twice as many decimal places as there are in the root? Should we ever have less? Why not?

Raise to the third power:

1. 1	5. 10	9. 100	13. 1000	17. 10,000
2. 9	6. 99	10. 999	14. 9999	18. 99,999
3. .1	7. .01	11. .001	15. .0001	19. .00001
4. .9	8. .99	12. .999	16. .9999	20. .99999

Note.—The square of .999 may be conveniently found by subtracting .999 from 999. Why? The same method may be applied to any other number expressed by 9's.

Write answers to the following questions:

1. How many places in the third power of an integer expressed by one figure? By two figures? By three figures? By four figures?

2. How many times as many figures in the power as there are in the root? Are there ever more than three times as many? Are there ever less? How many less may there be?

3. How many places are there in the third power of a decimal of one place? Of two places? Of three places? etc. In the cube of a decimal, should we ever obtain either more or less than three times as many decimal places as there are in the root? Why not?

364. Another method of raising numbers to the second and third power.

It will be found useful to note carefully how the tens and units are combined in the process of raising a number, expressed by two or more figures, to its second or third power. For illustration, let us take the example:

Raise 43 to its second power.

Analytical Method.

$$\begin{array}{ccccccc}
 & & 40 & & +3 \\
 & & 40 & & +3 \\
\hline
 & & 40 \times 3 & + & 3 \times 3 \\
40 \times 40 & + & 40 \times 3 & & \\
\hline
\underbrace{40 \times 40}_{\text{sq. of the tens.}} & + & \underbrace{2 \times (40 \times 3)}_{\text{twice the tens by the units.}} & + & \underbrace{(3 \times 3)}_{\text{sq. of the units.}}
\end{array}$$

Little explanation is here necessary, except that, in the analytical method, instead of actually multiplying and adding, as in the common process, we only *indicate* the multiplications and additions by means of signs. This we do, that we may trace the tens and units separately through the process, to see where we may find them in the product. Thus, in this case we see that

365. *The square of 43 is equal to the square of the tens, plus twice the product of the tens by the units, plus the square of the units.*

It may be shown that the same is true of any number.

Raise the following numbers to the second power, using the analytical method:

21. 26	**25.** 64	**29.** 76	**33.** 94
22. 35	**26.** 78	**30.** 23	**34.** 72
23. 45	**27.** 59	**31.** 54	**35.** 63
24. 53	**28.** 82	**32.** 83	**36.** 49

Note.—The pupil should be able to perform these operations without the aid of the pencil. For the last he would say *1600*, *720*, *2320*, *2401* the square of 49. If required to write out the work, he would write as in the margin.

$$\begin{array}{rr}
40^2 = & 1600 \\
2\,(40 \times 9) = & 720 \\
9^2 = & 81 \\
\hline
49^2 & 2401
\end{array}$$

A like process may be used in raising numbers, expressed by three or more figures, to the second power.

Example.—1. Raise 493 to the second power.

The square of 490 is obtained by annexing two ciphers to the square of 49, as already found	=	240100
Twice the product of 490 × 3	=	2940
The square of the units	=	9
The square of 493		243049

Note.—The pupil will do well to familiarize himself with this process, not for its own sake, but that he may be the better prepared for the demonstration of the process of extracting the square root, which is exactly the reverse of this.

In like manner raise the following numbers to the second power:

2. 528 **3.** 732 **4.** 236 **5.** 429 **6.** 523

366. If a number be divided into *any* two parts, it may be shown that the square of the whole number is equal to the square of the first part + twice the product of the first by the second + the square of the second.

Example.—7. Find the square of 16.

Solution.—$16 = 7 + 9$; according to the formula, therefore,

$$16^2 = 49 + 126 + 81 = 256.$$

In the same way compute the squares of the following numbers:

8. 17 **10.** 35 **12.** 81 **14.** 126 **16.** 839

9. 23 **11.** 46 **13.** 94 **15.** 348 **17.** 476

We shall find it useful to observe also how the tens and units of the root are combined in its third power.

Process of computing the Third Power analyzed.

Example.—What is the cube of 43?

Solution.—Multiplying the square of 43, as already found, by 40 + 3, we have

$$\begin{array}{rcccccc}
 & & (40\times40) & + 2 & (40\times3) & + & (3\times3) \\
 & & & & 40 & + & 3 \\
\hline
 & & (40\times40\times3) & + 2 & (40\times3\times3) & + & (3\times3\times3) \\
(40\times40\times40) & + 2 & (40\times40\times3) & + & (40\times3\times3) & & \\
\hline
\underbrace{(40\times40\times40)}_{\text{cu. of tens.}} & + 3 & \underbrace{(40\times40\times3)}_{3\times\text{sq. of tens by units.}} & + 3 & \underbrace{(40\times3\times3)}_{3\times\text{sq. of units by tens.}} & + & \underbrace{(3\times3\times3)}_{\text{cu. of units.}} \\
\text{Or, } 64000 & + & 14400 & + & 1080 & + & 27
\end{array}\Bigg\} =$$

Whereby we find that

367. *The cube of 43 is equal to the cube of the tens + three times the square of the tens multiplied by the units + three times the square of the units multiplied by the tens + the cube of the units.*

The same may be shown to be true of any number whatsoever.

In like manner find the cubes of

1. 36 **2.** 27 **3.** 92 **4.** 85 **5.** 73 **6.** 95

Note.—Test the accuracy of these results by the common process.

7. Calculate the cube of 47, and write the solution in the following form :

The cube of the tens........................	$40 \times 40 \times 40 =$	64000
Three times the square of the tens by the units.	$3 \times 40 \times 40 \times 7 =$	33600
Three times the tens by the square of the units.	$3 \times 40 \times 7 \times 7 =$	5880
The cube of the units......................	$7 \times 7 \times 7 =$	343
		103823

Write out the solution of the following examples in the same way :

Find the third powers or cubes of

8. 37 **10.** 45 **12.** 65 **14.** 83 **16.** 28

9. 54 **11.** 23 **13.** 71 **15.** 92 **17.** 74

368. If a number be divided into *any* two parts whatsoever, it may be shown that the cube of the first part + three times the square of the first multiplied by the second + three times the square of the second by the first + the cube of the second is equal to the cube of the number itself.

Example.—Find the cube of 82.

Solution.—$82 = 28 + 54$; according to the formula, therefore, we have

$$82^3 = (28 + 54)^3 = 28^3 + 3\ (28^2 \times 54) + 3\ (28 \times 54^2) + 54^3,\ \text{or,}$$

Cube of the first..................................	$28^3 =$	21952
Three times the square of the first by the second ..	$3 \times 28^2 \times 54 =$	127008
Three times the square of the second by the first ..	$3 \times 28 \times 54^2 =$	244944
Cube of the second.............................	$54^3 =$	157464
Third power or cube of 82................	$=$	551368

Definitions.

369. Evolution is a process of finding the root of a given number.

Evolution is the converse of *Involution.* In the latter the root is given to find the power; in the former the power is given to find the root.

370. Square root is indicated by the sign $\sqrt{\ }$; thus, $\sqrt{16} = 4$ is read, "The square root of 16 is equal to 4." The cube root is indicated by the same sign with the aid of a small figure 3 placed above it; thus, $\sqrt[3]{27} = 3$.

371. The sign of evolution ($\sqrt{\ }$) is called the ***radical sign,*** from the word ***radix,*** which means root. The figure which indicates the ***degree*** of the root to be extracted is called the ***index*** of the root.

372. The root of a number is indicated also by a fractional exponent, the denominator of which is the index of the root. Thus, $16^{\frac{1}{2}}$ means the same as $\sqrt{16}$. $27^{\frac{1}{3}}$ is only another expression for $\sqrt[3]{27}$.

To find the Square Root of a Number.

Example.—1. Let it be required to extract the square root of 1849.

It is to be remembered that the square of any number expressed by tens and units is equal to

The square of the tens + twice the product of the tens by the units + the square of the units.

Let it also be remembered that

No part of the square of tens can be found in tens or units' place, and that no part of the product of tens and units can be found in units' place.

The process of extracting the root consists in obtaining the tens of the root from the square of the tens, and the units of the root from the remaining parts of the power.

First Step.—To find the tens' figure of the root.—Since the square of the tens can not contain anything less than hundreds, the two figures at the right contain no part of the square of the tens, and are therefore disregarded for the present. The greatest square in 1800 is 1600, the square root of which is 40.

$$\begin{array}{r|l} 1849 & \underline{40} + 3 = 43 \\ 40 \times 40 = \underline{1600} & \\ 249 & \\ 80 \times \ 3 = \underline{\ 240} & \\ 3 \times \ 3 = \quad 9 & \end{array}$$

We place 40 to the right of the given number and subtract 1600 (40×40) from 1849.

Second Step.—To find the units' figure of the root.—Since no part of the product of tens by any integer whatsoever can be less than ten, the right-hand figure of the remainder can contain no part of the product of the tens by the units, and hence it is disregarded for the present. The 24 tens then being twice the product of the tens by the units, we obtain the units by dividing 24 by twice the tens. The quotient is 3, which we add to the 40. Multiplying twice the tens, or 80, by 3, we have twice the product of the tens by the units, and subtracting this we have remaining only the square of the units. Subtracting the square of the units nothing remains, and thus 43 is found to be the square root of 1849.

Notes.—1. Instead of multiplying 80 and 3 separately by 3 and adding the products, the work is somewhat shortened by multiplying the sum of 80 and 3 by 3. The work would then stand as in the margin.

$$\begin{array}{r|l} 1849 & \underline{43} \\ 1600 & \\ 83)\overline{249} & \\ \underline{249} & \end{array}$$

2. When the number whose root is to be found is expressed by five or six figures, and it is thus known that the root must contain three figures, the work may be commenced with the two left-hand periods as if they were the only ones, and when the root of these has been so obtained the operation may be completed as though the two figures of the root already found were so many tens, as they really are.

3. The only difficulty in the extraction of the square root is met with when, on multiplying twice *the tens* + *the units* by the units, the product is found to be too great. This difficulty arises from the trial-divisor being sometimes considerably increased by the tens that come from the square of the units.

Thus, in the example here given, if the 54 tens expressed by the first two figures of the remainder were the product of only the tens by the units, we should obtain the exact number of the units by dividing it by twice the tens. But, in this case, we should have 9 for the quotient, which is evidently too great, since by forming the sum of the product of twice *the tens* + *the units* by the units we get 621, which is greater than the dividend.

```
    1444 | 38
    9
68)544
   544
```

In such cases as this we have to diminish the units of the root, not forgetting to change the right-hand figure of the partial divisor at the same time, until we obtain a product not greater than the tens. The following example presents an extreme case of this nature:

2. Required the square root of 321735969.

The number being divided into periods of two figures each, the first is 3 (hundred millions). The greatest square in 3 is 1. Subtracting this, and annexing the next period to the remainder, we have 22(1) for a partial dividend, and, doubling the root already found, we have 2 for the partial divisor.

But the next term of the root can not be greater than 9. We try it, and obtain a result too great. We next try 8, and again the product is greater than the partial dividend. Finally, by trying 7, we obtain a product less than the partial dividend. Having subtracted this, we proceed with the solution.

Note.—In this example we have also a case which is of common occurrence, that is, the increase of the tens of the previous divisor by the doubling of the units. The pupil can avoid any confusion by observing *that the entire part of the root already found is always doubled before the new figure of the root can be found.*

First Trial.

```
   321735969 | 19
   1
29)221
   261
```

Second Trial.

```
   321735969 | 18
   1
28)221
   224
```

Third Trial.

```
     321735969 | 17937
     1
  27)221
     189
 349)3273
     3141
3583)13259
  *    *    *    *
```

A like difficulty will be found in each one of the following:

Find the square roots of

3. 310993225 **4.** 738534976 **5.** 27950824225

In the foregoing examples some of the remainders are large. A few are here given in which some very small remainders occur.

6. Let it be required to extract the square root of 731864809.

On bringing down the third period, we find that the dividend does not contain twice the root already found; in such a case we write a cipher in the root and also one to the right of the divisor. We then bring down the next period and proceed as before.

```
       7̇31̇86̇48̇09̇ | 27053
       4
   47)331
      329
 5405)28648
      27025
54103)162309
      162309
```

Find the first powers or roots of the following squares:

7. 1855197184 8. 36125464489 9. 4901120064

Rule for extracting the Square Root.

373. ***Rule.*—1. Separate the given number into periods of two figures each by placing a dot over the units' place and every second figure to the left, and in case of decimals to the right also, annexing a cipher, if necessary to complete a decimal period.**

2. Find by trial the greatest square in the left-hand period, and place its root in the form of a quotient at the right.

3. Subtract the square of the root thus found from the first period, and to the remainder bring down the second period for a dividend.

4. Double the root already found for a trial divisor; divide the dividend by it, and write the quotient for the second term of the root.

5. Annex the second figure of the root to the trial divisor. The result will be the complete divisor. Multiply this by the second term of the root, and subtract the product from the dividend.

6. Repeat the operation as in 4 and 5 until the periods are all brought down.

Notes.—1. When a partial divisor is not contained in a dividend, annex a cipher to the root already obtained, and also to the partial divisor. Bring down the next period, and proceed as in 4 and 5.

2. When the given number is not a perfect square, and hence a remainder occurs after the last period has been used, one or more periods of decimal ciphers may be annexed, and the operation continued as before. The figures in the root corresponding to the decimal periods will be decimals.

3. It must be kept in mind that no period should contain an integer and decimal, and that, if there is an odd number of decimal places in the given number, the last period must be completed by annexing a cipher.

To find the Square Root of a Common Fraction.

374. *Rule.*—1. Reduce the common fraction to a decimal, and extract the square root. Or,

2. Extract the square root of the numerator and of the denominator. The result will be the terms of the root.

Note.—If only the denominator of the fraction is a perfect square, the latter is the more convenient method. If the denominator is not a perfect square, it may be made so by multiplying both terms of the fraction by the denominator.

EXAMPLES.

Find the square root of

1. 36864	**5.** 244036	**9.** 579121	**13.** 966289
2. 81225	**6.** 258064	**10.** 734449	**14.** 1081600
3. 168921	**7.** 396900	**11.** 820836	**15.** 1177225
4. 212521	**8.** 499849	**12.** 950625	**16.** 1234321

Find one of the two equal factors of

17. 6838225	**20.** 296356225	**23.** 44502241
18. 9048064	**21.** 3196944	**24.** 61685316
19. 6885376	**22.** 19228225	**25.** 179586801

Extract the square root of

26. .0961	**30.** 28867	**34.** 3819.24	**38.** 5416.96
27. 15.21	**31.** 33489	**35.** 1.338649	**39.** 50.1264
28. 22.09	**32.** 4.2849	**36.** 226.8036	**40.** .00720801
29. .0004	**33.** 17.3056	**37.** .00001024	**41.** 290.225296

Extract the square root of

42. 5	**46.** 2	**50.** $20\frac{1}{4}$	**54.** $3\frac{3}{4}$
43. .5	**47.** .6	**51.** $153\frac{7}{9}$	**55.** $35\frac{7}{8}$
44. .05	**48.** 26	**52.** $1\frac{56}{169}$	**56.** $27\frac{3}{5}$
45. .005	**49.** .02	**53.** 23.1	**57.** $36\frac{4}{9}$

Find the square root of

58. $\frac{2}{5}$	**61.** $\frac{625}{676}$	**64.** $\frac{2}{25}$	**67.** $\frac{3}{8}$
59. $\frac{1}{3}$	**62.** $\frac{3136}{5329}$	**65.** $17\frac{16}{25}$	**68.** $5\frac{3}{7}$
60. $\frac{2}{3}$	**63.** $\frac{34681}{119025}$	**66.** $11\frac{37}{49}$	**69.** $38\frac{15}{16}$

375. To find the Cube Root of a Number.

Example.—1. Let it be required to find the root of which 42875 is the third power.

It must be kept in mind that the cube of any number composed of tens and units is equal to

The cube of the tens + three times the product of the square of the tens by the units + three times the product of the tens by the square of the units + the cube of the units.

Since the cube of tens can never fall short of a thousand, we shall not find any part of it in the first three figures to the right, hence they are disregarded in finding the tens.

$$\begin{array}{rl} & 42875 \mid \underline{3} \\ 30 \times 30 \times 30 = & \underline{27000} \\ & 15875 \end{array}$$

Evidently the root of 42000 can not be so great as 40, since the cube of 40 is 64000; but, the cube of 30 being only 27000, the real root must be between 30 and 40, and hence 3 must be the tens' figure sought for. Subtracting the cube of 30 from 42875 we have 15875 remaining.

But having taken the ***cube of the tens*** from 42728, the remainder must contain

(1.) 3 times the square of the tens × the units +
(2.) 3 times the tens × the square of the units +
(3.) the cube of the units.

Now, since we know that the tens' figure is 3, and hence that three times the square of the tens (3 times 30 × 30) is 2700; and since we know also that the remainder, 15875, contains the product of this 2700 by the *units*, we next *try* to find the units by dividing 15875 by 2700. But, since 15875 contains something more than 3 times the square of the tens by the units, our quotient is very likely to be too great. If it contained nothing more, it would be easy to find the units by division. Yet we may be sure that it can not be 9, nor 8, nor 7, nor 6, since 6 times 2700 alone is greater than 15875. The figure may be 5, but we can not be sure that even 5 is not too great, until we find that the sum of the three items is not greater than 15875. Let us try it, however, by completing the work as if it were the right figure, thus:

3 times the square of the tens × the units...	3 × 30 × 30 × 5 =	13500
3 times the tens × the square of the units...	3 × 30 × 5 × 5 =	2250
The cube of the units....................	5 × 5 × 5 =	125
		15875

The sum of all these parts being equal to the remainder, 15875, it is clear that 5 is the correct units' figure, and that 35 is the cube root of 42875.

The whole work of solution may be put into this form:

$$\begin{array}{r|l}
 & 42875 \;\underline{|\; 35} \\
30 \times 30 \times 30 = & 27000 \\
\hline
3 \times 30 \times 30 \times 5 = 13500 & 15875 \\
3 \times 30 \times 5 \times 5 = 2250 & \\
5 \times 5 \times 5 = \underline{125} & \underline{15875}
\end{array}$$

But, instead of multiplying three times separately by the factor 5, and adding the products, we may multiply the *sum* of the products *of the other factors* by 5 in one operation. The work would then stand as here given.

$$\begin{array}{r|l}
 & 42875 \;\underline{|\; 35} \\
30 \times 30 \times 30 = & 27000 \\
\hline
3 \times 30 \times 30 = 2700 & 15875 \\
3 \times 30 \times 5 = 450 & \\
5 \times 5 = \underline{25} & \\
3175 \times 5 & \overline{15875}
\end{array}$$

Note.—If, on completing the work, we had found that 5 was too great, we should have had to take 4 as the units' figure, and possibly even that might have been found too great. In that case, we should have had to take 3, and try again. The finding of the square or cube root of a number is often a process of guessing, and testing the correctness of the guess.

2. Find the cube root of 22906304.

Here the partial divisor is contained 11 times in the first remainder, but inasmuch as we know that the next figure of the root can not be greater than 9, we try 9, and, finding it to be too great, we try 8, which proves to be the right figure.

$$\begin{array}{r|l}
 & \dot{2}290\dot{6}30\dot{4} \;\underline{|\; 284} \\
2^3 = & 8 \\
\hline
3 \times 20^2 = 1200 & 14906 \\
3 \times 20 \times 8 = 480 & \\
8^2 = \underline{64} & \\
1744 & 13952 \\
\hline
3 \times 280^2 = 235200 & 954304 \\
3 \times 280 \times 4 = 3360 & \\
4^2 = \underline{16} & \\
238576 & 954304
\end{array}$$

3. Extract the cube root of 28372625.

On bringing down the second period, it is found that the partial divisor (2700) is not contained in the dividend, hence we place a cipher in the root, bring down the next period, and proceed again according to the rule (4 and 5).

$$\begin{array}{r|l}
 & 2\dot{8}37\dot{2}62\dot{5} \;\underline{|\; 305} \\
3^3 = & 27 \\
\hline
3 \times 300^2 = 270000 & 1372625 \\
3 \times 300 \times 5 = 4500 & \\
5 \times 5 = \underline{25} & \\
274525 & 1372625
\end{array}$$

Rule for extracting the Cube Root.

376. ***Rule.*** **— 1. Separate the given number into periods of three figures each by placing a dot over the units' place and every third figure to the left, and, if there be decimals, to the right also. Annex one or two ciphers if necessary to complete a decimal period.**

2. Find the greatest cube in the left-hand period, and place its root at the right for the first term of the root sought.

3. Subtract the cube of the first term of the root from the first period, and to the remainder annex the second period for a dividend.

4. Take three times the square of the root, already found, as a trial divisor, ascertain how many times it is contained in the dividend, and annex the result to the root as a trial term.

5. Find the sum of

3 times the square of the first term of the root,
+ 3 times the product of the first by the trial term,
+ the square of the trial term, for

a complete divisor. Multiply the sum by the trial term of the root, and subtract the product, if not too great, from the dividend.

6. If the product be greater than the dividend, diminish the trial term of the root, and proceed as before.

7. The remainder, if there be any, with the next period, will form another partial dividend with which we proceed again, as directed in 4 and 5.

Notes.—1. When a partial divisor is not contained in a dividend, annex a cipher to the root obtained, annex two ciphers to the partial divisor, bring down the next period, and proceed as directed in 4 and 5.

2. When the given number is not a perfect cube, and hence a remainder occurs after the last period has been used, one or more periods of decimal ciphers may be annexed, and the operation continued as before. The figures in the root corresponding to the decimal periods will be decimals.

To extract the Cube Root of a Common Fraction.

377. ***Rule.***—**1. Reduce the common fraction to a decimal, and extract the cube root.**

2. Or, if the numerator and denominator are perfect cubes, extract the cube roots of the terms separately. The results will be the terms of the root.

Note.—If only the denominator of the given fraction is a perfect cube, the latter is the more convenient method.

SLATE EXERCISES.

Find the cube root of

1. 6859	4. 2406104	7. 49027896
2. 12167	5. 3869893	8. 66430125
3. 27000	6. 5545233	9. 929714176

Extract the cube root of

10. 1412467848	12. 3341362375	14. 3616805375
11. 1865409391	13. 2857243059	15. 4065356736

Find the cube root of

16. 830.584	18. 1.092727	20. .000175616
17. .970299	19. .002197	21. .007645373

Find the cube root of the following numbers, carrying incomplete roots to three or five decimal places, as may be required:

22. 1.	24. .01	26. .001	28. $^2/_3$	30. $^7/_9$
23. 2.	25. .02	27. .002	29. $^3/_4$	31. $^1/_7$

The Extraction of Roots of Perfect Powers.

378. The Square Root.—The square root of a number being one of two equal factors, the square root of a perfect power may be found by resolving the power into its prime factors, separating them into two identical sets, and finding the product of one set.

Example.—1. Let it be required to find the square root of 3136.

The prime factors of 3136 are 2, 2, 2, 2, 2, 2, 7 and 7. These are separable into two sets of factors, each containing 2, 2, 2 and 7. The product of $2 \times 2 \times 2 \times 7 = 56$. Hence, 56 is the square root of 3136. In like manner find the

2. $\sqrt{484}$ 3. $\sqrt{1024}$ 4. $\sqrt{16384}$ 5. $\sqrt{234256}$

379. The Cube Root.—On the same principle, the cube root of a perfect power may be obtained by resolving the power into its prime factors, separating them into three identical sets, and finding the product of one set.

Example.—6. Find the cube root of 13824.

The prime factors of 13824 are 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3. These are separable into three sets of factors, each containing 3, 2, 2 and 2. The product of $3 \times 2 \times 2 \times 2$ is 24. Hence 24 is the cube root of 13824. In like manner find the

7. $\sqrt[3]{3375}$ 8. $\sqrt[3]{9261}$ 9. $\sqrt[3]{35937}$ 10. $\sqrt[3]{250047}$

Constructive or Geometric Solution of the Problem of the Square Root.

Problem.—Let it be required to find the side of a square which shall contain 3249 sq. in.

Solution.—Suppose that we have 3249 pieces of card-board, each one inch square, and let it be required to arrange them so that they shall together make one complete square. The number of pieces on one side will be the length of the side in inches.

First let us try one hundred pieces to the side. To make a square, we must have as many rows of square pieces as there are pieces in a row; hence, with this beginning, we would need 10,000 pieces. But, inasmuch as we have not so many, we plan to make our square a smaller one. Let us try 60. This we would find would require 3600 pieces, which again is more than the given number. Trying 50, we find that we can complete a square of this size, and have some pieces left. The required square will therefore contain between 50 and 60 pieces, and hence 5 must be the tens' figure of the root.

Having completed the square of 50 pieces to the side, we ascertain, as in the margin, that there are 749 pieces left with which the size of the square is to be increased.

$$\begin{array}{r|l} \text{No. pieces.} & \\ 3249 & 5\cdot \\ 50 \times 50 = \underline{2500} & \\ 749 & \end{array}$$

Laying down a row on each one of two adjacent sides (50 to the side)—for we must increase the length and breadth equally—we require twice 50, or 100 pieces. If we add two rows to each side, we shall require 2 times 100 pieces, and, if three rows be added, we shall require 3 times 100 pieces, that is, without filling the corner. To fill the corner, we shall need as many more pieces in each row as there are additional rows. Thus, if there are three rows added to each side, we must extend each row of one side by the addition of 3 pieces. Thus we may proceed, making successive additions of one row at a time to each of two sides until all the pieces are taken, or until the largest possible square is made out of the 3249 pieces given. But the process can be somewhat shortened by ascertaining at once how many rows are to be added. This we can do very nearly by dividing 749 by the number of pieces in two rows, exclusive, of course, of the number of pieces necessary to complete the rows when the corner is filled.

Dividing 749 by 100, the number required to make a row on each of the two sides, we ascertain that about 7 additional rows can be made out of the remaining pieces. But we can not complete the square by adding these seven rows, unless we can add also seven squares to each row, for the corner must be filled in order that we may have a square. Hence, we add 7 to 100 = 107, and divide by that as a complete divisor. Uniting the two parts of the arithmetical operation, we have the work on the left.

$$\begin{array}{r|l} 107)749 & 7 \\ \underline{749} & \end{array}$$

$$\begin{array}{r|l} 3249 & 57 \\ 5 \times 5 = 25 & \\ 107)749 & \\ \underline{749} & \end{array}$$

Constructive or Geometric Solution of the Problem of the Cube Root.

Problem.—Let it be required to find the edge of a cube which shall contain 175616 cu. inches.

Solution.—Suppose that we have 175616 cubic blocks, each measuring an inch, and let it be required to make of them one cubic block as large as possible.

If we lay down 100 blocks in one row, we will need a hundred rows for the first layer of our large cube. 100 in a row and 100 rows would require 10000 blocks, and the hundred layers necessary to complete the work would take 1000000 blocks; clearly, then, our cube can not be 100 inches in length, breadth, and thickness. If we take 90, 80, 70, or 60 for a first row, and try to complete a cube of so many inches, we shall fall short of blocks. Such repeated trials with the blocks, however, will hardly be necessary. A few trials with the slate and pencil would lead the beginner to discover that the number of blocks in a row must be somewhere between 50 and 60, and hence that the tens in the root can not exceed 5.

To build up a cube 50 inches in length will take 125000 blocks ($50 \times 50 \times 50$). How many blocks shall we have left. The computation in the margin, which will be readily understood, tells us that we shall have 50616.

$$\begin{array}{rr|l} & 175616 & 5\cdot \\ 50 \times 50 \times 50 = & 125000 & \\ \hline & 50616 & \end{array}$$

In making additions to a cube for the purpose in view, it is most convenient to make them to three sides.

Remembering that the block already constructed measures 50 inches long and wide and high, we know that we need 50×50 blocks for an addition of one layer to one side, and 3 times $50 \times 50 = 7500$ for an addition of one layer to each of the three sides. Can we add 7 such layers? Evidently we can not, for 7 times 7500 blocks would be 52500. Can we add six? Six times $7500 = 45000$. This leaves us $50616 - 45000 = 5616$ blocks.

Will 5616 blocks be enough to complete the edges and the corner? To test this we first fill the upper front edge by placing one layer after another on the top of the addition already made to that side. For one layer we need 50×6, and for six layers we need $50 \times 6 \times 6$, or 1800 blocks; for three edges we need 3 times $50 \times 6 \times 6$, or 5400 blocks. Taking these from the number of blocks remaining, we have 216 still left with which to complete the work.

$$\begin{array}{rr} & 5616 \\ 3 \times 50 \times 6 \times 6 = & 5400 \\ \hline & 216 \end{array}$$

Will 216 blocks be enough to fill out the corner? We can readily see that we need 6 rows of 6 blocks each for one layer upon the upper end of the addition made in the front right-hand edge, and that we must have 6 such layers to complete the block; $6 \times 6 \times 6 = 216$, exactly the number of blocks we had left.

Thus out of 175616 small cubic blocks, each measuring one inch in length, breadth, and thickness, we have constructed one large block measuring $50 + 6$ inches on each edge. Hence 56 is the cube root of 175616.

Applications of Square and Cube Root.

1. 5041 slabs of marble 9 inches square will pave a square court. How many slabs on each side?

2. How many rods long is the side of a square field containing 10 acres? 40 acres? 90 acres? 490 acres? 640 acres?

Note.—There being no linear unit corresponding to the acre, acres must be reduced to other denominations before we attempt the solution of such a problem as the above.

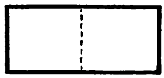
3. Being told that a certain cubic block of marble contained $91\frac{1}{8}$ cubic feet, I measured an edge and found it to be 4 ft. $5\frac{1}{2}$ in. How much more should it have measured?

4. What must be the inner dimensions, in feet and inches, of a cubic bin containing 25,000 bushels of grain?

5. The product of three equal numbers is 3189506048. What are the numbers?

6. A rectangular court that is twice as long as it is wide contains 31,250 square feet. How long and wide is it?

Suggestion.—If the rectangle were divided into two equal squares, how many square feet would there be in each?



7. A block of stone three times as long as it is wide and high is represented to contain $823\frac{7}{8}$ cubic feet. How wide and high should it be? (See suggestion under Ex. 6.)

8. What must be the dimensions in feet and inches of a square garden-lot, which shall be equal to two rectangular ones measuring respectively 8 by 10 and 8 by 18 rods?

9. What are the dimensions of a cubic bin which will hold as much as three bins measuring respectively 12 by 18 by 9 ft., 18 by 27 by 6 ft., and 12 by 9 by 9 ft.?

10. A bar of metal 5.75 ft. long, 3.9 in. wide, and .7 in. thick being melted and cast into cubic form, what was the edge of the cube?

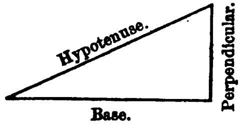
11. If one face of a cube contains 11 sq. ft. 16 sq. in., what are the contents of the cube?

Right-Angled Triangles.

380. A ***right-angled triangle*** is a triangle that has one right angle.

381. The side opposite the right angle is called the ***hypotenuse.***

Of the two sides forming the right angle, either one may be taken as the ***base,*** and the other as the ***perpendicular.***



The pupil should become thoroughly familiar with the following proposition, proved in geometry:

382. *The square of the hypotenuse is equal to the sum of the squares of the other two sides.*

Example.—1. The base of a right-angled triangle is 4 inches; the perpendicular is 3 inches. What is the length of the hypotenuse?

Solution.—The square of the base is 9, the square of the perpendicular is 16. Hence, to obtain the square of the hypotenuse, we add 9 and 16 = 25, the square root of which is 5 = the hypotenuse.

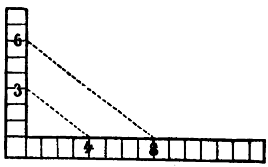
$$3^2 = 9$$
$$4^2 = 16$$
$$3^2 + 4^2 = 25$$
$$\sqrt{25} = 5 \text{ hyp.}$$

Note.—To ascertain whether this result is correct, measure the distance from 3 on either arm of a carpenter's square to 4 on the other.

Example.—2. The hypotenuse of a right-angled triangle is 10 inches; the perpendicular, 8 inches. What is the base?

Solution.—The square of the hypotenuse is 100, the square of the perpendicular is 64. To obtain the square of the base we subtract 64 from 100 (100 — 64 = 36). The sq. root of 36 is the base, $\sqrt{36} = 6$ *Ans.*

$$10^2 = 100$$
$$8^2 = 64$$
$$10^2 - 8^2 = 36$$
$$\text{Base} = \sqrt{36} = 6 \textit{ Ans.}$$



Note.—Test correctness of the answer by measuring from 6 on one arm to 8 on the other arm of a carpenters' square.

383. The carpenters' square is an instrument used in measuring and testing the work of the carpenter, stone-mason, etc.

EXERCISES.

1. If the width of a book is 9 inches, and the length 12, how many inches between the opposite corners?

2. Two dimensions of right-angled triangles being given, as follows, find the third dimension of each:

	Base.	Perpendicular.	Hypotenuse.		Base.	Perpendicular.	Hypotenuse.
3.	$1\frac{1}{2}$	2	—	6.	9	—	$11\frac{1}{4}$
4.	6	$4\frac{1}{2}$	—	7.	—	$7\frac{1}{2}$	$12\frac{1}{2}$
5.	$2\frac{1}{4}$	—	$3\frac{3}{4}$	8.	$8\frac{1}{4}$	11	—

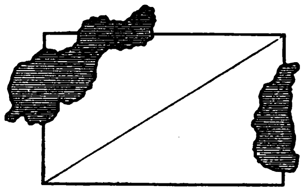
9. If a room is 21 ft. long and 20 ft. wide, how long is the diagonal? If it is 45 ft. long and 28 ft. wide? 45 ft. long and 24 ft. wide? 24 ft. long and 10 ft. wide?

10. Find the length and width of a square box which shall contain as much as two boxes, one 2 ft. and the other $2\frac{2}{3}$ ft. square, the three boxes being of the same height. If one is 4.2 and the other is 5.6 in. square.

11. What is the diameter of the largest circular saw that can be taken through a doorway $8\frac{1}{2}$ ft. high and $6\frac{3}{8}$ feet wide? If it is $7\frac{1}{2}$ ft. high and $5\frac{5}{8}$ ft. wide?

12. On a level play-ground there is a rope, $11\frac{1}{4}$ ft. long, fastened to a ring at the top of a pole 9 ft. high. How far from the foot of the pole will the rope reach the ground?

13. A horse is to be tethered in the center of a rectangular lot 240 ft. long by 238 ft. wide. How long must the rope be which will allow him to graze in the corners of the lot?



14. The figure here represents a rectangular farm. The only dimensions given are 1984 rods, one of the longer sides, and 2434 rods, the diagonal line. How many acres does the farm contain?

CHAPTER XIX.

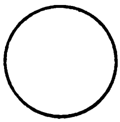
MENSURATION.

I. Plane Surfaces.

384. If a straight-edge laid anywhere upon a surface touches at every point the surface is a plane surface.

This is the practical test of a plane. It can be tried on the surface of a desk, table, floor, wall, or any other surface with an ordinary straight-edge rule. The carpenter uses the edge of his plane for the purpose.

385. A portion of a plane bounded by one or more lines is a ***plane figure.***



386. A ***Circle*** is a plane figure bounded by a curved line, every point of which is equally distant from a point within, called the *center.*

387. The boundary line of a circle is called the ***circumference.***

388. A straight line drawn through the center and terminating at the circumference on both sides is called a ***diameter.***

389. A straight line drawn from the center to the circumference is called a ***radius.*** A radius is one half of the diameter. (The plural of *radius* is *radii.*)

With a narrow slip of paper measure the circumference of a dinner plate, then measure the distance across it, and you will find the circumference a little more than three times the length of the diameter. If the plate is ten inches in diameter, and you have taken the measurements carefully, you will find the slip of paper to be very nearly $31\frac{1}{2}$ inches long. This result agrees very nearly with the great truth, proved in geometry, that

390. The circumference of any circle is ***3.14159*** times the length of its diameter. Hence, having the diameter,

391. ***To find the circumference of a circle:* Multiply the diameter by 3.14159.**

Conversely, having the circumference,

392. ***To find the diameter of a circle:* Divide the circumference by 3.14159.**

Note.—The *improper fraction* corresponding to 3.14159 may be remembered by referring to the series of figures, ***113355*** (the first three digits representing odd numbers written doubly). Taking the last three for the numerator and the first three for the denominator, thus, $\frac{355}{113}$, we have the ratio of the circumference to the diameter. The reciprocal, $\frac{113}{355}$, represents the ratio of the diameter to the circumference. For mere approximations, the circumference may be said to be $3\,^1/_7$ times the diameter.

SLATE EXERCISES.

1. If the diameter of an iron column is 3.5 in., what is the circumference? If the girth of a tree is 5 ft. 9 in. what must be its diameter?

2. If the equatorial diameter of the earth is 7925 miles, how long in miles and rods is the equator?

3. The distance from the center of the hub of a wheel to the outer edge of the felly is 15 in. How long must the tire be?

4. If the length of an oar from the thole-pin to the end of the blade is 5 ft., how many feet would the end of the blade travel in the water during 6000 strokes, each describing an arc of 60°? (60° = $^1/_6$ of the circumference.)

5. If the circumference of a circular pond is 628.318 rods, what part of a mile must I row to pass from shore to shore across the center of the pond?

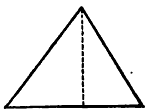
6. If a horse is tethered to the middle post of a fence, from which he can graze out into the field in a curved line 78.539314 ft. long, how long is the tether?

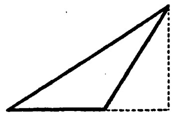
7. What will be the circumference of the largest circle that can be drawn on a sheet of paper 12 in. wide and 18 in. long?

393. A plane figure bounded by three straight lines is a ***triangle.***

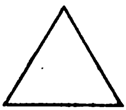
394. The ***base*** of a triangle is the side on which it is supposed to rest. (Any side of a triangle may be taken for its base.)

395. The ***altitude*** of a triangle is the perpendicular distance from the angle opposite the base, to the base, or to the base produced. (*Produced*—continued in the same direction.)



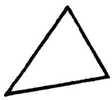


396. Triangles take different names, according to the relations of their sides. If the *sides of a triangle are equal,* it is an ***eqilateral triangle.*** If only *two sides of a triangle are equal,* it is an ***isosceles triangle.*** If *no two sides are equal,* it is a ***scalene triangle.*** If one of the angles is a *right angle,* it is a ***right-angled triangle,*** or a ***right triangle.***

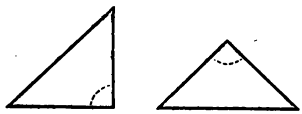


Equilateral Triangle.

Isosceles Triangle.



Scalene Triangle.



Right-angled Triangles.

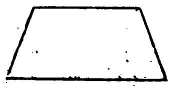
397. ***Parallel lines*** are straight lines that have the same direction but do not coincide, and can never meet, however far they may be produced.

398. A plane figure bounded by four straight lines is a ***quadrilateral.*** (*Quadrilateral* means four-sided.)

399. Quadrilaterals take different names from their angles and from the relation of the sides to each other.

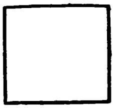
400. A quadrilateral which has no two sides parallel is a ***trapezium.***

401. A quadrilateral which has only two sides parallel is a ***trapezoid.***

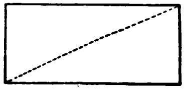


402. A quadrilateral that has its opposite sides parallel is a ***parallelogram.***

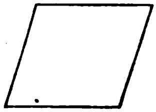
403. Parallelograms take different names from their angles and the relation of the sides to each other.



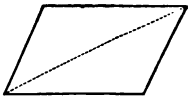
404. A parallelogram that has all its angles right angles, and all its sides equal, is a ***square.***



405. A parallelogram that has all its angles right angles, and only its opposite sides equal, is called a ***rectangle.***



406. A parallelogram that has its sides all equal, but whose angles are not right angles, is called a ***rhombus.***

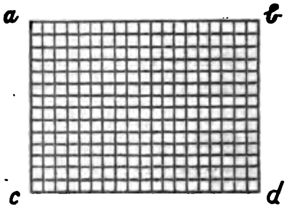


407. A parallelogram that has only its opposite sides equal, and whose angles are not right angles, is called a ***rhomboid.***

408. A straight line that joins the vertices of two angles, not adjacent, is a ***diagonal.***

To find the Areas of Quadrilaterals and Triangles.

The Rectangle, including the Square.—We have already found that to compute the area of a rectangle, we must multiply the number of the proposed square units of measure which can be placed on one side of the rectangle by the number of corresponding linear units in the adjacent side.



Example.—Let the figure represent a rectangle 14 inches wide and 19 inches long. What is the area?

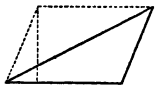
Solution.—19 square inches can be placed on the side *c d*, and, since there are 14 such rows, there will be 14 times 19 sq. in. in the whole rectangle = 266 sq. in.

The Rhomboid and Rhombus.—Example.—Let it be required to compute the area of a rhomboid, 10 in. long and 6 in. wide.

Solution.—$6 \times 10 \ \square$ in. $= 60 \ \square$ in. *Ans.*

If from either end of a rhomboid we cut a right-angled triangle, and add it to the other end, as indicated by dotted lines in the figure, we should form a rectangle equivalent to the rhomboid; hence,

409. ***To find the area of a rhomboid:*** **Multiply the length of one of two parallel sides by the distance between them.**



The rule for the rhombus is the same.

It is to be observed that, to obtain the width of a rhombus or rhomboid, we do not measure a side, but the perpendicular distance between parallel sides.

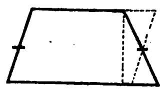
The Triangle.—Example.—Given the base of a triangle 14 yd. and the altitude 9 yd., to find the area.

The triangle is one half of a parallelogram having the same base and altitude, as may be seen by the above diagram. Hence,

410. ***To find the area of a triangle:*** **Find the area of a rectangle of the same base and altitude, and take one half of it.**

411. The following rule is sometimes necessary:

When the three sides of a triangle are given, to find the area: **From half the sum of the three sides subtract each side separately. Multiply the half sum and the three remainders together, and extract the square root of the product.**



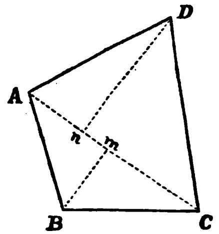
The Trapezoid.—Example.—Given the length of each of the two parallel sides of a trapezoid, 6 and 10 feet, and the distance between them, 5 feet, to find the area.

Solution.—6 ft. + 10 ft. = 16 ft. $^1/_2$ of 16 ft. = 8 ft. 5×8 sq. ft. = 40 sq. ft. *Ans.*

By inspection of the figure it will be seen that, by the aid of dotted lines, we have constructed a rhomboid equal in area to the trapezoid whose area is required; and, further, it is plain that the side of this rhomboid is equal to half the sum of the two parallel sides of the trapezoid. Hence,

412. ***To find the area of a trapezoid:*** **Multiply one half the sum of the parallel sides by the distance between them.**

413. The Trapezium.—The surface of a trapezium may be found by dividing it into two triangles; then having measured the length of the diagonal and the two perpendiculars, we calculate the area of each triangle separately. The sum of the areas of the two triangles is the area of the trapezium.



Example.—In a trapezium *A B C D*, we measure the diagonal *A C*, and find it to be 24 feet; also the perpendiculars, and find one to be 18, the other 9 feet. What is the area of the trapezium?

Solution.

Area of the triangle *A B C* = 24 × 9 = 216 sq. ft., 1/2 of 216 = 108 sq. ft.
Area of the triangle *A D C* = 24 × 18 = 432 sq ft., 1/2 of 432 = 216 sq. ft.
Area of *A B C* + *A D C*, or the whole tm. = 648 sq. ft., 1/2 = 324 sq. ft. *Ans.*

SLATE EXERCISES.

1. How many acres in a piece of woodland 220 yd. in length and 1 furlong in width?

2. How many square miles in a township 5 miles and 40 chains square?

3. How many square feet in a floor 20 ft. long and 5 yd. wide?

4. Find the surface of a pane of glass measuring $37\frac{1}{2}$ in. long and 23 in. wide.

5. How many square yards in the four walls of a room 15 ft. 6 in. high and 80 ft. in compass?

6. A rectangular pavement, 50 ft. 9 in. long and 12 ft. 6 in. wide, was laid with a central line of stone 5 ft. wide at $1.75 a running foot; the sides were flanked with brick at 80¢ per square yard. What did the paving cost?

7. How many square feet in a surface 24 ft. long 20 ft. wide? How many in another surface of half these dimensions?

8. I have a box without a lid; it is 5 ft. long, 4 ft. wide, and 3 ft. deep, interior dimensions. How many square feet of zinc will it take to line the bottom and sides of this box?

9. Find the area of a rhomboid whose length is 1 yd. 2 ft. 6 in., and whose width is 2 ft. 3 in. Draw this figure on your slate, with the scale reduced by 12.

10. What is the height of a rhomboid whose area is 12 A. and its length $13\frac{1}{3}$ chains?

11. The four eaves of a pyramidal roof measure each 44 ft. 3 in., and the common peak of the four triangles has a perpendicular distance of 24 ft. from the eaves. What is the area in slaters' squares (1) of one triangle? (2) of the roof?

12. I have a triangular garden containing $233\frac{1}{3}$ square yards. The perpendicular distance from the apex to the base is 20 ft. What is the length of the base?

13. A triangular field, whose sides are unequal, contains 5 acres. The base-line measures $\frac{1}{4}$ mile. What is the altitude in chains?

14. What is the area of a triangle whose three sides are 13, 14, and 15 ft.?

15. What is the area in acres of a triangular field whose three sides measure respectively 47, 58, and 69 rods?

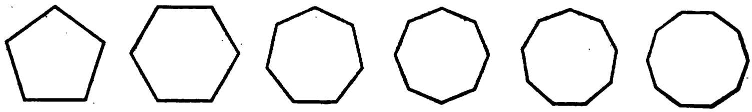
16. The parallel sides of a trapezoid measure respectively $3\frac{1}{3}$ ft. and 6 ft. 8 in.; the perpendicular distance between them is 2 ft. What is the area?

17. Find the area of a trapezium whose diagonal is 168, and one perpendicular 42, the other 56.

18. How many centares in a trapezoid one side of which measures 50 meters, the perpendicular distance to the opposite side being 35 meters?

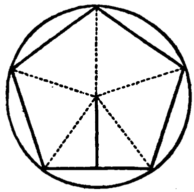
19. What is the area of a square field, the diagonal of which measures 174 meters?

Regular Polygons, having more than four sides, take different names according to the number of sides. The following are some of them :



Pentagon. Hexagon. Heptagon. Octagon. Nonagon. Decagon.

414. By dividing a regular polygon into triangles by lines drawn from the center to the several angles, it will be readily seen that the area of the polygon is equal to the sum of the areas of the triangles ; hence,

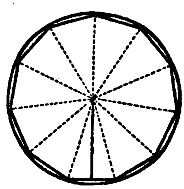


415. ***To find the area of a regular polygon :*** **Multiply the perimeter (the sum of all the sides) by one half the perpendicular distance from the center to one of the sides.**

Example.—The side of a pentagon measures 5 ft., and the perpendicular distance from the center to the side is $4\frac{1}{2}$ ft. What is the area of the polygon ?

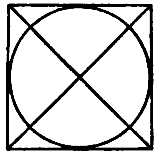
Solution.—The area of each triangle $= \frac{1}{2}(4\frac{1}{2} \times 5)$, that is, one half the product of the base by the altitude, and the area of the whole polygon $= \frac{1}{2}(4\frac{1}{2} \times 25) = 56\frac{1}{4}$ □ ft. *Ans.*

416. The Circle.—The calculation of the areas of polygons leads us by an easy step to the calculation of the area of a circle, for it is plain that, as the number of sides of the polygon is increased, the perimeter becomes more and more nearly equal to the circumference of the circumscribed circle, and the perpendicular distance from the center to the sides of the polygon becomes more and more nearly equal to the radius of the circle. Hence, having the circumference and the radius,



417. ***To find the area of a circle :*** **Multiply the circumference by one half of the radius.**

Another Method of finding the Area of a Circle.—If we have a square, and find the center of it by lines joining opposite corners, and with this center inscribe a circumference exactly touching the sides of the square, the corners outside of the circle will contain .2146, and the circle itself .7854 of the surface of the square. Hence, we have another rule,



418. ***To find the surface of a circle:*** **Multiply the square of the diameter by .7854.**

SLATE EXERCISES.

1. Required the area of a regular hexagon whose side is 73 ft. and the perpendicular is 63.2 ft.

2. How many acres in an octagonal section of land whose side is 1988.2 rods and perpendicular $7\frac{1}{2}$ miles?

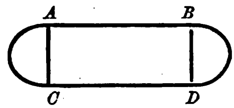
3. How many square yards are there in a circle whose diameter is 12 ft. 6 in.?

4. Draw a square containing 81 square inches; inscribe a circle in this square. What is the superficies of this circle in square inches?

5. A cow is tethered to a post driven in the center of a lot 100 ft. square; the tether is just long enough for her to reach the fence. How much of the surface of the field is she unable to crop?

6. Find the radius of a circle whose area is 95.0334 square ft.

7. Find the area of a tent floor, with semicircular ends, from the dimensions of the following diagram, in which the line *A B* equals 200 ft. and the line *A C* equals 90 ft.



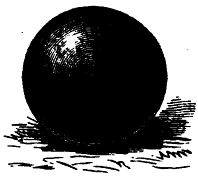
8. What is the difference in length between a fence around a circular lot 123 ft. in diameter and a square lot of the same width?

9. Find the difference in cost at $87\frac{1}{2}$¢ per rod between fencing a square field of 10 acres and a rectangular field 32 rods wide of the same area.

II. Mensuration of Solids.

419. A ***Solid*** is a limited portion of matter having length, breadth, and thickness.

420. A solid bounded by a curved surface, every point of which is equally distant from a point within, called the ***center,*** is called a ***globe,*** or ***sphere.***



Sphere.

A circle which divides the surface of a sphere into two equal parts is called a ***great circle*** of the sphere.

421. The surface of a sphere is exactly equal to four times the surface of a great circle of the sphere.

It would require just four times as much gold-leaf to cover a sphere as would cover one side of the section made by cutting the sphere into two equal parts (hemispheres).

It will be recollected that the surface of a circle is found by multiplying the circumference by one half the radius; hence, to find the surface of the sphere, we multiply its circumference by *four times one half the radius*, or, in other words, by the diameter. Hence,

422. ***To find the surface of a sphere:*** **Multiply the circumference by the diameter.**

Conversely, having the surface,

423. ***To find the diameter of a sphere:*** **Divide the surface of the sphere by 4 to obtain the surface of a great circle. Divide this by .7854 to find the area of the circumscribed square. Extract the square root to find the side of the square. The side of the circumscribed square is equal to the diameter of the circle.**

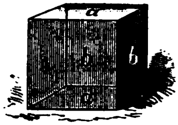
Applications.—1. If the earth were a sphere with a diameter of 7925 miles, what would its whole surface be?

2. How many square inches of leather would it require to cover a ball 3 in. in diameter?

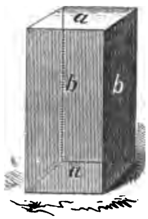
3. The surface of a sphere is 11170.12 square feet. What is its diameter?

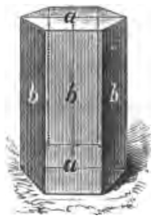
424. Prisms.—A prism is a solid whose bases are equal and parallel polygons, and whose faces are parallelograms.

425. Prisms take different names from the forms of their bases. It will be seen that *cubes* and *rectangular solids* are *prisms*. They are also called ***Parallelopipeds.***



A, a, a, a, are ***bases;*** b, b, b, b, are ***lateral faces;*** the edges between the faces are ***lateral edges*** (side edges); the edges between the faces and bases are ***basal edges*** (base edges). If the faces and edges are *perpendicular* to the bases, the prism is said to be a ***right prism,*** otherwise it is said to be ***oblique.***





426. The sum of all the lateral faces is the ***convex surface.*** The sum of bases and faces is the ***entire surface.*** The ***altitude*** is the shortest distance between the bases.

Since the faces of prisms are all parallelograms, having a common altitude, if we have a side of the base and the altitude given,

427. ***To find the convex surface of a prism:*** **Multiply the perimeter (sum of all the sides) of the base by the altitude of the prism.**

To find the entire surface: **Add the areas of the bases to the convex surface.**

Suggestion.—Suppose that you have a block of the shape of one of these prisms, and that you have fitted to it a piece of paper so as to exactly cover its convex surface. The paper will be a parallelogram, one side being equal to the height and the other side to the perimeter of the base. Hence the rule as given above.

428. Cylinders.—When the number of sides of the bases of a right prism is so increased that the bases become circles, the prism becomes a ***cylinder.***

In this case the prism loses the lateral edges, and the faces become *one*. Other terms used for prisms apply also to the corresponding dimensions of the cylinder.

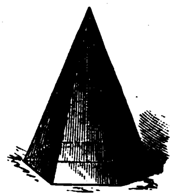
Cylinder.

429. ***To find the convex surface of a cylinder:*** **Multiply the circumference of the base by the altitude of the cylinder.**

430. ***To find the entire surface:*** **Add the areas of the bases to the convex surface.**

431. Pyramids.—A solid that has a polygon for its base, and triangles, meeting in a point, for its faces, is a ***pyramid.***

432. The ***vertex*** is the point in which the triangular faces meet. The ***altitude*** is the shortest distance from the vertex to the base. The ***slant height*** is the shortest distance from the vertex to one of the sides of the base. Other names applied to the parts of the prism apply also to the corresponding parts of the pyramid.



Pyramid.

433. Since the faces of a pyramid are equal triangles having the sides of the base for their bases, and the slant height for their common altitude,

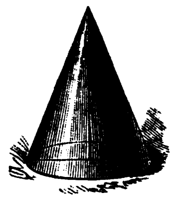
434. ***To find the convex surface of a pyramid:*** **Multiply the perimeter of the base by one half the slant height.**

Applications.—1. Find the entire surface of an octagonal prism, 12 in. high with 2 in. sides.

2. Find the convex surface of a piece of stove-pipe, 6 in. in diameter and 2 ft. in length.

3. Find the convex surface of a great pyramid, 764 feet square, and having a slant height of 451 feet.

435. The Cone.—When the number of the sides of the base of a pyramid is so increased that the base becomes a circle, the pyramid becomes a *cone.*

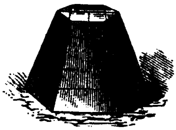


Cone.

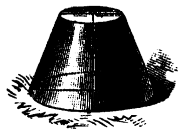
From the explanation of the rule for finding the convex surface of a pyramid, the reason of the following rule is plain. Having the circumference of the base and the slant height given,

436. ***To find the convex surface of a cone:* Multiply the circumference of the base by one half the slant height.**

437. Frustums of Pyramids and Cones.—If a part of a pyramid or cone be cut away, so that the section is parallel to the base, the portion between the section and the base is called the *frustum* of the pyramid or cone.



Frustum of a pyramid.



Frustum of a cone.

A face of a frustum of a pyramid is evidently a trapezoid, the surface of which is found by multiplying one half the sum of the parallel sides by the slant height of the frustum. Hence, having a side each of the upper and lower bases,

438. ***To find the convex surface of a frustum of a pyramid:* Multiply half the sum of the perimeters of the upper and lower bases by one half the slant height.**

Also, having the circumference of the upper and lower bases,

439. ***To find the convex surface of a frustum of a cone:* Multiply half the sum of the circumferences of the upper and lower bases by one half the slant height.**

Applications.—1. What is the convex surface of the frustum of a hexagonal pyramid, the side of whose greater end is 4 ft., that of the less end 3 ft., and the slant height 8 ft.?

2. What is the convex surface of the frustum of a cone, the diameter of whose greater end is 18 in., that of the less end 8 in., and the slant height $8\frac{1}{3}$ in.?

The Volume or Contents of Solids.

Contents of Prisms and Cylinders.—To find the volume or contents of rectangular solids, we have learned to multiply the number of solid units which can be laid upon the base by the number of layers necessary to complete a parallelopiped of the required height; or, as it is commonly expressed, we multiply the base by the altitude. The rule is the same for all prisms, and also for cylinders. Hence,

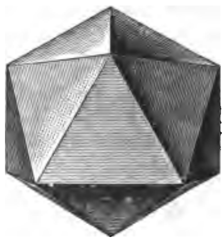
440. ***To find the contents of prisms and cylinders:*** **Having found the base of the prism or cylinder by the rules for plain surfaces, multiply the base by the altitude. The product will be the volume sought.**

Contents of Pyramids and Cones.—It is proved in geometry that the volume of a pyramid is exactly one third of that of a prism which has the same base and altitude, and that the volume of a cone is exactly one third of that of a cylinder having the same base and altitude.

If a solid iron cylinder of any dimensions were turned down to a cone, as represented in the cut, the cone would weigh just one half as much as the parts cut away, that is, only one third the solid contents of the cylinder would remain; hence we have the rule:

441. ***To find the contents of pyramids and cones:*** **Having found the base, multiply it by the altitude of the pyramid or cone, take one third of the product, and the result will be the contents required.**

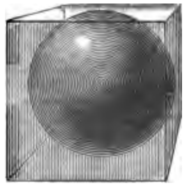
Contents of the Sphere.—As the mode of calculating the area of a triangle was applied to the calculation of the area of a circle, so the mode of finding the contents of a pyramid may be applied to the calculation of the contents of the sphere.



For, as the triangles into which a polygon can be divided may be regarded as being so increased in number that their bases finally become the circumference of a circumscribed circle, so the number of faces of a solid similar to the one here represented may be regarded as being so increased that they become the faces of a solid differing so little from a perfect sphere that the solid may be regarded as a sphere composed of a great number of pyramids, all the bases of which make up the surface of the sphere; hence,

442. ***To find the contents of a sphere:*** **Multiply the surface of the sphere by one third of the radius.**

Another Method for finding the Solid Contents of a Sphere.—The solid contents of any sphere are .5236 of a cube whose edges measure the same as the diameter of the sphere. (A base-ball that just touches every side of the box containing it occupies a little more than one half, or more nearly .5236, of the space in the box.) Hence, having the diameter,



443. ***To find the solid contents of a sphere:*** **Find the contents of a cube whose edges are equal to the diameter, and take .5236 of the result.**

Applications.—1. What is the solidity of a triangular prism whose length is 12 ft., and either of the equal sides of one of its equilateral ends is 3 ft.?

2. How many gallons of water would a cylindrical boiler 25 in. high and 12 in. wide contain?

3. Find the cubic inches in the largest cone that can be cut from a cylinder 2 ft. 6 in. high and 14 in. in diameter.

4. A sphere 8 in. in diameter is placed in a cubic box whose interior dimensions are 8 in. How much vacant space is left?

5. I have a cylindrical tank which contains 160 gallons; it is 6 ft. 5 in. in diameter. How deep is it?

6. Find the cubic feet in a log 30 ft. long and 2 ft. in diameter at the larger and 1 ft. 10 in. at the smaller end.

7. Find the cubic contents of the great pyramid mentioned in Problem 3, page 388.

8. How many cubic yards in a mound shaped like the frustum of a cone, and having a diameter of 85 ft. at the top, with circumference of 392.7 ft. at the bottom?

9. How many cubic miles in the earth, supposing it to be a perfect sphere 8000 miles in diameter?

10. How many barrels of oil in a tank 60 ft. in diameter if the oil is 5 ft. deep? (40 gal. to the barrel.)

11. Find how many cubic meters in a sphere, the surface of which contains 5682 □ meters.

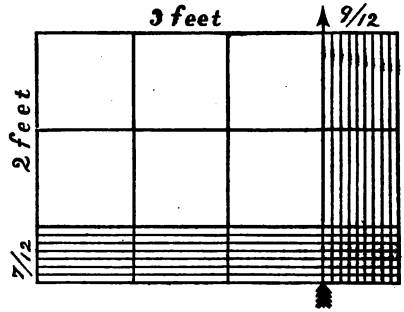
Duodecimals.

444. A scale of *twelfths,* called a ***Duodecimal Scale,*** is sometimes used in the measurement of surfaces and solids.

Example.—What is the area of a table top which is 3 ft. 9 in. long by 2 ft. 7 in. wide?

Ft.	12ths.	
2	7	
3	9	
7	9	
1	11	3
9.	8.	3

Solution.—Write one dimension under the other, calling the inches 12ths (of a foot). Then multiplying by 3, we have $3 \times {}^7/_{12} = {}^{21}/_{12} = 1$ sq. ft. and ${}^9/_{12}$ sq. ft. = the area of the narrow strip at the bottom and on the left of the arrow. Writing the 12ths under 12ths, and adding the units to the product of 3×2 (area of the large squares above) we have $3 \times 2 + 1 = 7$ sq. ft. Writing the 7 under ft., we have 7 sq. ft. and 9 twelfths of a sq. ft., the entire area of that part of the figure on the left of the arrow.



Next, ${}^9/_{12} \times {}^7/_{12} = {}^{63}/_{144} =$ 5 twelfths and ${}^3/_{144}$, the area of the small rectangle at the lower right-hand corner. Writing the 3 in the line below the first partial product, and to the right, as a lower denomination, and adding the 5 twelfths to ${}^9/_{12} \times 2$ (the area of the remaining part of the figure), we have ${}^{23}/_{12} = 1$ and ${}^{11}/_{12}$. Writing the ${}^{11}/_{12}$ under the 12ths, and the 1 under the ft., we have in the second part of the product the entire area of that part of the figure which is represented on the right of the arrow. The sum of the two parts thus found is the entire area required.

445. In duodecimals, the unit is a *linear* foot, a *square* foot, or a *cubic* foot, according as it is used to represent the length of a line, the area of a surface, or the contents of a solid.

One twefth of a foot is a ***prime*** ($'$), ${}^1/_{12}$ of a prime is a ***second*** ($''$), ${}^1/_{12}$ of a second is a ***third*** ($'''$), etc.

Hence, for linear, square, and cubic measure, we need only the following

Table:

$12'''' = 1'''$ $\quad 12''' = 1''$ $\quad 12'' = 1'$ $\quad 12' = 1$ foot.

Addition and subtraction of duodecimals are performed as in compound numbers. For multiplication, we observe the following

446. *Rule.*—1. Write the terms of the multiplier under the corresponding terms of the multiplicand.

2. Multiply each term in the multiplicand, beginning with the lowest, by the highest term of the multiplier, then by the next lower, etc., observing that 12 of any lower denomination make one of the next higher.

3. Add together the partial products thus obtained.

SLATE EXERCISES.

1. Find the area of a rectangle measuring 35 ft. 9′ by 10 ft. 6′.

Solution.

35 ft.	9′	
10 ft.	6′	
357 ft.	6′	
17 ft.	10′	6″
375 ft.	4′	6″

Or, all reductions may be made in the process of adding the partial products, as at the right.

35 ft.	9′	
10 ft.	6′	
350 ft.	90′	
	210′	54″
375 ft.	4′	6″

At 14¢ per □ yard, find the cost of painting

2. 64 ft. 3′ by 25 ft. 3′
3. 28 ft. 6′ by 17 ft. 9′
4. 108 ft. 9′ by 31 ft. 6′
5. 36 ft. 8′ by 38 ft. 8′

At 16¢ a □ yard:

6. 65 ft. 9′ by 1 ft. 1′ 6″
7. 34 ft. 10′ 6″ by 2 ft. 3′
8. 123 ft. 4′ 6″ by 1 ft. 8′ 3″

At the prices given per □ yard:

9. 198 ft. 9′ by 2 ft. 1′ at 10¢
10. 283 ft. 9′ by 3 ft. 4′ at 11¢
11. 114 ft. 6′ by 5 ft. 11′ at 12¢

Find the solid contents of blocks of marble measuring

12. 3 ft. 2′ by 2 ft. 1′ by 1 ft. 6′
13. 4 ft. 9′ by 1 ft. 9′ by 2 ft. 3′
14. 5 ft. 3′ by 5 ft. 2′ by 15 ft. 9′
15. 7 ft. 2′ by 4 ft. 5′ by 3 ft. 6′
16. 10 ft. 1′ by 3 ft. 2′ by 4 ft. 3′
17. 7 ft. 8′ by 8 ft. 7′ by 6 ft. 5′

447. The process of division being the reverse of multiplication, no rule is needed.

Example.—Divide 375 ft. 4′ 6″ by 10 ft. 6′.

```
10 ft. 6′ ) 375 ft.  4′ 6″ ( 35 ft. 9′
            367 ft.  6′
              7 ft. 10′ 6″
              7 ft. 10′ 6″
```

Original Problems.

1. Drop a plumb-line from a window; mark the distance from the ground, and find what else is needed to make a problem requiring the distance from the sill of the window to any distant object on the ground.

2. The masts used for electric lights in many cities will suggest good problems.

3. Several pieces of smooth, straight wire of uniform length, being bent into outlines of various geometric planes, will suggest many problems. How should one of these wires be bent to inclose the greatest space, in the shape of a triangle, of a square, of a circle, or what? (A good foot-rule should be used to take the necessary dimensions.)

4. Card-boards being placed in their hands, the members of the class may be asked to make boxes of them so that there shall be only one piece in a box; to make the convex surfaces of cylinders, cones, prisms, pyramids, etc. Ask how a circular card-board may be cut so that, when the cut edges are neatly joined edge to edge, the surface of the cone thus produced may contain $^3/_4$ or other specified fractional part of the surface of the circle.

5. Ask the class to take the necessary measurements and to calculate the contents of a block of stone, of a large water-pipe, of a log of wood, or other suitable object in the neighborhood.

6. Ask the diameter and solid contents of the largest cylinder, the largest cone, or of the largest sphere that can be cut from a rectangular block of wood which you may bring into the class-room, leaving the members of the class to make their own measments.

7. Construct a cubic box into which a sphere may be placed touching all the sides, and take the ball out and fill the box with sand; weigh that, and ask the class what the weight of the sand would be if it had been filled in around the ball. Weigh, and see how nearly the two results correspond.

CHAPTER XX.

EXCHANGE.—DUTIES OR CUSTOMS.—BONDS.

448. Exchange is a method by which one person may make a payment to another person at a distance without transmitting money.

Illustrations.—1. A familiar illustration of exchange is found in the use of Postal-Notes and Money-Orders for paying small sums to persons at a distance.

2. If larger ones are to be paid, the Government leaves the business to private persons, usually bankers, who have credit and the necessary understanding with each other. Thus, if a person in New York buys merchandise of another at New Orleans, he does not send gold or bank-notes to pay for it, but goes to a New York bank and buys an order, called a *draft*, on a bank in New Orleans, for the amount desired. This he sends to his creditor, who takes it to the banker on whom it is drawn and gets the money for it.

3. Postal-notes and money-orders are always payable at sight, that is, when presented, but, on such orders as those just mentioned, time is often allowed for payment. Hence, the banker in New York who gets "cash down" for a draft allows interest on the sum till the time set for its payment in New Orleans.

4. So long as New York is buying cotton and sugar from New Orleans, and New Orleans is buying manufactured goods from New York to about equal amounts, the sums to be paid in each city by the merchants of the other are about equal, and the number of drafts drawn in each city on the other is about the same; but, if New York were buying $10,000,000 worth per month and New Orleans only $5,000,000 worth, money would have to be sent from New York to New Orleans to pay the balance. In this case, the banks of New York, in selling drafts on New Orleans, would charge something to pay for the risk and expense of shipping money to New Orleans; and, on the other hand, the banks of New Orleans would be glad to sell drafts on New York for something less than their face, for thus they would be getting some of the balance due them.

5. The rates of premium on drafts for considerable sums can be but little greater than the charges made by the express companies for carrying the money, for, if a merchant in Chicago had to pay $10,075 for a draft of $10,000 on New York, and the charge for expressage were but $50, he would save $25 by sending the money directly.

Definitions.

449. A ***Draft*** is a written order directing or requesting the party to whom it is addressed to pay a certain sum to a certain person, or to his order, and to charge the same to the person who makes the request.

450. Drafts payable in the country in which they are drawn are called ***Domestic*** or ***Inland Bills;*** when payable in a foreign country, they are most commonly called ***Bills of Exchange.***

451. The party making the order is the ***Drawer;*** the party ordered or requested to pay is the ***Drawee;*** the party named to whom or to whose order the payment is to be made is the ***Payee.***

452. The payee may transfer a draft to another party in the same way that he would transfer a promissory note made payable to his order. (See page 321, note 2.)

453. A ***Sight Draft*** is payable when presented. A ***Time Draft*** is payable at the time named in the draft.

Three days' grace are allowed on time drafts and sometimes on sight drafts.

454. The following is the common form of domestic bills:

Sight Draft.

$500. *Cleveland, O., May 15, 1886.*

At sight, pay to the order of HENRY JAMES & SONS, *five hundred dollars, and charge to the account of*

M. W. CHESTER.

To WILSON & HEWITT, *Boston, Mass.*

Notes.—1. If time is allowed for the payment of a draft, it should be made to read *Thirty days after sight*, *Sixty days after sight*, etc., according to the time agreed upon. In this case it would be called a ***Time Draft.***

2. When a time draft is received, the payee should at once present it to the drawee, who writes the word *Accepted*, with *the amount*, *date*, and his own *name*, across the face of the draft, if he is willing to ***honor*** it, that is, pay the sum called for. The time in which it matures is then reckoned from the date of ***acceptance.***

SLATE EXERCISES.

On May 14, 1886, exchange on New York was quoted as below in the several cities named:

Chicago,	50 premium.	Charleston,	$^1/_8$ and $^1/_4$% premium.
St. Louis,	25 and 50 "	New Orleans,	100 "
Savannah,	$^3/_{16}$ and $^1/_4$% "	San Francisco,	15 and 20 "

Boston, 17 and 20 premium.

Note.—When exchange is quoted at a given sum, it means so much on \$1000. 50¢ on \$1000 is equivalent to $^1/_{20}$%.

1. Find the cost in Milwaukee of a sight draft on New York for \$600, exchange being $^3/_4$% premium.

Analysis.—Since the draft is sold at a premium of $^3/_4$%, each dollar costs \$1.00$^3/_4$, and \$600 costs 600 times 1.00$^3/_4$ = \$604.50 *Ans.*

2. How much must be paid in Chicago for a sight draft on New York for \$11,200, the discount being $^3/_{16}$% ?

Analysis.—Since the discount on the face of the draft is $^3/_{16}$%, each dollar costs \$.99$^{13}/_{16}$, and \$11,200 costs 11,200 times \$.99$^{13}/_{16}$ = \$11,179 *Ans.*

What must be paid for a draft drawn at

3. New York	on Cleveland	for \$1500	at $^1/_8$%	discount?
4. Detroit	" San Francisco	" \$500	" 1%	premium?
5. Cincinnati	" Louisville	" \$1725	" $^1/_4$%	discount?
6. Charleston, S. C.,	" New York	" \$850	" \$2.50	discount?
7. Savannah	" Chicago	" \$1600	" \$1.25	premium?
8. Philadelphia	" New Orleans	" \$2500	" \$1.50	discount?

9. A dealer of New York buys 500 barrels of pork in Chicago at \$12.50 per barrel, and pays by draft at 30 days after date. What does the draft cost him, exchange being 50¢ discount ?

Solution.—The interest on \$6250 for 33 days at 6% is \$34.375, and the discount for exchange ($^1/_{20}$% of \$6250) is \$3.125; hence the total discount on the face of the bill = \$37.50. This being deducted from \$6250, the cost of the draft is found to be \$6212.50.

Or,

Discount on \$1 for 33 days		= .0055
Exchange discount	$^1/_{20}$%	= .0005
Total discount on \$1		.006

Hence \$1 of exchange costs \$.994, and \$6250 costs 6250 times \$.994 = \$6212.50.

What must be paid for a draft drawn at

10. St. Louis on Atlanta for \$525, payable 60 d. after date, at $^1/_2\%$ discount?

11. St. Augustine on Philadelphia for \$400, payable 90 d. after date, at $^1/_8\%$ premium?

12. New Orleans on Buffalo for \$1950, payable 30 d. after date, at $^1/_8\%$ discount?

13. Philadelphia on Baltimore for \$275, payable 60 d. after date, at $^4/_5\%$ discount?

14. New York on Charleston, S. C., for \$3000, payable 90 d. after date, at \$1.25 premium?

15. What is the face of a sight draft bought for \$7500 at a premium of \$2.50? (\$2.50 on \$1000 = $^1/_4\%$.)

Analysis.—At a premium of $^1/_4\%$, \$1 exchange can be bought for \$1.00$^1/_4$, and as many dollars of exchange can be bought for \$7500 as \$1.00$^1/_4$ is contained times in \$7500. \$7500 ÷ 1.00$^1/_4$ = \$7481.05; hence \$7500 will purchase a draft for \$7481.05.

What is the face of a sight draft bought for

16. \$1000, premium being			$1^1/_2\%$.	**21.** \$650, discount	being		$^1/_2\%$.
17. \$250,	"	"	1%.	**22.** \$1225, premium	"		$^3/_4\%$.
18. \$4300, discount	"		$1^1/_4\%$.	**23.** \$360, discount	"		$^2/_5\%$.
19. \$2900,	"	"	\$1.75.	**24.** \$500, premium	"		\$2.
20. \$2625, premium	"		\$1.25.	**25.** \$3115,	"	"	\$2.25.

26. Find the largest draft payable 30 days after date that can be bought for \$4985.00, exchange being at a premium of $^1/_4\%$.

Solution.—The cost of \$1 exchange, on the conditions given above, is found as already shown. From \$1, the interest for 33 days at 6% = \$.0055 is deducted, and to the remainder the premium at $^1/_4\%$ = .0025 is added. The sum is the cost of \$1 exchange. Hence, as many dollars exchange can be bought for \$4985 as \$.997 is contained times in \$4985 = 5000. Hence a draft for \$5000 can be bought for \$4985.

$$\begin{array}{r} 1. \\ .0055 \\ \hline .9945 \\ .0025 \\ \hline .997 \end{array}$$

\$4985 ÷ \$997 = 5000

Note.—The result of the above process is exact, but business men would greatly shorten the work by adding $^3/_{10}$ of 1% to \$4985, and thus obtain a result correct to within $4^1/_2$¢, which would be considered sufficiently accurate. The answers given to the following examples are found by the process given in the solution.

SLATE EXERCISES.

Interest being 6%, find the face of a 30-day draft that can be bought for

27. $500, ex. being $\frac{1}{8}\%$ premium.

28. $325, " " $\frac{1}{4}\%$ discount.

29. $90, " " 1% "

30. $720, " " $\frac{1}{2}\%$ premium.

31. $1216, ex. being $\frac{3}{4}\%$ premium.

32. $1925, " " $\frac{7}{8}\%$ discount.

33. $2500, " " $\frac{5}{8}\%$ premium.

34. $1650, " " $\frac{3}{5}\%$ discount.

At the same rate of interest, find the face of a 60-day draft that can be bought for

35. $375, ex. being $\frac{7}{8}\%$ discount.

36. $1465, " " $\frac{1}{4}\%$ "

37. $1390, " " $\frac{3}{8}\%$ premium.

38. $1500, " " $\frac{9}{10}\%$ discount.

39. $695, " " $\frac{1}{2}\%$ premium.

40. $750, ex. being $\frac{5}{6}\%$ premium.

41. $2000, " " $\frac{3}{5}\%$ discount.

42. $5650, " " $\frac{1}{4}\%$ "

43. $560, " " $\frac{3}{4}\%$ premium.

44. $225, " " 1% discount.

The rate of interest being 4%, find the face of a 90-day draft that can be bought for

45. $2600, ex. being $\frac{1}{2}\%$ discount.

46. $700, " " $\frac{1}{4}\%$ premium.

47. $1950, " " $1\frac{1}{8}\%$ "

48. $5000, ex. being $\frac{3}{5}\%$ discount.

49. $3750, " " $\frac{5}{6}\%$ premium.

50. $9000, " " $1\frac{1}{4}\%$ dis't.

Applications.—1. A merchant in St. Louis ordered his broker in New York to purchase $5000 worth of merchandise for him. On shipping the goods, the broker draws on the merchant for the amount, with commission at 3%. What should be the face of the draft, the premium on St. Louis being $\frac{1}{2}\%$?

2. Find the face of a 60-day draft, bought for $620.75, if exchange is $2.50, and interest 6%.

3. A merchant in Chicago pays $1075 in Kansas City by a 30-day draft. What does he pay for the draft, exchange being $\frac{4}{5}\%$?

4. What per cent. of its face is the cost of a 90-day draft, if exchange is 1% premium, and interest is allowed at 4%?

5. If exchange is $2.50 premium and interest 6%, what will be the cost in Philadelphia of a draft on New Orleans payable 60 days after date?

Foreign Exchange.

455. Foreign bills of exchange are drafts drawn in one country and payable in another.

Note.—Bills drawn in one State, and payable in another, are termed foreign bills in the laws of some of the States.

Foreign Bill of Exchange.

1

£656. *Milwaukee, Dec. 12, 1886.*

*Sixty days after sight of this **First** of Exchange, Second and Third of the same tenor and date unpaid, pay to the order of* *David Mason & Co.,*

Six Hundred Fifty-six Pounds Sterling.

Value received, and charge the same to the account of

To Gilmore, Grace & Co. *Gilmore, Hope & Co.*

No. *1262* *London.*

In making foreign bills, it is customary to draw three of the same tenor and date, called ***a set of exchange,*** each of which contains a clause rendering all of them worthless except the one first presented for payment. These bills, or two of them, are sent by different mails to avoid the inconvenience of delay if one should be lost. The third is sometimes retained by the ***buyer or remitter.***

456. The ***Par of Exchange*** between two countries is the value of the standard coins of one country in the standard coins of the other.

Thus, the par of exchange between the United States and England is $4.8665; that is, the value of the gold in the sovereign of Great Britain is found by careful analysis to be worth $4.8665 in the gold of United States currency. (For par of exchange between the United States and other foreign countries, see page 415.)

457. In consequence of fluctuations in demand for bills of exchange (as explained in § 4, Art. 448), they are commonly at a slight premium or discount, that is, a little above or below par.

The following were the rates posted in the offices of dealers in foreign exchange in New York city, May 14, 1886:

	60 days.	3 days.
Sterling	$4.87\frac{1}{2}$	4.90
Paris (francs)....................	$5.15\frac{5}{8}$	$5.12\frac{1}{2}$
Hamburg (reichsmarks)...........	$.95\frac{3}{4}$	$.96\frac{3}{8}$
Berlin (reichsmarks)..............	$.95\frac{7}{8}$	$.96\frac{3}{8}$
Amsterdam (guilders).............	$.40\frac{1}{2}$	$.40\frac{3}{4}$

Quotations for 3 days refer to sight exchange, on the theory that 3 days' grace are allowed on sight drafts, though custom varies in this respect. The reason for charging less for time drafts than for sight bills is, that the banker who sells them has the use of the money from the time the draft is drawn till it is paid, as already explained. Sight drafts are sometimes called "short" exchange, and sixty-day drafts "long" exchange.

1. At the rates quoted above, find the cost of a draft on London for £1896 10. 6.

$$£1896\ 10.\ 6 = £1896.525$$
$$1896.525 \times \$4.87\tfrac{1}{2} = \$9245.56-$$

Explanation.—Reducing 10*s.* 6*d.* to the decimal of a pound sterling, we find £1896 10. 6 to be equal to £1896.525, and, since the cost of £1 is $\$4.87\frac{1}{2}$, the cost of £1896.525 will be 1896.525 times $\$4.87\frac{1}{2} = \$9245.56-$ *Ans.*

2. Find the cost in New York of a draft on Paris for 80,568.25 francs, at the rate quoted.

$$80{,}568.25 \text{ fr.} \div 5.12\tfrac{1}{2} \text{ fr.} = \$1572.06+$$

Explanation.—Since $5.12\frac{1}{2}$ francs = \$1, as many dollars must be paid for the draft as there are times $5.12\frac{1}{2}$ francs in 80,568.25 francs = \$1572.06+ *Ans.*

At the rates quoted, find the cost of a sight draft in New York on

3. London	for	£200.	6. Paris	for	500 fr.
4. Paris	"	\$1000.	7. Amsterdam	"	\$720.
5. Hamburg	"	300 marks.	8. Berlin	"	\$2300.

9. Find the face of a bill on London that can be bought in New York for \$793, the rate of exchange being \$4.88.

$$\$793 \div \$4.90 = £161.8367$$
$$£161.8367+ = £161\ 16.10-$$

Explanation.—Since \$4.90 will buy £1 of exchange, \$793 will buy as many pounds exchange as \$4.90 is contained times in \$793 = £161 16. 10— *Ans.*

10. Find the face of a 3-day (sight) bill on Hamburg, bought in New York for \$100,000, exchange being $96\frac{3}{8}$.

$$\$100,000 \div 96\tfrac{3}{8} = 103,761.35$$

$$103,761.35 \times 4 \text{ marks} = 415,045.4 \text{ marks.}$$

Explanation.—Since 4 marks can be bought for \$.96 $\frac{3}{8}$, as many times 4 marks can be bought for \$100,000 as \$.96 $\frac{3}{8}$ is contained times in \$100,000 = 103,761.35 = 415,045.4 marks *Ans.*

At the same rates, find the face of a 60-day draft, bought in New York, on

11. Paris for \$5000.	**14.** Antwerp for \$900.
12. London " \$6500.	**15.** Paris " \$2175.
13. Berlin " \$8000.	**16.** Hamburg " \$1900.

Applications.—1. A merchant wishes to send \$2200 to his correspondent in Paris. Find the face of a sight draft, exchange being $5.15\frac{1}{2}$.

2. Find the cost of a bill of exchange on Berlin for 2500 marks, the rate being .96. (The quotation is for 4 marks.)

3. Find the face of a sight draft on London, purchased in Boston for \$5222.75, exchange at $4.87\frac{1}{2}$.

4. A merchant purchased a sight draft on London for £625 10. 6. Exchange being 4.90, what did he pay for it?

5. Find the value of a 60-day draft on Paris for 5000 francs, exchange being $5.14\frac{1}{2}$.

6. I paid \$2575 for a draft on Hamburg. How many marks did the draft call for, exchange being $.95\frac{1}{2}$?

7. If I paid \$580.80 for a draft of £120 on London, what was the rate of exchange?

8. A hardware merchant in Louisville purchased cutlery in Sheffield, England, the bill for which was £785 15. What was the cost of the draft sent in payment, exchange being 4.82?

9. How much must be paid for a bill of exchange on Amsterdam for 15,640 guilders at 3 days' sight, exchange being $40\frac{3}{8}$ and brokerage $\frac{1}{8}\%$?

Duties or Customs.

458. Duties or customs are taxes upon imported goods.

Notes.—1. Places designated by government for the collection of duties are called ***Ports of Entry.*** Each port of entry has a ***Custom House***, which is in charge of the ***Collector of Customs*** for that district.

2. A tax called ***tonnage*** is levied upon vessels according to the number of tons they are estimated to carry. The collection of this tax belongs to the customs officers.

459. A duty fixed at a certain per cent. on the cost of an imported article is called an ***ad valorem duty*** (i. e., duty based on value or cost).

The cost on which an ad valorem duty is calculated is the net cost of the merchandise in the country from which it is imported, as ascertained from an invoice, which is required to be exhibited by the importer when he applies for a permit to land his goods. This invoice or manifest must be accompanied by a consular certificate that the prices of the goods given in the manifest are the prices that prevail in the market where they were purchased. This precaution is taken to prevent undervaluation, whereby the revenues are sometimes defrauded.

460. A duty assessed on each article, or on each pound, yard, etc., of imported goods, is called a ***specific duty.***

On some goods, both specific and ad valorem duties are required to be paid. Thus, a specific duty of 35¢ per lb., and an ad valorem duty of 40 per cent., are both assessed on all imported woolen knit-goods.

In assessing specific duties, the collector must make allowance "for the difference between the invoiced and ascertained quantity," duties being exacted on "the quantity of merchandise which arrives in the United States, not on the quantity shipped at the foreign port."—(General Regulations under the Customs and Navigation Laws, Art. 604.)

SLATE EXERCISES.

1. Find the duty on 5000 lb. raisins at 2¢ per lb.

2. Find the duty paid on 5300 boxes of oranges, invoiced at 15*s*. per box, at 20 % ad valorem.

3. What is the duty on 1000 yards of carpet, invoiced at 4*s*. per yard, at 30¢ per yard and 30 % ad valorem.

4. At 5¢ per yard and 35 % ad valorem, what is the duty on 500 yards of dress-goods invoiced at 20¢ per yard?

5. At \$2 per ton, what is the duty on 7565 lb. of hay?

6. Find the duty paid on a shipment of knit-goods weighing 623 lb., and valued at \$2340, at 35¢ per lb. and 40% ad valorem.

7. Find the duties paid on the following importation: 500 boxes of oranges at \$4 per box, at 20% ad valorem; 700 lb. raisins at 2¢ per lb.; 1000 lb. figs at 2¢ per lb.; 750 boxes lemons at 30¢ per box; and \$250 worth of preserved fruit at 35% ad valorem.

8. What was the duty paid on 12 pianos, valued at \$200 each, at 25% ad valorem?

9. The duty on steel bars being 45% ad valorem, what was the duty paid on 8500 lb., invoiced at 4¢ per lb.?

10. A lady, on returning from Europe, brings with her laces and insertings for which she had paid 350 marks; gloves, for which she had paid 60 francs; a picture, for which she had paid 1200 francs; and three unused ready-made dresses, which cost her 780, 930, and 800 francs. How much duty did she have to pay, the duty on laces and insertings being 30%, gloves 50%, pictures 30%, and ready-made clothing 50%? (See Table, page 415)

Bonds.

461. A bond is a written instrument given to secure the discharge of an obligation.

Bonds issued by a government or corporation to secure the payment of money correspond to promissory notes issued by individuals. They are made payable at a certain time, and bear a specified rate of interest. The interest on bonds is commonly made payable annually, semi-annually, or quarterly.

Note.—An individual who wishes to borrow money for his own use, can do so from any one who will lend it to him. If he pays more interest than he needs to, it is his own loss. But one who borrows for others must borrow at the lowest possible rates of interest, or else there is good ground for complaint.

Hence, when the United States Government, a State, county, city, or incorporated company, finds it necessary to borrow money, bonds are prepared, and these are sold to the highest bidders; that is, those who desire to loan the money, and will pay the highest premium for the privilege of doing it. This effects the same result as if the bonds were offered to those who would lend the money at the lowest rates of interest.

462. ***Registered bonds*** are recorded, with the names of their owners, and can not be transferred from one party to another without a change of the record.

463. ***Coupon bonds*** are bonds to which certificates are attached calling for the payment of certain interest at specified times.

These certificates called *coupons* are cut off and presented for payment when they become due. No record is made of the holders of coupon bonds, hence they may be transferred from one person to another by delivery as bank-notes.

464. Bonds issued by the United States Government (sometimes called Government securities), and State bonds (called State securities), are distinguished by their rates of interest, dates at which they are made payable, etc.

Thus, in the daily papers of May 20, 1886, we find mentioned "U. S. $4^1/_2$'s, '91, reg.," that is, United States registered bonds bearing $4^1/_2$% interest and payable in 1891. (See quotations, Art. 465.)

465. Bonds, like stocks, are bought and sold at the Stock Exchanges of all the principal cities, the brokerage on the purchase and sale of both being the same. (See Art. 291, page 291.)

The following are the quotations for the United States securities, in the market May 22, 1886:

	Bid.	Asked.		Bid.	Asked.
U. S. New 3	$100^5/_8$	—	U. S. Currency 6's, 1895..	$127^5/_8$	—
U. S. $4^1/_2$, 1891, regist'd.	$111^1/_8$	$111^3/_8$	" 1896..	$130^1/_8$	—
U. S. $4^1/_2$, 1891, coupon.	$112^3/_8$	$112^1/_2$	" 1897..	$132^3/_8$	—
U. S. 4, 1907, registered.	$125^3/_4$	$125^7/_8$	" 1898..	$135^1/_8$	—
U. S. 4, 1907, coupon ...	$125^3/_4$	$125^7/_8$	" 1899..	$137^3/_8$	—

Note.—Currency 6's are bonds issued by the United States Government in aid of the trans-continental railways, bearing 6% interest, and payable in currency at the times specified in the quotations.

SLATE EXERCISES.

In the following problems, $100 bonds are referred to, and brokerage is reckoned at $^1/_8$% on *par values*, unless otherwise stated.

1. Find the cost to the buyer of 38 bonds at $97^1/_2$.

2. If a person sells 38 bonds at $97^1/_2$, what will he receive from his broker?

3. What amount in bonds at $112^1/_2$ can be bought for \$10,586.75 ?

The brokerage being added to the price of the bond, the cost of 1 bond is found to be $112^5/_8$. At this rate, how many can be bought for the given sum ?

Note.—No principles are involved in the solution of problems like the foregoing but such as are applicable to the purchase and sale of stocks; but when questions arise as to the advantage of investing in one kind of bonds rather than another, such problems as the following occur:

4. If a person buys 5% stock at 125, what rate of interest does he receive on the money invested ?

Whatever he *pays* for the bond, the interest he *receives* is always the interest specified in the bond. Hence he receives \$5 interest on his investment. What per cent. of \$125 is \$5 ?

Solution.

$$5 \div 125 = {}^5/_{125} = {}^1/_{25} = 4\% \textit{ Ans.}$$

Or, if stock at 100 yields 5%, at 125 it must yield $^{100}/_{125}$, or $^4/_5$ of $5\% = 4\%$.

What % interest will be realized on money invested

5. In 4% bonds at 80.		**9.** In 8% bonds at 160.	
6. In 6% " " 120.		**10.** In $5^1/_2$% " " 110.	
7. In $4^1/_2$% " " 90.		**11.** In $6^1/_4$% " " 125.	
8. In $3^3/_4$% " " 75.		**12.** In 8% " " 140.	

How much money must be invested

13. In 4% bonds at $92^3/_8$ to produce \$352 income.
14. In 3% " " $100^1/_8$ " " \$540 "
15. In $3^1/_2$% " " 108 " " \$630 "
16. In $4^1/_2$% " " 108 " " \$630 "

17. That I may receive 6% on the money invested, what price may I pay for 8% bonds ?

What would be the difference between the income from an investment

18. Of \$8400 in $4^1/_2$'s at 120, and in $3^1/_2$'s at 112.
19. Of \$2700 " 6's " 135, " " $4^1/_2$'s " 90.
20. Of \$4500 " $3^1/_2$'s " 90, " " 3's " 75.

21. If a person invest \$4488 in 3% stocks at 70, and \$5505 in currency 6's at $137^1/_2$, paying the usual brokerage, what per cent. will his income be on the sum invested ?

APPENDIX.

Testing the Accuracy of Addition, Subtraction, Multiplication, and Division.

466. There is no better way of making sure of the correctness of an addition than adding "both ways"; and in subtraction the best test of accuracy is that the sum of the subtrahend and remainder is equal to the minuend.

In multiplication, the most thorough proof of accuracy is found if the product of the multiplier by the multiplicand is equal to the first product. In division, if the sum of the remainder added to the product of the quotient by the divisor is equal to the dividend, the work may be relied upon as correct. But these methods of proof require as much time as the original operation, hence the common use of the method called

Casting out Nines.

To cast out the nines of a number we may add all the terms of the number, and divide the sum by 9; the remainder will be the result sought. Thus, the sum of the terms of 4787763 is 42. Dividing this by 9, we obtain 6 as the ***excess of nines.*** But, since we wish to know only what the remainder is, we may drop the nines from the results as we proceed.

Thus, in the operation of casting out nines from 4787763, we may *think* the process indicated in light-faced italics, and speak the numbers printed in full-faced type, as follows:

4 + 7 = 11, 11 − 9 = ***2****; 2 + 8 = 10, 10 − 9 =* ***1****; 1 + 7 + 7 = 15, 15 − 9 =* ***6****; 6 + 6 = 12, 12 − 9 =* ***3****; 3 + 3 =* ***6***. 6 being the excess of nines, is written as it is pronounced.

In this process we *skip* the 9's, for there is no use of adding a nine and at once dropping it.

Applied to Multiplication.

Written Work.

```
 3885—6
  647—8
 27195  48—3
 15540
23310
2513595—3
```

Explanation.

Remainder after casting out nines from 3885.
" " " " 647.

Multiplying the two remainders and casting out nines from the product, we have *3* for a remainder; and the remainder found by casting out nines from the product being the same, we judge the work to be correct.

Applied to Division.

```
  8         6
647)2514157(3885
     1941
      5731
      5176
       5555
       5176
8      3797
6      3235
48—3 —> 562—7
```

Explanation.—Casting out the nines from the divisor and quotient separately, we find the remainder written over the last figure of each. We then multiply the one remainder by the other, cast out the nines of the product, and carry the excess to the remainder found by the division 562, and think $3 + 5 + 6 = 14$, $14 - 9 = \mathit{5}$, and $5 + 2 = \mathit{7}$. We finally cast out the nines from the dividend, and since we obtain *7* from this also we judge the work to be correct.

467. The principle on which this method is based is, that *The remainder arising from dividing any number by 9* is always the same as the remainder that arises from dividing the sum of all its terms by 9.

That this must be so is evident from the fact that, on being divided by 9, there is a remainder of 1 for every ten, hundred, or thousand that go to make up a number, thus:

Dividing 10, 100, or 1000 by 9, the remainder is always 1. Dividing 20, 200, or 2000 by 9, the remainder is always 2, etc. Hence, if we divide separately the parts of a number represented by its digits by 9, the remainders will be expressed by those digits: e. g., if we divide the 2000, 400, 70, and 8, in 2478, separately by 9, the remainders will be *2, 4, 7,* and *8,* the excess of 9's in the sum of which is evidently the same as in the sum of the digits or *in the number itself.*

468. ***Another excellent*** test of the correctness of an operation in division is that the remainder after division added to all the subtrahends produces a sum equal to the dividend.

Greatest Common Divisor. (*See Art. 141.*)

Example.—Let it be required to find the greatest common divisor of 91 and 224.

The process as given in Art. 141, page 142, is as follows:

We divide the greater number by the less, and the divisor by the remainder, and so on till we find that 7 will exactly divide the preceding divisor or remainder, as we choose to regard it. By trial, we find that 7 is a common divisor of 91 and 224. But, by reasoning, we might conclude that it must always be the case that the last remainder in such a succession of divisors will be a common divisor of the given numbers. The reasoning would be based on two principles:

```
91)224(2
   182
    42)91(2
       84
        7)42(6
          42
```

1. That an exact divisor of any number must be an exact divisor of any multiple (number of times) that number.

If there is an exact whole number of times 7 apples in one heap, there would be an exact whole number of times 7 apples of the same size in any number of equal heaps.

2. That an exact divisor of two numbers will be an exact divisor of their sum or difference.

If there are 5 times 12 buttons on one string, and 2 times 12 buttons on another, there will be an exact whole number of times 12 buttons on one string more than the other, and an exact whole number of times 12 buttons on both.

With these principles in view, to show that the last divisor is a common divisor, we would reason thus:

Seven being a divisor of 42, it is, according to principle 1, a divisor of 84; and being a divisor of 84, it is (principle 2) a divisor of 91 $(84 + 7)$; and being a divisor of 91, it is a divisor (principle 1) of 182 (2×91); and being a divisor of 182 and 42, it is (principle 2) a divisor of their sum, 224. Hence, 7 *must* be *an* exact divisor of 91 and 224.

And to show that the last divisor is the *greatest* common divisor, we would reason as follows:

According to principle 1, the common divisor of 91 and 224 must be an exact divisor of 182 (2×91); and hence, being a common divisor of 182 and 224, it must be, according to principle 2, an exact divisor of their difference, 42; and, reasoning in a similar manner, being a divisor of 42, it must be a divisor of $91 - 84$, or 7. Hence, the *greatest* common divisor can not be greater than 7.

Thus, having found that 7 is *a* common divisor, and that *the* common divisor can not be greater than 7, we conclude that *7 is the greatest common divisor.*

Repetends, or Circulating Decimals.

1. Decimals, equivalent to the common fractions, given in exercises *10* and *11*, page 149, were easily found, but in *12*, the division of the numerators by the denominators was interminable. The decimals produced might therefore be called ***interminate*** (not terminating).

2. But, if the division were carried far enough (never to a number of places in the quotient greater than the number represented by the divisor), a remainder would be obtained which had occurred before, and hence a figure or set of figures would be repeated in the same order in never-ending succession. Such a figure, or set of figures, was called a circulating decimal or repetend.

3. Let the pupil now reduce $^1/_9$, or any number of 9ths less than 9, to a decimal; also $^1/_{99}$, or any number of 99ths less than 99, to a decimal; also $^1/_{999}$, or any number of 999ths less than 999, to a decimal; also any number of 9999ths less than 9999, to a decimal, and let him note the several results. He will in every case obtain a repetend consisting of the *same digits as the numerator.*

4. From these experiments it may be safely concluded that any repetend is equivalent to a common fraction having the repetend for its numerator, and a number of 9's in the denominator equal to the number of places in the repetend. Hence the method of reducing repetends to common fractions becomes evident.

Example.—1. Reduce $\dot{1}\dot{8}$ to a common fraction.

Process.

$$\dot{1}\dot{8} = {}^{18}/_{99} = {}^{2}/_{11}.$$

Change to common fractions:

2. $.\dot{2}8571\dot{4}$ 3. $.\dot{5}3846\dot{1}$ 4. $.\dot{1}5384\dot{6}$ 5. $.\dot{0}4\dot{5}$

6. Change $.17\dot{2}$ to a common fraction.

$$.17\dot{2} = .17^2/_9 = \frac{17^2/_9}{100} = \frac{155}{900} = \frac{31}{180}$$

Change to common fractions and mixed numbers:

7. $.2\dot{4}\dot{5}$ 8. $17.\dot{5}3\dot{1}$ 9. $.24\dot{1}6\dot{2}$ 10. $15.18\dot{9}\dot{3}$

Progression.

469. An ***Arithmetical Progression*** is a series of numbers, increasing or decreasing by a constant difference.

1, 3, 5, 7, 9, 11, is an increasing, or ascending series.

12, 10, 8, 6, 4, 2, is a decreasing, or descending series. The constant difference in each is 2.

To find any term of an Arithmetical Series.

1. If the first term of an arithmetical series is 2, and the constant difference is 3, what is the fifth term?

Analysis.—Since the constant difference is 3, the terms of the series are

$$\underbrace{2}_{\text{1st}} \quad \underbrace{2+3}_{\text{2d}} \quad \underbrace{2+3+3}_{\text{3d}} \quad \underbrace{2+3+3+3}_{\text{4th}} \quad \underbrace{2+3+3+3+3}_{\text{5th}}$$

Here we see that the fifth term is equal to the first term + the constant difference multiplied by a number one less than the number of the term required.

$$2 + (4 \times 3) = 14, \text{ the fifth term.}$$

2. If the first term of a descending arithmetical series is 30, and the constant difference is 5, what is the fourth term?

Analysis.—Since the constant difference is 5, the terms of the series are

$$\underbrace{30}_{\text{1st}} \quad \underbrace{30-5}_{\text{2d}} \quad \underbrace{30-5-5}_{\text{3d}} \quad \underbrace{30-5-5-5}_{\text{4th}}$$

Here we have the fourth term equal to the first term less the constant difference multiplied by a number one less than the number of the term required.

$$30 - (3 \times 5) = 15, \text{ the fourth term.}$$

Hence, having the first term and the constant difference, to find any required term we have the

470. ***Rule.***—**To the first term add the product of the constant difference by a number one less than the term required, if the series is ascending; or, if the series is descending, *subtract* the product from the first term.**

3. If the first term of an ascending series is 6, the constant difference 3, and the number of terms 300, what is the last term? The seventh term? The tenth term?

4. The first term of a descending series is 110, constant difference 6. What is the seventh term?

5. If the first term of a descending series is 72, and the constant difference 6, what is the seventh term? The ninth term? The twelfth term?

6. A bicyclist travels 5 miles the first hour, and increases his speed $^3/_4$ mile each hour for fifteen hours. How far does he travel during the last hour?

To find the sum of an Arithmetical Series.

7. The first term of an ascending series is 5, the constant difference is 3, and the number of terms 8. What is the sum?

The complete series	5,	8,	11,	14,	17,	20,	23,	26.
The inverted series	26	23	20	17	14	11	8	5
	31,	31,	31,	31,	31,	31,	31,	31.

From this we see that the sum of the two series, or twice the sum of one series, is equal to the sum of the first and last terms taken as many times as there are terms; hence the sum of either series is equal to $^1/_2$ the sum of the first and last terms taken as many times as there are terms. Therefore, having the first term, the constant difference, and the number of terms, to find the sum of the terms, we have the following

471. *Rule.*—Find the last term, according to the previous rule; multiply the sum of the first and last terms by the number of terms, and divide the product by 2.

8. The extremes of a series are 3 and 78, the number of terms 16. What is the sum?

9. How many strokes does a clock make in 12 hours? How many would it make if it struck the hours from 1 to 24.

10. To cancel a debt, A agrees to pay B \$100 a year for five years, with an increase, after the first year, of \$5 per month. What was the debt?

11. The first term of a series is 3, the constant difference is 5, and the number of terms 99. What is the twelfth term? The ninety-eighth term? The sum?

12. A railroad train ran 26 miles the first hour, and increased its speed 4 miles each hour. How fast was it running the fifth hour after starting? What was the entire distance traveled?

472. It can be readily seen that an arithmetical progression has five elements which enter into the solution of the various kinds of problems coming under this head, viz.: The first term; the constant difference; the number of terms; the last term; and the sum of all the terms. If any three of these elements be given, the other two can readily be found.

Pupils should make other problems and deduce rules for finding other elements than those required in the foregoing exercises.

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473. A ***Geometrical Progression*** is a series of numbers which increase or decrease by a constant ratio.

2, 6, 18, 54, 162 is a geometrical progression, the ratio of which is 3.

64, 32, 16, 8, 4, 2, 1 is a decreasing geometrical progression, the ratio of which is $\frac{1}{2}$.

To find any term of a Geometrical Series.

13. The first term of a geometrical series is 2, and the ratio 3. What is the fourth term?

$$\underbrace{2}_{\text{1st}} \qquad \underbrace{3\times2}_{\text{2d}} \qquad \underbrace{3\times3\times2}_{\text{3d}} \qquad \underbrace{3\times3\times3\times2}_{\text{4th}}$$

Or,

$$\underbrace{2}_{\text{1st}} \qquad \underbrace{3\times2}_{\text{2d}} \qquad \underbrace{3^2\times2}_{\text{3d}} \qquad \underbrace{3^3\times2}_{\text{4th}}$$

Thus we see the *fourth* term is equal to the first term multiplied by the *third* power of the ratio.

Hence, for finding any term of a geometrical series, we have the

474. ***Rule.*—Multiply the first term by the ratio raised to a power *one* less than the number of the term required.**

14. A father once made the following contract with his joking son. For certain services rendered, he agreed to pay him one cent the first month, two cents the second, four cents the third, eight cents the fourth, and so on at the same rate for a term of five years. If the contract could have been carried out, what would have been the last payment?

To find the sum of a Geometrical Series.

15. What is the sum of a geometrical series whose first term is 5, ratio 4, number of terms 6?

The sum of the series would be

$$5 + 20 + 80 + 320 + 1280 + 5120.$$

If, now, we multiply this by the ratio 4, we have the sum of a new series. Subtracting from this the sum of the first series, we have left a result equal to *three* times the sum of the first series, viz.:

$$\begin{array}{rlr} & \cancel{20} + \cancel{80} + \cancel{320} + \cancel{1280} + 20480 & \text{Sum of new series.} \\ 5 + & \cancel{20} + \cancel{80} + \cancel{320} + \cancel{1280} & \text{Sum of first series.} \\ \hline & \text{Result,} \qquad 20480 - 5 = & \text{to three times the sum of first series.} \end{array}$$

$$20475 \div 3 = 6825 = \text{Sum of first series.}$$

Hence, having the first term, the number of terms, and the ratio of an increasing geometrical series, to find the sum of the terms we have the

475. ***Rule.*—Find the last term; multiply the last term by the ratio, from the product subtract the first term, and divide the remainder by the ratio − 1.**

To find the sum of a decreasing arithmetical series, the following is the

476. ***Rule.*—Find the last term; multiply the last term by the ratio, subtract the product from the first term, and divide the remainder by 1 − the ratio.**

Thus it is seen that in the last rule the last two steps are the reverse of the last two in the first rule.

16. The first term of a geometrical series is 5, the ratio 3, number of terms 10. What is the last term?

17. What is the sum of a geometrical series whose first term is 5, ratio 6, and number of terms 8?

18. The first term of a geometrical series is 3125, the ratio is $\frac{1}{5}$. What is the fourteenth term?

19. What is the sum of the infinite series 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, etc.?

Note.—As the number of terms in a descending series increases the last term decreases, and when the number of terms becomes infinite, the last term becomes 0. Hence the sum = the first term ÷ (1 − the ratio).

Values of Foreign Coins

Announced by the United States Treasury Department, January 1, 1886.

Country.	Monetary unit.	Standard.	Value in U. S. money.
Argentine Republic	Peso	Gold and silver.	.96,5
Austria	Florin	Silver	.37,1
Belgium	Franc	Gold and silver.	.19,3
Bolivia	Boliviano	Silver	.75,1
Brazil	Milreis of 1000 reis	Gold	.54,6
British possessions in N. A.	Dollar	Gold	$1.00
Chili	Peso	Gold and silver.	.91,2
Cuba	Peso	Gold and silver.	.93,2
Denmark	Crown	Gold	.26,8
Ecuador	Peso	Silver	.75,1
Egypt	Piaster	Gold	.04,9
France	Franc	Gold and silver.	.19,3
German Empire	Mark	Gold	.23,8
Great Britain	Pound sterling	Gold	4.86,6$^1/_2$
Greece	Drachma	Gold and silver.	.19,3
Hayti	Gourde	Gold and silver.	.96,5
India	Rupee of 16 annas	Silver	.35,7
Italy	Lira	Gold and silver.	.19,3
Japan	Yen	Silver	.81,0
Liberia	Dollar	Gold	1.00
Mexico	Dollar	Silver	.81,6
Netherlands	Florin	Gold and silver.	.40,2
Norway	Crown	Gold	.26,8
Peru	Sol	Silver	.75,1
Portugal	Milreis of 1000 reis	Gold	1.08
Russia	Rouble of 100 copecks.	Silver	.60,1
Spain	Peseta of 100 centimes.	Gold and silver.	.19,3
Sweden	Crown	Gold	.26,8
Switzerland	Franc	Gold and silver.	.19,3
Tripoli	Mahbub of 20 piasters.	Silver	.67,7
Turkey	Piaster	Gold	.04,4
United States of Columbia.	Peso	Silver	.75,1
Venezuela	Bolivar	Gold and silver.	.19,3

Note.—Let the student observe that the monetary unit of the States in the northwestern part of Europe is the *crown;* of the States in the south and southwestern part of Europe is the *franc,* under different names; and of the northwestern part of South America the *peso,* and he will have no difficulty in remembering nearly one half of this table.

Table showing Legal Rates of Interest in the several States. (*See Art. 300, page 300.*)

[From "The Bankers' Almanac and Register," 1886.]

STATES.	Rate %.		STATES.	Rate %.		STATES.	Rate %.	
Alabama	8	..	Kentucky	6	6	North Carolina	6	8
Arizona	10	*	Louisiana	5	8	Ohio	6	8
Arkansas	6	10	Maine	6	*	Oregon	8	10
California	7	*	Maryland	6	6	Pennsylvania	6	6
Colorado	10	*	Massachusetts	6	*	Rhode Island	6	*
Connecticut	6	6	Michigan	7	10	South Carolina	7	10
Dakota	7	12	Minnesota	7	10	Tennessee	6	6
Delaware	6	6	Mississippi	6	10	Texas	8	12
Dist. Columbia	6	10	Missouri	6	10	Utah	10	*
Florida	8	*	Montana	10	*	Vermont	6	7
Georgia	7	8	Nebraska	7	10	Virginia	6	6
Idaho	10	18	Nevada	10	*	Wash. Ter.	10	*
Illinois	6	8	New Hampshire	6	6	West Virginia	6	6
Indiana	6	8	New Jersey	6	6	Wisconsin	7	10
Iowa	6	10	New Mexico	6	12	Wyoming	12	*
Kansas	7	12	New York	6	6			

The legal rate is to be found in the first column. The second column gives the rate that may be collected if agreed to in writing. * No limit.

Edda [illegible]